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A *F*-OPERATOR INTUITIONISTIC FUZZY VERSION OF THE NEAREST NEIGHBOR CLASSIFIER

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ABSTRACT: A F-operator intuitionistic fuzzy version of one of the basic statistical nonparametrical methods, the nearest prototype (respectively nearest neighbor - NN) classification method, is proposed. It is based on a procedure of adjusting the degrees of membership, nonmembership and indeterminacy to the nearest class by means of the F operator. The parameter values of that operator iteratively take into account the distances to the prototype (mean) vector of the class, to the 1- NN which belongs to that nearest class, to the 3- NN, to the 5- NN, etc. The procedure stops when the degree of indeterminacy is considerably diminished and the degree of membership reaches a sufficiently high value and in this way increases the confidence in the classification decision.

1. INTRODUCTION

The nearest prototype (NP) method is one of the often applied and accurate methods in pattern recognition [1,3,4,5,9,11,12]. This method is a simplification of the K-NN (nearest neighbors) method. The basic assumption of this statistical and nonparametrical method is that inside of the small hypersphere around a given pattern the probability density function is approximately constant [4]. For this reason the classifiers have to be restricted to a small value of K. However, in cases of small sample size and overlapping classes this requirement causes accuracy decreasing. A lot of modifications of the method have been published, which improve it with respect to increasing the classification accuracy and minimization of the calculations. Several fuzzy versions have been developed [1,3,9,10,11,12]. Recently intuitionistic fuzzy modifications of this method have also been elaborated [15,16] and they showed promising results. Here a new intuitionistic version is proposed. It is based on a procedure of adjusting, getting the degrees of membership and nonmembership to the nearest class more precise by means of the *F* operator. The parameter values of that operator iteratively take into account the distances to the prototype (mean) vector of the class, to the one NN which belongs to that nearest class, to the three NN, to the five NN, etc. The procedure stops when the degree of indeterminacy is considerably diminished and the degree of membership reaches a sufficiently high value and in this way increases the confidence in the classification decision.

2. THEORETICAL BACKGROUND

Let the sample (training, reference) set of patterns(vectors) is:

$$X = \{x_1, x_2, \ldots x_N\};\ \ x_l \in \Re^n,\ l = 1,2,\ldots,N \tag{1}$$

Each pattern $x_l \in \Re^n$ is described by ***n*** features :

$$x_l = (x_{l1}, x_{l2}, \ldots, x_{ln}) \tag{2}$$

The set of the classes defined over $\Re^n$ is:

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_M\};\ \ \omega_i \in \Omega;\ \ i = 1,2,\ldots,M \tag{3}$$

Each class ω_i contains $N_i,\ i = 1,2,\ldots,M$ patterns (vectors), $\sum_i N_i = N$. On the base of the training set for each class ω_i the prototype (mean) vector x_{i_m} is evaluated according to

$$x_{i_m} = \frac{1}{N_i}\sum_{l_i=1}^{N_i} x_{l_i} \tag{4}$$

The fuzzy logic and description can be used not only in cases of insufficient and small training set but they can be used also in case of large training set in order to improve the classification accuracy. What is proposed here is the use of the intuitionistic fuzzy sets (IFSs) for classification purposes is proposed, i.e. of the degrees of membership $\mu(x)$ and nonmembership $\nu(x)$. It is known that the IFS A* in E is an object having the form [13,14]:

$$A^* = \{< x, \mu(x), \nu(x) > / \, x \in E\}, \tag{5}$$

where $\mu_A : E \to [0,1]$, $_A : E \to [0,1]$ define the degrees of membership and nonmembership of the element $x \in E$ to the set A which is a subset of E, respectively, and for every $x \in E$

$$0 \le \mu_A(x) + \nu_A(x) \le 1 \tag{6}$$

In case of a strong inequality, i.e. $0 \le \mu_A(x) + \nu_A(x) < 1$

then

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1 \tag{7}$$

and

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \tag{8}$$

where $\pi_A(x)$ is the degree of indeterminacy.

We consider the degree of membership as confidence for membership to the given class; the degree of nonmembership as diffidence for membership; and the degree of indeterminacy as an uncertainty, vagueness which should be considerably diminished, preferably to zero, or possibly to be eliminated.

In the next sections a procedure for decrease and possible elimination of $\pi_A(x)$ is described.

2.1. Evaluation of the degrees of membership, nonmembership and indeterminacy

The implementation of IFS in a nearest prototype rule is possible if $\mu(x)$ and $\nu(x)$ for a pattern x_u with unknown classification are evaluated. These memberships can be determined in several different ways: on the basis of the geometrical properties of the classes [1,2,5,6,12]; on the basis of the probabilistic measures and expert knowledge [1,2,4,11,12].

We propose the following way for evaluating of degrees of membership and nonmembership to the nearest class ω_i [15]:

1. The distances between a vector with unknown classification x_u and the prototype (mean) vector x_{i_m} for a class ω_i are calculated according to:

$$d_{i_m} = \left\| x_u - x_{i_m} \right\| = \sqrt{\sum_{j=1}^{n} (x_{uj} - x_{i_m j})^2} \ , i = 1,2,\ldots,M \tag{9}$$

These distances are ordered in a sequence in ascending way $D = (d_{i_m}^{\min},\ldots,d_{i_m}^{\max})$.

2. The degrees of membership and nonmembership to the nearest class are evaluated by means of:

$$\mu_{\omega_i}(x_u) = e^{-d_{i_m}^{\min}}, \tag{10}$$

$$\nu_{\omega_i}(x_u) = e^{-d_{iNm}}, \qquad i \in \{1,2,\ldots,M\} \tag{11}$$

where d_{iN_m} is the second one distance in the sequence D, i.e. the distance to the next, to the competitive class.

Another way for determination of $\mu(x)$ and (x) could be [16]:

$$\mu_{\omega_i}(x_u) = \frac{\left(\left\| x_u - x_{i_m} \right\|^2 \right)^{\left(-\frac{1}{q-1}\right)}}{\sum_{k=1}^{M} \left(\left\| x_u - x_{k_m} \right\|^2 \right)^{\left(-\frac{1}{q-1}\right)}}, \tag{12}$$

$$\nu_{\omega_i}(x_u) = \frac{\left(\left\| x_u - x_{iNm} \right\|^2 \right)^{\left(-\frac{1}{q-1}\right)}}{\sum_{k=1}^{M} \left(\left\| x_u - x_{k_m} \right\|^2 \right)^{\left(-\frac{1}{q-1}\right)}}, \tag{13}$$

where q>1 is a degree of fuzziness.

In both cases $\pi_{\omega_i}(x_u)$ is evaluated according to (8).

In such a way the nearest class ω_i; $i \in \{1,2,\ldots,M\}$ is presented as an IFS. If (6) is not satisfied then $\nu_{\omega_i}(x_u)$ is evaluated according to:

$$\nu_{\omega_i}(x_u) = \min\{(1-\mu_{\omega_i}(x_u)), \nu_{\omega_i}(x_u)\} \tag{14}$$

After the evaluation of $\mu(x)$ and $\nu(x)$, it is checked if $\mu_{\omega_i}(x_u) > \mu_{thres}(x)$. If so, then the classification is done to the nearest class. Otherwise, a procedure of applying the F-operator F, i.e. a procedure of adjusting $\mu(x)$, $\nu(x)$ and diminishing $\pi\ (x)$ is proposed.

2.2. Decrease of the indeterminacy

In order to decrease and possibly to eliminate the indeterminacy we use the following operators for IFS, defined in [17]:

$$D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x) > / x \in E\}, \tag{15}$$

where $\alpha \in [0,1]$ is a fixed number. From this definition it follows that Dα(A) is a fuzzy set.
The next operator is;

$$F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) > / x \in E\}, \tag{16}$$

where $\alpha,\beta \in [0,1]$, $\alpha+\beta \le 1$.

After the evaluation of $\mu_{\omega_i}(x_u)$, $\nu_{\omega_i}(x_u)$ and $\pi_{\omega_i}(x_u)$, for example, in case of two classes, in fact we obtain the following Figure 1.

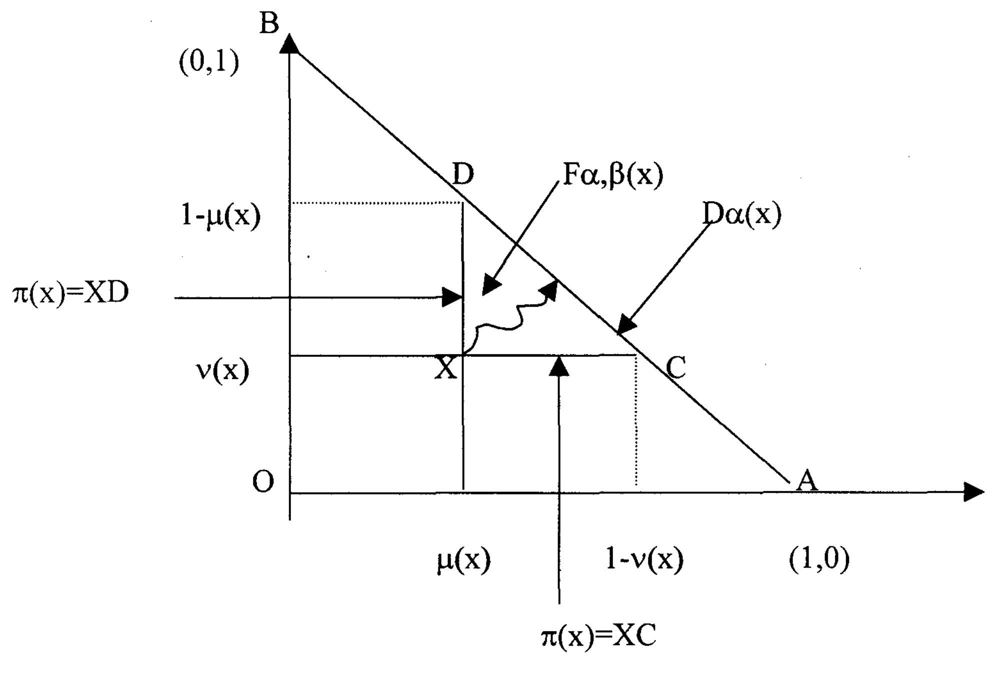

Fig.1.

Operator Dα(x) transforms and projects point X onto hypotenuse AB. It should be noted that on AB, $\pi_{\omega_i}(x_u) = 0$. Operator Fα,β(x) transforms and moves point X within triangle

XCD. Our goal is to reduce and if possible, to eliminate the indeterminacy $\pi_{\omega_i}(x_u)$ finally. This could be achieved by iterative application of the F-operator. For the determination α and β some statistical and geometrical information is taken into account. The combination of statistical and fuzzy approaches in pattern recognition intuitively leads to an improvement of the recognition results. In this way we can change $\mu_{\omega_i}(x_u)$, $\nu_{\omega_i}(x_u)$ and $\pi_{\omega_i}(x_u)$, aiming to reach hypotenuse AB. If we have sufficient and confident information at our disposal, we can project *Xu* onto AB (actually CD) in one step. In fact, we do not have such information and that is why we suggest for the moving to be done stepwise, iteratively by means of applying the F operator. We propose the following algorithm for α and β determination:

1. j=0
2. $$\alpha = e^{-d_{i_m}^{averaged\ (1+2j)NN}}, \quad (17)$$

$$\beta = e^{-d_{i_{Nm}}^{averaged\ (1+2j)NN}}, \quad (18)$$

where the superscript "*averaged (1+2j)NN*" means that the given distance is the average one over (1+2j) number of distances, i.e. over the distances between *Xu* and the nearest neighbors (patterns) (1+2j) in number. In (17), these nearest neighbors(NN) belong to the nearest class and in (18) to the competitive class, respectively.

3. With these α and β, the F-operator (16) is applied, and the new values $\mu_{\omega_i}^{new}(x_u)$, $\nu_{\omega_i}^{new}(x_u)$ and $\pi_{\omega_i}^{new}(x_u)$ are obtained, having in mind (6) and (14).

4. If $\mu_{\omega_i}^{new}(x_u) > \mu_{thres}(x)$ then the algorithm stops, otherwise – next Step 5 is to be made. This is equal to check if $\pi_{\omega_i}(x_u^{new}) < \pi_{thres}(x)$.

5. j=j+1. Go to Step 2.
6. Stop

In fact, at the first iteration α and β are determined having taken into account the 1-NNs, belonging to the nearest class and to the competitive class, respectively. Then at the second having taken into account the average distance of the 3 NN, then the average distances of the 5 NN, etc.

The moving procedure stops when $\pi_{\omega_i}(x_u^{new}) < \pi_{thres}(x)$, i.e. it is smaller than a preset threshold. At each Step 3 the validation of (6) is checked and if it is not satisfied, (14) is applied and the moving procedure stops.

3.CONCLUSION

A *F*-operator intuitionistic fuzzy version of the prototype and NN classification method is proposed. This version combines the statistical and fuzzy approaches. Using *F*-operator with various values of the parameters α and β, dependent on various set of nearest neighbors distances, the values of $\mu_{\omega_i}(x_u)$, $\nu_{\omega_i}(x_u)$ and $\pi_{\omega_i}(x_u)$ have been adjusted and are getting more precise. As a result the confidence in the classification decision is enhanced.

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