"Twenty years later" or some words about the intuitionistic fuzzy sets

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Like an instant passed twenty years since February 1983 when I was distracting from an illness with the well-known Kaufmann's book *"Introduction a la theorie des sour-ensembles flous"*, which inspired me to define the Intuitionistic Fuzzy Sets (IFSs). Two months later I discussed my idea with Georgi Gargov (1947-1996), who suggested the name of these sets; other two months later I presented IFSs for the first time ...

These 20 years witnessed the foundation-laying of the IFS theory. Intuitionistic fuzzy sets were shown to be a proper generalisation of ordinary fuzzy sets, to be equivalent to interval-valued fuzzy sets, their relationship with L-fuzzy sets was made clear. In turn, IFS themselves became object of generalisations - now there are Temporal IFSs, Interval values IFSs, Intuitionistic L-fuzzy sets, IFSs over different universes, Rough IFSs, Intuitionistric fuzzy rough sets,IFSs from a second type, etc.

The bibliography of M. Nikolova, N. Nikolov, C. Cornelis, G. Deschrijver *"Survey of the research on intuitionistic fuzzy sets"* (Advanced Studies in Contemporary Mathematics, Vol. 4, 2002, No. 2, 127-157) comprises more than 400 papers, while by now their number exceeds 500 and their authors live in more than 40 countries. The International Conferences on IFS has gathered during the last 7 years researchers from Australia, Belgium, Bulgaria, China, Greece, India, Iran, Italy, Jordan, Morocco, Poland, Portugal, Romania, Russia, South Korea, Spain, Turkey, United Kindom, USA. A workshop on IFS is organized yearly in Poland, and special sections on IFS have been included in larger conferences held in France, Germany, Portugal, and USA.

Over the years there appeared sometimes works reinventing the concept of IFS under different names - bifuzzy sets, vague sets, neutrosophic sets etc. We regard this as an evidence that the IFS idea is indeed timely and useful. Criticisms were expressed concerning their name. It was Georgi Gargov who proposed it, and it was not long before it has sticked so much that it was too late to be changed. The name, however, corresponds well to the Brouwer's understanding of mathematics. I am going to dwell a little longer on this point.

Set theory, proposed in the 1880's by Georg Cantor, is at first welcomed by the majority of mathematicians - only a few (such as Kronecker) reject it. Basing on set

theory, in 1898 Guiseppe Peano axiomatizes the arithmetics, and soon after that David Hilbert does the same with geometry. An year later he presents his famous 23 problems to the mathematical community - even the first problem being related to set theory. Just when it has seemed that all is well, deep paradoxes start to shatter the fundaments of mathematics - Burali-Forti's, Russell's, etc. It was Hilbert who thought first of a solution to this situation. Several years later another approach was proposed by Russell, and yet another arises in 1913, when L. Brouwer introduces the term 'intuitionism'. He proposes that mathematics gets rid of the law of the excluded middle, that is, to stop inferring $\neg A$ from the information that A is false. This rule is extremely convenient in theorem proving. It is used in almost all branches of mathematics since Aristotle, but it is exactly his rule that Brouwer identifies as a primary source of problems. Gödel's incompleteness theorems of 1930 put an end to both Hibert's and Russell's approaches. Brouwer's platform is the only one to survive, but the newly emerged teories of algorithms, finite automata and Post's and Turing's machines clearly show the need for a new mathematics - the constructive mathematics as it is known today. In a sense, it is the heir of Brouwer's intuitionism. Meanwhile the first multi-valued (in the beginning, three-valued) logics have appeared due to Lukasiewicz, and his follower Lotfi Zadeh proposed the notion of fuzzy set in 1965. Now for a statement p can be estimated by a valuation function V so that $V(p) \in [0, 1]$. It may not be estimated as either true or false, as it should be according to Aristotle - Leibnitz - Frege - Whitehead - Russell - Tarski logic. For such a statement we would have that its negation $\neg p$ deviates from the standard case as well:

$$V(p \vee \neg p) = V(p) \vee V(\neg p) \in [0, 1],$$

and moreover, for p with the property that $0 < V(p), V(\neg p) < 1$ is valid that

$$0 < V(p \vee \neg p) < 1.$$

We thus can see Brouwer's intuitionism behind Zadeh's fuzzy logic (at the level of valuations). The case with IFS is similar, but with the following important addition: Brouwer's idea is present here not only at the valuation level but also at the semantical level. Questions like 'What is the degree of falsity of the statement p?', that is, 'What is the degree of truth of the statement $\neg p$?', can now be put explicitly. Just like Brouwer's idea, with IFS it is possible not to know which of the two statements, p and $\neg p$, is true and which is false. We can now speak of a degree of indeterminacy as well. This degree (it can be explicitly calculated for every statement) is the essentially new feature of IFS that does not occur in fuzzy sets; it is present, although implicitly, in interval valued fuzzy sets and L-fuzzy sets. That is why the name of IFS, cumbersome as it may be, is actually correct. Of course, the word 'intuitionistic' should always be followed by 'fuzzy', or otherwise we risk referring to intuionistic mathematics or logic.

It must be added that the definition of IFS leads not only to operations and relations analogous to those defined over ordinary fuzzy sets, but also to operators of modal type,

as well as their extensions. Recently various appications of IFS to artificial intelligence have appeared - intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations etc.; relations to other areas of mathematics were found - algebra, geometry, analysis, number theory, topology.

The warm reception of IFS by a large number of researchers worldwide are a promise for their future and I hope after 30 years to celebrate a more significant anniversary.