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On intuitionistic fuzzy hyperstructure with T-norm

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Abstract: In this paper, we redefine T-intuitionistic fuzzy H_ν-subring of R and investigate some related properties. Some fundamental relation properties are studied.

Keywords: H_ν-rings, Fuzzy H_ν-group, Fundamental definition of H_ν-rings, Intuitionistic fuzzy H_ν-ideal, T-norm.
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1 Introduction

Since the concept of a fuzzy subset of a non-empty set was first introduced by Zadeh [15] in 1965, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets, first introduced by Atanassov [1], is one among them.

The theory of hyperstructures was introduced by Marty [11] in 1934. Marty introduced the notion of a hypergroup and then many researchers have been working on this new field of modern algebra and have been developing it. H_ν-rings first were introduced by Vougiouklis [14] in 1990. The largest class of algebraic systems satisfying ring-like axioms is the H_ν-ring. So, he defined the fundamental definition of H_ν-rings theory.

The concept of the fuzzy subhypergroup as well as of the fuzzy H_ν-group were introduced by Davvaz [3] in 1999. Davvaz [4] defined the concept of fuzzy H_ν-ideal of an H_ν-ring, which is a generalization of the concept of fuzzy ideal. The notion of intuitionistic fuzzy H_ν-ideal of an H_ν-ring were introduced by Davvaz, Dudek [5] in 2006.

Definition 1. *[15] Let X be a nonempty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.*

Definition 2. *[1] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A", $\nu_A(x)$, $(\nu_A : X \to [0,1])$ is called the "degree of non-membership of x in A", and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\}$.

Definition 3. *[**?**] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then*

1. *$A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$*
2. *$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$*
3. *$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$*
4. *$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$*
5. *$A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$*

Definition 4. *[11] A hyperstructure is a non-empty set H together with a mapping $* : H \times H \to P^*(H)$ which is called hyperoperation, where $P^*(H)$ denotes the set of all non-empty subsets of H. The image of pair (x,y) is denoted by $x * y$. If $x \in H$ and $A, B \subseteq H$, then by $A * B$, $A * x$ and $x * B$ we mean, respectively,*

$$A * B = \bigcup_{a \in A, b \in B} a * b, A * x = A * \{x\} \text{ and } x * B = \{x\} * B.$$

Definition 5. *[3] A hyperstructure $(H, *)$ is called a hypergroup if the following axioms hold:*
(i) $(H, *)$ is a semihypergroup, that is, $\forall x, y, z \in H, (x * (y * z)) = ((x * y) * z)$;
(ii) $x * H = H * x = H$ for all $x \in H$.

Definition 6. *[13] An H_ν-ring is a system $(R,+,\cdot)$ with two hyperoperations satisfying the following ring-like axioms:*
(i) $(R,+,\cdot)$ *is an H_ν-group, that is,*

$$\forall a \in R,\ a+R=R+a=R$$
$$\forall x,y,z \in R,\ ((x+y)+z)\cap(x+(y+z)) \neq \varnothing;$$

(ii) $(R,\cdot)$ *is an H_ν-semigroup, that is,*

$$\forall x,y,z \in R,\ ((x\cdot y)\cdot z)\cap(x\cdot(y\cdot z)) \neq \varnothing;$$

(iii) $(\cdot)$ *is weak distributive with respect to* $(+)$ *, that is, for all* $x,y,z \in R$,

$$\begin{aligned} ((x+y)\cdot z)\cap(x\cdot z+y\cdot z) &\neq \varnothing \\ (x\cdot(y+z))\cap(x\cdot y+x\cdot z) &\neq \varnothing. \end{aligned}$$

Definition 7. *[3] Let $(H,\cdot)$ be a hypergroup (or H_ν-group) and let μ be a fuzzy subset of H. Then, μ is said to be a fuzzy subhypergroup (or fuzzy H_ν-subgroup) of H if the following axioms hold:*
(i) $\min\{\mu(x),\mu(y)\} \leq \inf\limits_{\alpha\in x\cdot y}\{\mu(\alpha)\}$, $\forall x,y \in H$;
(ii) *for all $x,a \in H$ there exists $y \in H$ such that $x \in a\cdot y$ and* $\min\{\mu(a),\mu(x)\} \leq \mu(y)$.

Definition 8. *[3] Let $(H,\cdot)$ be an H_ν-group and let μ be a fuzzy subset of H. Then, μ is said to be a T-fuzzy H_ν-subgroup of H with repect to T-norm T if the following axioms hold:*
(i) $T(\mu(x),\mu(y)) \leq \inf\limits_{\alpha\in x\cdot y}\{\mu(\alpha)\}$, $\forall x,y \in H$;
(ii) *for all $x,a \in H$ there exists $y \in H$ such that $x \in a\cdot y$ and* $T(\mu(a),\mu(x)) \leq \mu(y)$.

Definition 9. *[5] An intuitionistic fuzzy set $A=(\mu_A,\nu_A)$ in R is called a left (resp., right) intuitionistic fuzzy H_ν-ideal of R if*
1) $\min\{\mu_A(x),\mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x+y\}$, *for all* $x,y \in R$;
2) *for all $x,a \in R$ there exist $y,z \in R$ such that $x \in (a+y)\cap(z+a)$ and*

$$\min\{\mu_A(a),\mu_A(x)\} \leq \min\{\mu_A(y),\mu_A(z)\};$$

3) $\mu_A(y) \leq \inf\{\mu_A(z) : z \in x\cdot y\}$*(resp.,*$\mu_A(x) \leq \inf\{\mu_A(z) : z \in x\cdot y\}$*) for all* $x,y \in R$;
4) $\sup\{\nu_A(z) : z \in x+y\} \leq \max\{\nu_A(x),\nu_A(y)\}$, *for all* $x,y \in R$;
5) *for all $x,a \in R$ there exist $y,z \in R$ such that $x \in (a+y)\cap(z+a)$ and*

$$\max\{\nu_A(y),\nu_A(z)\} \leq \max\{\nu_A(a),\nu_A(x)\};$$

6) $\sup\{\nu_A(z) : z \in x\cdot y\} \leq \nu_A(y)$ *(resp.,*$\sup\{\nu_A(z) : z \in x\cdot y\} \leq \nu_A(x)$*) for all* $x,y \in R$.

Definition 10. *[10] By a t-norm T, we mean a function $T : [0,1]\times[0,1] \to [0,1]$ satisfying the following conditions:*
1) $T(x,1)=x$;
2) $T(x,y) \leq T(x,z)$ *if* $y \leq z$;
3) $T(x,y)=T(y,x)$;
4) $T(x,T(y,z))=T(T(x,y),z)$;
for all $x,y,z \in [0,1]$.

For a t-norm T on $[0,1]$, denote by Δ_T the set of element $\alpha \in [0,1]$ such that $T(\alpha,\alpha)=\alpha$, i.e., $\Delta_T := \{\alpha \in [0,1] \ : \ T(\alpha,\alpha)=\alpha\}$.

Definition 11. *[10] Let T be a t-norm. A fuzzy subset μ of R is said to satisfy the idempotent property if $\Im(\mu) \subseteq \Delta_T$.*

2 Main results

Definition 12. *Let $(R,+,\cdot)$ be an H_ν-ring and $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of R. Then, $A=(\mu_A,\nu_A)$ is said to be a T-intuitionistic fuzzy H_ν-subring of R with respect to t-norm T if the following axioms hold:*
1) $T(\mu_A(x),\mu_A(y)) \le \inf\{\mu_A(z) : z \in x+y\}$, *for all* $x,y \in R$;
2) $\sup\{\nu_A(z) : z \in x+y\} \le 1-T(1-\nu_A(x),1-\nu_A(y))$, *for all* $x,y \in R$;
3) *for all* $x,a \in R$ *there exist* $y,z \in R$ *such that* $x \in (a+y)\cap(z+a)$ *and*

$$T(\mu_A(a),\mu_A(x)) \le T(\mu_A(y),\mu_A(z));$$

4) $T(\mu_A(x),\mu_A(y)) \le \inf\{\mu_A(z) : z \in x\cdot y\}$, *for all* $x,y \in R$;
5) $\sup\{\nu_A(z) : z \in x\cdot y\} \le 1-T(1-\nu_A(x),1-\nu_A(y))$, *for all* $x,y \in R$;
6) *for all* $x,a \in R$ *there exist* $y,z \in R$ *such that* $x \in (a+y)\cap(z+a)$ *and*

$$T(1-\nu_A(a),1-\nu_A(x)) \le T(1-\nu_A(y),1-\nu_A(z)).$$

Theorem 1. *Let T be an t-norm and $A=(\mu_A,\nu_A)$ be an T-intuitionistic fuzzy H_ν-subring of R. Let μ_A, $1-\nu_A$ have the idempotent property. Then, the following sets are H_ν-subring of R*

$$R^w=\{x\in R : \mu_A(x) \ge \mu_A(w)\} \ , \ \ L^w=\{x\in R : \nu_A(x) \le \nu_A(w)\}.$$

Proposition 1. *Let $x,y \in R^w$. Then, $\mu_A(x) \ge \mu_A(w)$ and $\mu_A(y) \ge \mu_A(w)$.*

Since $A=(\mu_A,\nu_A)$ be a T-intuitionistic fuzzy H_ν-subring of R and μ_A have the idempotent property, it follows that

$$\begin{aligned}\inf\{\mu_A(z) : z \in x+y\} &\ge T(\mu_A(x),\mu_A(y))\\ &\ge T(\mu_A(x),\mu_A(w))\\ &\ge T(\mu_A(w),\mu_A(w)) = \mu_A(w)\end{aligned}$$

Hence, $x+y \subseteq R^w$ implies $x+y \in P^*(R^w)$. Similarly, we have $x\cdot y \subseteq R^w$ and $x\cdot y \in P^*(R^w)$. Hence, $a+R^w \subseteq R^w$ and $R^w+a \subseteq R^w$ for all $a \in R^w$.

Now, let $x \in R^w$. Then, there exist $y,z \in R$ such that $x \in (a+y)\cap(z+a)$ and

$$T(\mu_A(a),\mu_A(x)) \le T(\mu_A(y),\mu_A(z)).$$

Since a,$x \in R^w$, we have

$$\mu_A(w) = T(\mu_A(w),\mu_A(w)) \le T(\mu_A(a),\mu_A(x))$$

and so

$$\mu_A(w) \le T(\mu_A(y), \mu_A(z)) \le \min\{\mu_A(y), \mu_A(z)\},$$

which implies $y \in R^w$ and $z \in R^w$.

This proves that $R^w \subseteq a + R^w$ and $R^w \subseteq R^w + a$.

Since $(R, +, \cdot)$ is an H_ν-group and $R^w \subseteq R$ then for all $x, y, z \in R^w$,

$$\begin{aligned}
&((x+y)+z) \cap (x+(y+z)) \ne \varnothing;\\
&((x+y)\cdot z) \cap (x\cdot z + y\cdot z) \ne \varnothing;\\
&(x\cdot(y+z)) \cap (x\cdot y + x\cdot z) \ne \varnothing;\\
&((x\cdot y)\cdot z) \cap (x\cdot(y\cdot z)) \ne \varnothing.
\end{aligned}$$

Consequently, R^w is an H_ν-subring of R.

If $x, y \in L^w$, then $\nu_A(x) \le \nu_A(w)$ and $\nu_A(y) \le \nu_A(w)$. Since $A = (\mu_A, \nu_A)$ is a T-intuitionistic fuzzy H_ν-subring of R and $1 - \nu_A$ has the idempotent property, it follows that

$$\begin{aligned}
\sup\{\nu_A(z) : z \in x+y\} &\le 1 - T(1-\nu_A(x), 1-\nu_A(y))\\
&\le 1 - T(1-\nu_A(w), 1-\nu_A(w)) = \nu_A(w).
\end{aligned}$$

Hence, $x + y \subseteq L^w$. Similarly, we have $x \cdot y \subseteq L^w$. Hence, $a + L^w \subseteq L^w$ and $L^w + a \subseteq L^w$ for all $a \in L^w$.

Let $x \in L^w$. Then, there exist $y, z \in R$ such that $x \in (a+y) \cap (z+a)$ and

$$T(1-\nu_A(a), 1-\nu_A(x)) \le T(1-\nu_A(y), 1-\nu_A(z)).$$

Since a,$x \in L^w$, we have

$$\begin{aligned}
1-\nu_A(w) &= T(1-\nu_A(w), 1-\nu_A(w))\\
&\le T(1-\nu_A(w), 1-\nu_A(x)) \le T(1-\nu_A(a), 1-\nu_A(x)),
\end{aligned}$$

and so

$$1-\nu_A(w) \le T(1-\nu_A(y), 1-\nu_A(z)) \le \min\{1-\nu_A(y), 1-\nu_A(z)\},$$

which implies $y \in L^w$ and $z \in L^w$.

This proves that $L^w \subseteq a+L^w$ and $L^w \subseteq L^w+a$. Since $(R, +, \cdot)$ is an H_ν-group and $L^w \subseteq R$, then for all $x, y, z \in L^w$,

$$\begin{aligned}
&((x+y)+z) \cap (x+(y+z)) \ne \varnothing;\\
&((x+y)\cdot z) \cap (x\cdot z + y\cdot z) \ne \varnothing;\\
&(x\cdot(y+z)) \cap (x\cdot y + x\cdot z) \ne \varnothing;\\
&((x\cdot y)\cdot z) \cap (x\cdot(y\cdot z)) \ne \varnothing.
\end{aligned}$$

Consequently, L^w be an H_ν-subring of R.

Proposition 2. *Let H be a non-empty subset of an H_ν-ring R and let the fuzzy sets μ, ν in R be defined by*

$$\mu(x) = \begin{cases} \alpha_0, & x \in H \\ \alpha_1, & \textit{otherwise} \end{cases}, \quad \nu(x) = \begin{cases} \beta_0, x \in H \\ \beta_1, \ \textit{otherwise} \end{cases},$$

where $0 \leq \alpha_1 < \alpha_0$, $0 \leq \beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$.

Let $\mu, 1 - \nu$ have the idempotent property. Then, $A = (\mu, \nu)$ is a T-intuitionistic fuzzy H_ν-subring of $R \Leftrightarrow H$ is an H_ν-subring of R.

Proof. Suppose that $A = (\mu, \nu)$ is a T-intuitionistic fuzzy H_ν-subring of R. Let $x, y \in H$. Then,

$$\inf \{\mu(z) : z \in x + y\} \geq T(\mu(x), \mu(y)) = T(\alpha_0, \alpha_0) = \alpha_0.$$

It follows that $x + y \subseteq H$. Similarly, we have $x \cdot y \subseteq H$.

Hence, $a + H \subseteq H$ and $H + a \subseteq H$ for all $a \in H$. Let $x \in H$. Then, there exist $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$T(\mu(a), \mu(x)) \leq T(\mu(y), \mu(z)).$$

Since a,$x \in H$, we have

$$\alpha_0 = T(\mu(a), \mu(x)) \leq T(\mu(y), \mu(z)) \leq \min\{\mu(y), \mu(z)\},$$

which implies $y \in H$ and $z \in H$.

This proves that $H \subseteq a + H$ and $H \subseteq H + a$. Since $(R, +, \cdot)$ is an H_ν-group and $H \subseteq R$ then for all $x, y, z \in H$,

$$\begin{aligned} &((x + y) + z) \cap (x + (y + z)) \neq \varnothing; \\ &((x + y) \cdot z) \cap (x \cdot z + y \cdot z) \neq \varnothing; \\ &(x \cdot (y + z)) \cap (x \cdot y + x \cdot z) \neq \varnothing; \\ &((x \cdot y) \cdot z) \cap (x \cdot (y \cdot z)) \neq \varnothing. \end{aligned}$$

Therefore H is an H_ν-subring of R.

Conversely suppose that H is an H_ν-subring of R. Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\mu(x) = \alpha_1$ or $\mu(y) = \alpha_1$ and so

$$\inf \{\mu(z) : z \in x + y\} \geq \min\{\mu(x), \mu(y)\} = \alpha_1 \geq T(\mu(x), \mu(y)).$$

Assume that $x \in H$ and $y \in H$. Then, $x + y \subseteq H$ and hence

$$\inf \{\mu(z) : z \in x + y\} \geq \min\{\mu(x), \mu(y)\} = \alpha_0 \geq T(\mu(x), \mu(y)).$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\nu(x) = \beta_1$ or $\nu(y) = \beta_1$ and so

$$\begin{aligned} \sup \{\nu(z) : z \in x + y\} &\leq \beta_1 = \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1 - \nu(x), 1 - \nu(y)\} \leq 1 - T(1 - \nu(x), 1 - \nu(y)). \end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then, $x + y \subseteq H$ and hence

$$\begin{aligned}\sup\{\nu(z) : z \in x + y\} \leq \beta_0 &= \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1 - \nu(x), 1 - \nu(y)\} \leq 1 - T(1 - \nu(x), 1 - \nu(y)).\end{aligned}$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\mu(x) = \alpha_1$ or $\mu(y) = \alpha_1$ and so

$$\inf\{\mu(z) : z \in x \cdot y\} \geq \min\{\mu(x), \mu(y)\} = \alpha_1 \geq T(\mu(x), \mu(y)).$$

Assume that $x \in H$ and $y \in H$. Then, $x + y \subseteq H$ and hence

$$\inf\{\mu(z) : z \in x \cdot y\} \geq \min\{\mu(x), \mu(y)\} = \alpha_0 \geq T(\mu(x), \mu(y)).$$

Let $x, y \in R$. If $x \in R \setminus H$ or $y \in R \setminus H$, then $\nu(x) = \beta_1$ or $\nu(y) = \beta_1$ and so

$$\begin{aligned}\sup\{\nu(z) : z \in x \cdot y\} \leq \beta_1 &= \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1 - \nu(x), 1 - \nu(y)\} \leq 1 - T(1 - \nu(x), 1 - \nu(y)).\end{aligned}$$

Assume that $x \in H$ and $y \in H$. Then, $x + y \subseteq H$ and hence

$$\begin{aligned}\sup\{\nu(z) : z \in x \cdot y\} \leq \beta_0 &= \max\{\nu(x), \nu(y)\} \\ &= 1 - \min\{1 - \nu(x), 1 - \nu(y)\} \leq 1 - T(1 - \nu(x), 1 - \nu(y)).\end{aligned}$$

Let $x, a \in R$. Since R is an H_ν-ring, then there exist $y, z \in R$ such that $x \in (a + y) \cap (z + a)$. If $x \in R \backslash H$ or $a \in R \backslash H$, then $\mu(x) = \alpha_1$ or $\mu(a) = \alpha_1$ and hence $\mu(x) \leq \mu(y), \mu(a) \leq \mu(z)$. And so

$$T(\mu(a), \mu(x)) \leq T(\mu(y), \mu(z)).$$

Assume that $x \in H$ and $a \in H$. Since H is an H_ν-subring of R, then there exist $y, z \in H$ such that $x \in (a + y) \cap (z + a)$. Then, $\mu(x) = \mu(y) = \mu(a) = \mu(z) = \alpha_0$ and so

$$T(\mu(a), \mu(x)) \leq T(\mu(y), \mu(z)).$$

Similarly, we have for all $x, a \in R$ there exist $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$T(1 - \nu(a), 1 - \nu(x)) \leq T(1 - \nu(y), 1 - \nu(z)).$$

Consequently $A = (\mu, \nu)$ be a T-intuitionistic fuzzy H_ν-subring of R. $\square$

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