Mathematical Techniques To Convert Intuitionistic Fuzzy Sets Into Fuzzy Sets

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ABSTRACT

Intuitionistic fuzzy set theory is an extension of fuzzy theory, which has been in existence since 1965 when Lotfi A. Zadeh presented the formal definition of fuzzy sets and their relevance. Later K.T. Atanassov proposed the concept of Intuitionistic fuzzy sets derived from fuzzy theory. Since then it has been studied and researched upon by various research scholars but no body has yet been able to back-trace Intuitionistic fuzzy values to equivalent fuzzy values. Here the authors have taken a bold step to propose certain mathematical approaches which could give a precise and approximate equivalent fuzzy values of Intuitionistic fuzzy sets.

KEYWORDS

Fuzzy sets, Intuitionistic fuzzy set, IF set, degree of memberships.

1. INTRODUCTION

Fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial but rather a matter of degree. When A is a fuzzy set and X is a relevant object, the proposition "X is a member of A" is not necessarily true or false but it may be only to some degree to which X is actually a member of the set A [4].

A Fuzzy Set can be represented as:

$$A = \{< X, \mu_A(X) > : X \in E\}$$

where E is defined as the universal set [5], and

$\mu_A : E \to [0,1]$ where μ_A defines the degree of membership.

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Intuitionistic Fuzzy Set theory is based on the extensions of corresponding definition of fuzzy set objects and definitions of new objects and their properties [1].

Mathematically an Intuitionistic Fuzzy Set (IFS) A is an object of the form [3] :

$$A = \{< X, \mu_A(X), V_A(X) > : X \in E\}$$

where E is the universal set, and

$$\mu_A : E \to [0,1]$$
$$V_A : E \to [0,1]$$

define the degree of membership and the degree of non-membership respectively of an element $X \in E$ to the set A, for every $X \in E$.
Also

$$0 < \mu_A(X) + V_A(X) \leq 1$$

and

$$\pi_A(X) = 1 - (\mu_A(X), V_A(X))$$

is called the hesitation part.

A special case of an Intuitionistic fuzzy set arises when the sum of the degree of membership and degree of non-membership adds up to 1 or in other words the value of $\pi_A(X)$ (the hesitation value) is equal to 0. This is a rare example of an Intuitionistic fuzzy set representing a fuzzy set indirectly.

2. THE NEED

The need of conversion from Intuitionistic Fuzzy Sets to Fuzzy Sets may arise in situations where the IF set values are available as data but implementation requirements and methods are fuzzy. This may happen due to two distinct reasons. One, because of the industrial popularity and acceptability of fuzzy theory and the faith of the masses on the success of fuzzy theory for efficiently and effectively solving real life problems. Secondly, the IF theory is relatively new with very little industrial applications.

The prevalent and popular technique of acquiring the data is, by taking views of experienced persons. These views when aggregated, give more of an IF values rather than fuzzy values, comprising only of membership degrees. Also, the views and thinking of masses does have a factor of hesitation, which makes their views a member of Intuitionistic fuzzy (IF) set, rather than the established Fuzzy Set.

The normal trend adopted by the implementers is to simply ignore the hesitation part, irrespective of its value/ ratio. A better and more precise result can be obtained by ignoring/ removing the hesitation part using certain mathematical methods and formulas. The authors have tried to put forward a few concrete mathematical methods to obtain fuzzy sets from Intuitionistic fuzzy sets.

3. THE METHODS

3.1 Method 1

As one is not at all sure about the hesitation zone as to where would it lead to. The hesitation zone can be divided into two equal parts and assigned to the degrees of membership and non-membership, irrespective of their degree values.

e.g.   For an IF set, A = (X, 0.6, 0.3)

$$\mu_A(X) = 0.6$$

$$V_A(X) = 0.3$$

Using this method, its equivalent fuzzy variable can be found as -

$$\pi_A(X) = 1 - (0.6 + 0.3) = 0.1$$

Dividing this by 2 we have, 0.1/2 = 0.05

therefore, $\mu_A(X) = 0.6 + 0.05 = 0.65$ and

$$V_A(X) = 1 - 0.65 = 0.35$$

Therefore, the equivalent fuzzy variable will be, (X, 0.65)

3.2 Method 2

This method converts the IF variable to fuzzy variable by simply merging of hesitation zone (value of the hesitation part of the given IF variable) with the value of degree of membership or non-membership, whichever is greater.

e.g.   For an IF set, A = (X, 0.2, 0.7)

$$\mu_A(X) = 0.2$$

$$V_A(X) = 0.7$$

Thus, $\pi_A(X) = 1 - (0.2 + 0.7) = 0.1$

According to this method, the equivalent fuzzy variable would have

$V_A(X) = 0.7 + 0.1$ (By adding the hesitation part to the non-membership value, being
greater than the membership value).

Thus, $V_A(X) = 0.8$. Correspondingly, $\mu_A(X) = 0.2$. Therefore, the equivalent Fuzzy variable would be, (X, 0.2).

A problem arises when the membership and non-membership degrees are same. In such situations, the hesitation value should be merged with either of the degrees arbitrarily or both degrees receiving half-half portion of $\pi_A(X)$, i.e. by using the method–1 described in section 3.1.

3.3 Method 3

This method is the most effective and appropriate method for the said conversion. It involves the calculation of the percentage of the degrees of membership and non-membership with respect to their sum, ignoring the hesitation zone.

The amount of percentage of degree is taken out from the hesitation value and added to the respective degree of membership and non-membership.

e.g.   for an IF set, A = (X, 0.4, 0.2)

$$\mu_A(X) = 0.4$$

$$V_A(X) = 0.2$$

Thus, $\pi_A(X) = 1 - (0.4 + 0.2) = 0.4$

% of $\mu_A(X)$ with respect to the given IF variable (after ignoring the hesitation zone)

would be = $\dfrac{\mu_A(X)}{\mu_A(X) + V_A(X)} \times 100$

$$= (0.4 / 0.6) * 100$$
$$= 66.66\% \ (\text{approx.})$$

Similarly, % of $V_A(X)$ = (0.2 / 0.6)*100
$$= 33.37\% \ (\text{approx.})$$

Using this method of conversion:

$$\pi_A(X) = 1 - (0.4 + 0.2) = 0.4$$

Now, 66.66 % of 0.4   = (0.4 * 66.66) / 100 = 0.266

The value of $\mu_A(X)$ for the equivalent fuzzy variable would be = 0.4 + 0.266 = 0.666

and the value of $V_A(X)$ would be = 1 − 0.666 = 0.334.

Therefore the equivalent fuzzy variable would be, (X, 0.666).

4. CONCLUSION:

In this paper we have presented three different mechanisms for converting inherently Intuitionistic Fuzzy values to Fuzzy values. We hope that this humble effort would enable the users of Fuzzy Logic to remove some of the arbitrariness in the conversion of hesitation zone for use in fuzzy sets and provide rigorous mathematical techniques for it. The different methods depicted here have distinct trade-offs in terms of processing time

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and accuracy of the final result. We here conclude that out of the three methods explained in the paper, the third method is the one, which would give the most exact fuzzy variable that would be equivalent to the converted IF variable.

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