Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy completely weakly generalized continuous mappings

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Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy completely weakly generalized continuous mappings in intuitionistic fuzzy topological space. We investigate some of their properties.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy weakly generalized closed set, Intuitionistic fuzzy weakly generalized continuous mappings, Intuitionistic fuzzy completely weakly generalized continuous mappings.

AMS Classification: 03E72.

1 Introduction

Fuzzy set (FS) proposed by Zadeh [18] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang [3] in 1968. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Çoker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy completely weakly generalized continuous mappings in intuitionistic fuzzy topological space and studied some of its properties. We provide some characterizations of intuitionistic fuzzy completely weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic mappings.

2 Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An *intuitionistic fuzzy topology* (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$;

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$$int(A) = \cup \{ G \mid G \text{ is an IFOS in X and } G \subseteq A \};$$

$$cl(A) = \cap \{ K \mid K \text{ is an IFCS in X and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5: An IFS A = {⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ | x ∈ X} in an IFTS (X, τ) is said to be an

(a) *intuitionistic fuzzy semi closed set* [7] (IFSCS in short) if int(cl(A)) ⊆ A;

(b) *intuitionistic fuzzy α-closed set* [7] (IFαCS in short) if cl(int(cl(A))) ⊆ A;

(c) *intuitionistic fuzzy pre-closed set* [7] (IFPCS in short) if cl(int(A)) ⊆ A;

(d) *intuitionistic fuzzy regular closed set* [7] (IFRCS in short) if cl(int(A)) = A;

(e) *intuitionistic fuzzy γ closed set* [6] (IFγCS in short) if int(cl(A)) ∩ cl(int(A)) ⊆ A;

(f) *intuitionistic fuzzy generalized closed set* [16] (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS;

(g) *intuitionistic fuzzy generalized semi closed set* [14] (IFGSCS in short) if scl(A) ⊆ U , whenever A ⊆ U and U is an IFOS;

(h) *intuitionistic fuzzy α generalized closed set* [12] (IFαGCS in short) if αcl(A) ⊆ U, whenever A ⊆ U and U is an IFOS.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy γ open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set* (IFSOS, IFαOS, IFPOS, IFROS, IF*γO*S IFGOS, IFGSOS and IFαGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFγCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6: [8] An IFS A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if cl(int(A)) ⊆ U whenever A ⊆ U and U is an IFOS in X .

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7: [8] An IFS A is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Result 2.8: [8] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9: [10] Let (X, τ) be an IFTS and A = ⟨x, μ_A, ν_A⟩ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

wgint(A) = ∪ { G | G is an IFWGOS in X and G ⊆ A },
wgcl (A) = ∩ { K | K is an IFWGCS in X and A ⊆ K }.

Definition 2.10: [4] Let f be a mapping from an IFTS X to an IFTS Y. If B = {⟨y, $\mu_B(y)$, $\nu_B(y)$⟩ | y ∈ Y} is an IFS in Y, then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B)$ = {⟨x, $f^{-1}(\mu_B(x))$, $f^{-1}(\nu_B(x))$ ⟩ | x ∈ X}.

If A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ | x ∈ X} is an IFS in X, then the *image* of A under f denoted by f(A) is the IFS in Y defined by f (A) = {⟨y, f($\mu_A(y)$), f_($\nu_A(y)$)⟩ | y ∈ Y} where f_(ν_A) = 1 – f(1 – ν_A).

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be an:

(a) [5] *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every B ∈ σ;

(b) [7] *intuitionistic fuzzy semi continuous* (IFS continuous in short) if $f^{-1}(B)$ ∈ IFSO(X) for every B ∈ σ;

(c) [7] *intuitionistic fuzzy α continuous* (IFα continuous in short) if $f^{-1}(B)$ ∈ IFαO(X) for every B ∈ σ;

(d) [7] *intuitionistic fuzzy pre continuous* (IFP continuous in short) if $f^{-1}(B)$ ∈ IFPO(X) for every B ∈ σ;

(e) [6] *intuitionistic fuzzy completely continuous* if $f^{-1}(B)$ ∈ IFRO(X) for every B ∈ σ.

(f) [6] *intuitionistic fuzzy γ continuous* (IFγ continuous in short) if $f^{-1}(B)$ ∈ IFγO(X) for every B∈ σ;

(g) [16] *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B)$ ∈ IFGO(X) for every B∈σ;

(h) [14] *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ ∈ IFGSO(X) for every B ∈ σ;

(i) [13] *intuitionistic fuzzy* α *generalized continuous* (IFαG continuous in short) if $f^{-1}(B)$ ∈ IFαGO(X) for every B ∈ σ;

(j) [9] *intuitionistic fuzzy weakly generalized continuous* (IFWG continuous in short) if $f^{-1}(B)$ ∈ IFWGO(X) for every B ∈ σ;

(k) [11] *intuitionistic fuzzy weakly generalized closed mapping* (IFWGCM in short) if f(A) is an IFWGCS in Y for every IFCS A in X.

Result 2.12: [7] Every IF continuous mapping is an IFα continuous mapping and every IFα continuous mapping is an IFS continuous mapping as well as an IFP continuous mapping but the separate converses need not be true, in general.

Result 2.13: [16] Every IF continuous mapping is an IFG continuous mapping but the converse need not be true, in general.

Definition 2.14: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $_wT_{1/2}$ (IF$_wT_{1/2}$ in short) space if every IFWGCS in X is an IFCS in X.

Definition 2.15: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{wg}T_q$ (IF$_{wg}T_q$ in short) space if every IFWGCS in X is an IFPCS in X.

3 Intuitionistic fuzzy completely weakly generalized continuous mappings

In this section, we introduce intuitionistic fuzzy completely weakly generalized continuous mappings and studied some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an *intuitionistic fuzzy completely weakly generalized continuous* (IF completely WG continuous in short) if $f^{-1}(B)$ is an IFRCS in (X, τ) for every IFWGCS B of (Y, σ).

Theorem 3.2: Every IF completely WG continuous mapping is an IF continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFCS, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. □

Example 3.3: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.2, 0.4), (0.8, 0.6)⟩, T_2 = ⟨y, (0.2, 0.4), (0.8, 0.6)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y,σ) by f(a) = u and f(b) = v. Then, f is an IF continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.6, 0.7), (0.3, 0.3)⟩ is an IFWGCS in Y but $f^{-1}(A)$ = ⟨x, (0.6, 0.7), (0.3, 0.3)⟩ is not an IFRCS in X, since $cl(int(f^{-1}(A))) = T_1^C \neq f^{-1}(A)$.

Theorem 3.4: Every IF completely WG continuous mapping is an IFα continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFαCS, $f^{-1}(B)$ is an IFαCS in X. Hence f is an IFα continuous mapping. □

Example 3.5: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.2, 0.3), (0.8, 0.6)⟩ and T_2 = ⟨y, (0.2, 0.3), (0.8, 0.6)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTS on X and Y, respectively. Define a mapping f: (X, τ) → (Y,σ) by f(a) = u and f(b) = v. Then f is an IFα continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.9, 0.7), (0.1, 0.3)⟩ is an IFWGCS in Y but $f^{-1}(A)$ = ⟨x, (0.9, 0.7), (0.1, 0.3)⟩ is not an IFRCS in X, since $cl(int(f^{-1}(A))) = T_1^C \neq f^{-1}(A)$.

Theorem 3.6: Every IF completely WG continuous mapping is an IFG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFGCS, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping. □

Example 3.7: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.5, 0.4), (0.5, 0.6)⟩, T_2 = ⟨x, (0.5, 0.3), (0.5, 0.7)⟩ and T_3 = ⟨y, (0.5, 0.3), (0.5, 0.7)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y, respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFG continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.6, 0.8), (0.3, 0.1)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.6, 0.8), (0.3, 0.1)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.8: Every IF completely WG continuous mapping is an IFP continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFPCS, f^{-1}(B) is an IFPCS in X. Hence f is an IFP continuous mapping. ☐

Example 3.9: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩, T_2 = ⟨x, (0.2, 0.3), (0.8, 0.7)⟩ and T_3 = ⟨y, (0.2, 0.3), (0.8, 0.7)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y, respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFP continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.9, 0.3), (0.1, 0.6)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.9, 0.3), (0.1, 0.6)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.10: Every IF completely WG continuous mapping is an IFαG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFαGCS, f^{-1}(B) is an IFαGCS in X. Hence f is an IFαG continuous mapping. ☐

Example 3.11: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.2), (0.6, 0.6)⟩, T_2 = ⟨x, (0.3, 0.1), (0.6, 0.8)⟩ and T_3 = ⟨y, (0.3, 0.1), (0.6, 0.8)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y, respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαG continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.7, 0.7), (0.2, 0.1)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.7, 0.7), (0.2, 0.1)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.12: Every IF completely WG continuous mapping is an IFS continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFSCS, f^{-1}(B) is an IFSCS in X. Hence f is an IFS continuous mapping. ☐

Example 3.13: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.3, 0.3), (0.5, 0.7)⟩ and T_2 = = ⟨y, (0.3, 0.3), (0.5, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFS continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.7, 0.6), (0.2, 0.4)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.7, 0.6), (0.2, 0.4)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.14: Every IF completely WG continuous mapping is an IFGS continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFGSCS, f^{-1}(B) is an IFGSCS in X. Hence f is an IFGS continuous mapping. ☐

Example 3.15: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.2), (0.6, 0.6)⟩, T_2 = = ⟨x, (0.1, 0.1), (0.6, 0.8)⟩ and T_3 = ⟨y, (0.1, 0.1), (0.6, 0.8)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y,σ) by f(a) = u and f(b) = v. Then f is an IFGS continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.6, 0.8), (0.1, 0.1)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.6, 0.8), (0.1, 0.1)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.16: Every IF completely WG continuous mapping is an IFγ continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFγCS, f^{-1}(B) is an IFγCS in X. Hence f is an IFγ continuous mapping. ☐

Example 3.17: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.2), (0.6, 0.6)⟩, T_2 = ⟨x, (0.2, 0.1), (0.6, 0.8)⟩ and T_3 = ⟨ y, (0.2, 0.1), (0.6, 0.8)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y,σ) by f(a) = u and f(b) = v. Then f is an IFγ continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.5, 0.8), (0.2, 0.1)⟩ is an IFWGCS in Y but f^{-1}(A) = ⟨x, (0.5, 0.8), (0.2, 0.1)⟩ is not an IFRCS in X, since cl(int(f^{-1}(A))) = $T_1^C \neq f^{-1}$(A).

Theorem 3.18: Every IF completely WG continuous mapping is an IFWG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y,σ) be an IF completely WG continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFWGCS, B is an IFWGCS in Y. Then f^{-1}(B) is an IFRCS in X, by hypothesis. Since every IFRCS is an IFWGCS, f^{-1}(B) is an IFWGCS in X. Hence f is an IFWG continuous mapping. ☐

Example 3.19: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.2, 0.3), (0.8, 0.7)⟩, T_2 = ⟨x, (0.8, 0.7), (0.2, 0.2)⟩ and T_3 = ⟨y, (0.8, 0.7), (0.2, 0.2)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} and σ = {$0_\sim$, T_3, $1_\sim$} are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFWG continuous mapping but not an IF completely WG continuous mapping since IFS A = ⟨y, (0.8, 0.9), (0.2, 0.1)⟩ is an IFWGCS in Y but $f^{-1}(A)$ = ⟨x, (0.8, 0.9), (0.2, 0.1)⟩ is not an IFRCS in X, since $cl(int(f^{-1}(A))) = T_1^C \neq f^{-1}(A)$.

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts map' means continuous mapping.

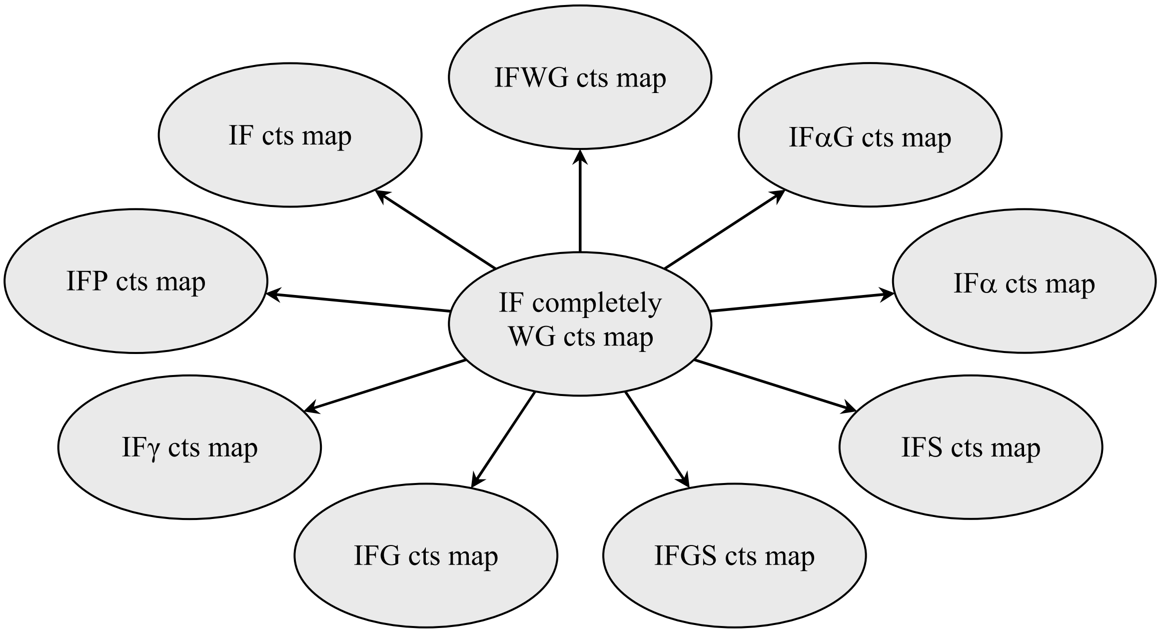

Fig. 1 Relation between intuitionistic fuzzy completely weakly generalized continuous mappings and other existing intuitionistic fuzzy continuous mappings

In this diagram by "A → B" we mean A implies B but not conversely. None of them is reversible.

Theorem 3.20: A mapping f: X → Y is an IF completely WG continuous mapping if and only if the inverse image of each IFWGOS in Y is an IFROS in X.

Proof:
Necessity: Let A be an IFWGOS in Y. This implies that A^c is an IFWGCS in Y. Since f is an IF completely WG continuous, $f^{-1}(A^c)$ is IFRCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFROS in X.
Sufficiency: Let A be an IFWGCS in Y. This implies that A^c is an IFWGOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFROS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRCS in X. Hence, f is an IF completely WG continuous mapping. □

Theorem 3.21: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be any two mappings where (Z, δ) is an $IF_wT_{1/2}$ space. Then the following statements hold.

(i) Let f : (X, τ) → (Y, σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IF continuous mapping. Then their composition g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping.

(ii) Let f : (X, τ) → (Y,σ) be an IF completely WG continuous mapping and g : (Y,σ) → (Z, δ) an IFα continuous mapping. Then their g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping

(iii) Let f : (X, τ) → (Y,σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IFαG continuous mapping. Then their composition g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping.

(iv) Let f : (X, τ) → (Y, σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IFP continuous mapping. Then their composition g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping.

(v) Let f : (X, τ) → (Y, σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IFG continuous mapping. Then their composition g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping.

(vi) Let f : (X, τ) → (Y, σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IFWG continuous mapping. Then their composition g∘f : (X, τ) → (Z, δ) is an IF completely WG continuous mapping.

(vii) Let f : (X, τ) → (Y,σ) be an IF completely WG continuous mapping and g : (Y, σ) → (Z, δ) an IF completely continuous mapping. Then their composition g∘f : (X, τ) →(Z, δ) is an IF completely WG continuous mapping.

Proof:

(i) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since every IFCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g∘f is an IF completely WG continuous mapping.

(ii) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFαCS in Y, by hypothesis. Since every IFαCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g∘f is an IF completely WG continuous mapping.

(iii) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFαG CS in Y, by hypothesis. Since every IFαGCS is an IFWGCS, $g^{-1}(A)$

is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping.

(iv) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFPCS in Y, by hypothesis. Since every IFPCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping.

(v) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFGCS in Y, by hypothesis. Since every IFGCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping.

(vi) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFWGCS in Y, by hypothesis. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X. Hence g_of is an IF completely WG continuous mapping.

(vii) Let A be an IFWGCS in Z. Since Z is an $IF_wT_{1/2}$ space, A is an IFCS in Z. Then $g^{-1}(A)$ is an IFRCS in Y, by hypothesis. Since every IFRCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping. ☐

Theorem 3.22: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be an IF completely continuous mapping and g : $(Y,\sigma) \rightarrow (Z, \delta)$ be an IF completely WG continuous mapping then their composition $g_of : (X, \tau) \rightarrow (Z, \delta)$ is an IF completely WG continuous mapping.

Proof: Let A be an IFWGCS in Z. Then $g^{-1}(A)$ is an IFRCS in Y, by hypothesis. Since every IFRCS is an IFCS, $g^{-1}(A)$ is an IFCS in Y. Therefore $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping. ☐

Theorem 3.23: Let f : $(X, \tau) \rightarrow (Y, \sigma)$ and g : $(Y, \sigma) \rightarrow (Z, \delta)$ be any two IF completely WG continuous mapping, then their composition $g_of : (X, \tau) \rightarrow (Z, \delta)$ is an IF completely WG continuous mapping.

Proof: Let A be an IFWGCS in Z. Then $g^{-1}(A)$ is an IFRCS in Y, by hypothesis. Since every IFRCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Therefore, $f^{-1}g^{-1}(A)$ is an IFRCS in X, by hypothesis. Hence g_of is an IF completely WG continuous mapping. ☐

Theorem 3.24: If f : X → Y is an IF completely WG continuous mapping then $int(cl(f^{-1}(int (B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFS. Then int(B) is an IFOS in Y and hence an IFWGOS in Y. By hypothesis, $f^{-1}(int (B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int (B)))) = f^{-1}(int (B)) \subseteq f^{-1}(B)$. ☐

Theorem 3.25: If f : (X, τ) → (Y, σ) is an IF completely WG continuous mapping, then wgcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for every IFS B of Y.

Proof: Let B be an IFS in Y. Then cl(B) is an IFCS in Y. Since every IFCS is an IFWGCS, cl(B) is an IFWGCS in Y. By hypothesis, f^{-1}(cl(B)) is a IFRCS in X and hence an IFWGCS in X. Clearly, B ⊆ cl(B). This implies f^{-1}(B) ⊆ f^{-1}(cl(B)). Therefore, wgcl(f^{-1}(B)) ⊆ wgcl(f^{-1}(cl(B))) = f^{-1}(cl(B)). Hence wgcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) for every IFS B of Y. ☐

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