

## A GENERALIZED NET DESCRIBING A PROCESS OF DECISION MAKING WITH INTUITIONISTIC FUZZY EXPERT SCORES

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### Abstract

We propose a description of a decision making process via a Generalized Net.

## 1 Generalized Nets and Decision Making

Let us have  $k$  experts  $E_1, \dots, E_k$  who are to evaluate some situation/object/event using given criteria. A Generalized net (GN, see [1]) describing their work is constructed and discussed. The process of Decision Making (DM) of the expert is described in [3].

Initially, token  $\alpha$  enters place  $a_1$  with initial characteristic

*“situation/object/event for estimation”*,

token  $\beta$  enters place  $b_1$  with initial characteristic

*“criteria for DM”*,

and tokens  $\gamma_1, \dots, \gamma_k$  ( $k \geq 1$ ) enter place  $c_1, \dots, c_k$  with initial characteristics

*“ $\langle E_i, \mu_i^E, \nu_i^E \rangle$ ”*,

where  $1 \leq i \leq k$  and for  $i$ -th expert  $E_i$   $\mu_i^E$  and  $\nu_i^E$  are degrees of correctness and incorrectness, i.e., degrees of its score. These degrees generate intuitionistic fuzzy couples, because for each  $i = 1, \dots, k$

$$\mu_i^E + \nu_i^E \leq 1$$

(for intuitionistic fuzziness see [2]).

The GN contains  $k + 4$  transitions that have the following forms.

$$Z_1 = \langle \{a_1, b_1, b_2, c_1, \dots, c_k\}, \{a_2, b_2, c_{k+1}, c_{2k}\} \rangle,$$

	$a_2$	$b_2$	$c_{k+1}$	$\dots$	$c_{2k}$
$a_1$	<i>true</i>	<i>false</i>	<i>false</i>	$\dots$	<i>false</i>
$b_1$	<i>false</i>	<i>true</i>	<i>false</i>	$\dots$	<i>false</i>
$b_2$	<i>false</i>	<i>true</i>	<i>false</i>	$\dots$	<i>false</i>
$c_1$	<i>false</i>	<i>false</i>	<i>true</i>	$\dots$	<i>false</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$c_k$	<i>false</i>	<i>false</i>	<i>false</i>	$\dots$	<i>true</i>

$\wedge(a_1, b_1, c_1, \dots, c_k) > .$

Token  $\alpha$  enters place  $a_2$ , without a new characteristic.

Token  $\beta$  enters place  $b_2$ , where it will unite with all next  $\beta$ -tokens and will enter the net, obtaining as a current characteristic

*"list of all criteria for decision making"*.

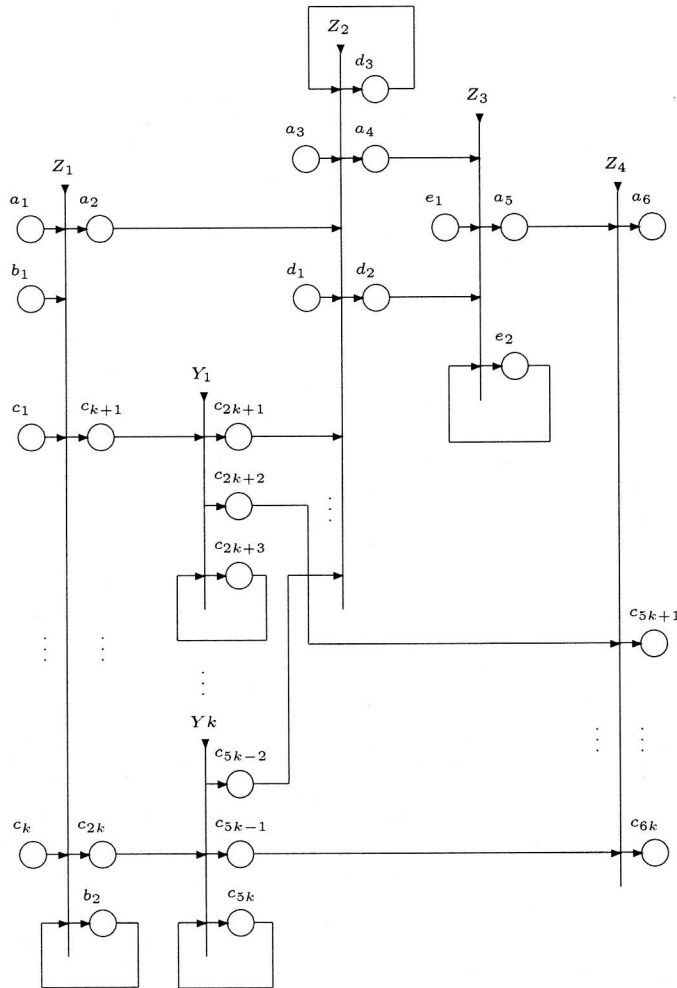


Fig.1 GN describing a process of decision making

The  $i$ -th token  $\gamma_i$  enters place  $c_i$  with a characteristic

*"list of criterie for DM, that will be used by expert  $E_i$ "*.

Below we shall describe the form of  $i$ -th transition  $Y_i$ , where  $1 \leq i \leq k$ .

We shall assume that each expert evaluates the given situation/object/event, using the criteria that he/she determine (as the characteristic from place  $c_{k+i}$  of the token that represents the respective expert's list of criteria). Therefore, the separate expert can use different list of criteria. We shall assume that each evaluation use one elementary GN-time-step. Hence, the number of tokens cycles in place  $l_{k+i}$  can be different.

$$Y_i = \langle \{c_{k+i}, c_{2k+3i}\}, \{c_{2k+3i-2}, c_{2k+3i-1}, c_{2k+3i}\},$$

$c_{k+1}$	$c_{2k+3i-2}$ <i>false</i>	$c_{2k+3i-1}$ <i>false</i>	$c_{2k+3i}$ <i>true</i>
$c_{2k+3i}$	$W_{2k+3i, 2k+3i-2}$	$W_{2k+3i, 2k+3i-1}$	$W_{2k+3i, 2k+3i}$

$$\vee (c_{k+i}, c_{2k+3i}) \rangle,$$

where

$W_{2k+3i, 2k+3i-2}$  = "the expert finishes his/her evaluation and token  $\delta$  does not enter place  $d_1$ ",

$W_{2k+3i, 2k+3i-1}$  = "the expert finishes his/her evaluation or token  $\delta$  enters place  $d_1$ ",

$W_{2k+3i, 2k+3i}$  = "the expert does not finish his/her evaluation and token  $\delta$  does not enter place  $d_1$ ",

where  $\neg P$  is the negation of predicate  $P$ .

The GN must be constructed so, that the place  $d_1$  has the higher priority than each one of places  $c_1, \dots, c_k$ . Therefore, token  $\delta$  will enters place  $d_1$  before  $\gamma$ -tokens and we can calculate correctly the above three predicates.

Token  $\gamma_i$  obtains the following characteristics. In place  $c_{2k+3i}$  it is

*"current evaluation of situation/object/event on the basis of the current used criterion".*

When the expert finishes evaluation of the situation/object/event using all the criteria that he/she wishes to use, and when there is not a token in place  $d_1$ , then the token  $\gamma_i$ , representing the expert splits to two tokens  $\gamma'_i$  and  $\gamma''_i$  that, respectively, enter places  $c_{2k+3i-2}$  and  $c_{2k+3i-1}$  obtaining characteristics

*"aggregated evaluation of situation/object/event on the basis of all used criteria by the  $i$ -th expert"*

and

$$\left\{ \begin{array}{ll} \text{"the } i\text{-th expert is not ready} & \text{if the expert does not finish} \\ \text{with his/her evaluation"} & \text{his/her evaluation or token } \delta \text{ enters place } d_1 \\ \\ \text{"the } i\text{-th expert is ready} & \text{if the expert finishes} \\ \text{with his/her evaluation"} & \text{his/her evaluation and token } \delta \text{ is not enters place } d_1 \end{array} \right.$$

In some moment token  $\alpha'$  enters place  $a_3$  with initial characteristic

*"information for changings of the situation/object/event".*

When the process of experts' evaluation must stop, token  $\delta$  enters place  $d_1$  without an initial characteristic.

$$Z_2 = \langle \{a_2, a_3, c_{2k+1}, \dots, c_{5k-2}, d_1, d_3\}, \{a_4, d_2, d_3\},$$

	$a_4$	$d_2$	$d_3$
$a_2$	$W_{2,4}$	<i>false</i>	<i>false</i>
$a_3$	$W_{3,4}$	<i>false</i>	<i>false</i>
$c_{2k+1}$	<i>false</i>	<i>false</i>	<i>true</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$c_{5k-2}$	<i>false</i>	<i>false</i>	<i>true</i>
$d_1$	<i>false</i>	<i>true</i>	<i>false</i>
$d_3$	<i>false</i>	$W_{3,2}$	$W_{3,3}$

$$\vee(\wedge(a_2, a_3), \vee(c_{2k+1}, \dots, c_{5k-2}, d_3), \wedge(d_1, d_3)) \rangle,$$

where

$W_{2,4}$  = "there is a token in place  $a_3$ ",

$W_{3,4}$  = "there is a token in place  $a_2$ ",

$W_{3,2}$  = "there is a token in place  $d_1$ ",

$W_{3,3} = \neg W_{3,2}$ .

Tokens  $\alpha$  and  $\alpha'$  unite in place  $a_4$  to token  $\alpha$ , that obtain characteristic

*"final form of the situation/object/event"*.

Each  $\gamma$ -token that is in the input place of transition  $Z_2$  (therefore, this is before the moment in which token  $\delta$  enters place  $d_1$ ) enters place  $d_3$  and all these tokens unite in one token  $\gamma$  with characteristic

*"current aggregation of the evaluations of the situation/object/event on the basis of the aggregated estimations of the experts, who finished his/her research"*.

Token  $\delta$  enters place  $a_2$  where it unites with token  $\gamma$  obtaining the characteristic

*"final aggregated evaluation of the situation/object/event on the basis of the aggregated estimations of all experts, who finished his/her research"*.

In some moment tokens (one or more)  $\varepsilon$  enter place  $e_1$  with initial characteristic

*"criterion for comparison of the experts' evaluations and the real informations for the situation/object/event"*.

$$Z_3 = \langle \{a_4, d_2, e_1, e_2\}, \{a_5, e_2\},$$

	$a_5$	$e_2$
$a_4$	<i>true</i>	<i>false</i>
$d_2$	<i>true</i>	<i>false</i>
$e_1$	<i>false</i>	<i>true</i>
$e_2$	<i>false</i>	<i>true</i>

$$\vee(\wedge(a_4, d_2), e_1, e_2) > .$$

Tokens  $\alpha$  and  $\delta$  enter place  $a_5$  and unite in token  $\alpha$  with a characteristic

*“results of the comparison of the experts’ evaluations and the real informations for the situation/object/event”.*

Token(s)  $\varepsilon$  are united in place  $e_2$  with a characteristic

*“list of criteria for comparison of the experts’ evaluations and the real informations for the situation/object/event”.*

$$Z_4 = < \{a_5, c_{2k+2}, \dots, c_{5k-1}\}, \{a_6, c_{5k+1}, c_{6k}\},$$

	$a_6$	$c_{k+1}$	$\dots$	$c_{2k}$
$a_1$	<i>true</i>	<i>false</i>	$\dots$	<i>false</i>
$c_1$	<i>false</i>	<i>true</i>	$\dots$	<i>false</i>
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$c_k$	<i>false</i>	<i>false</i>	$\dots$	<i>true</i>

$$\wedge(a_5, c_{2k+2}, \dots, c_{5k-1}) > .$$

Token  $\alpha$  go out GN through place  $a_6$  without a new characteristic, while token  $\gamma_i$  ( $1 \leq i \leq k$ ) enters place  $c_{5k+i}$  with characteristic

*“new expert’s score on the basis of the comparison of the expert evaluation and the real status of the situation/object/event”.*

## 2 Conclusions

We have shown the possibilities of Generalized Nets as far as description of the decision making processes is concerned.

## References

- [1] Atanassov K. *Generalized Nets*. Singapore, New Jersey, London, World Scientific, 1991.
- [2] Atanassov K. *Intuitionistic Fuzzy Sets*, Springer-Verlag, Heidelberg, 1999.
- [3] Atanassov, K., G. Pasi and R. Yager. Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making. *International Journal of Systems Research*, Vol. 36, 2005, No. 14,