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New operations over intuitionistic fuzzy index matrices

Veselina Bureva[1], Evdokia Sotirova[1] and Krassimir Atanassov[1,2]

[1] "Prof. Asen Zlatarov" University
1 "Prof. Yakimov" Blvd, Bourgas 8000, Bulgaria
e-mails: vesito_ka@abv.bg and esotirova@btu.bg

[2] Bioinformatics and Mathematical Modelling Department
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., Sofia 1113, Bulgaria
e-mail: krat@bas.bg

Abstract: In this paper, eight new operations over intuitionistic fuzzy index matrix are introduced and some of their basic properties are studied.
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1 Introduction

The concept of Index Matrix (IM) was discussed in a series of papers (see, e.g., [1, 2] and others) and it was extended to the concept of an Intuitionistic Fuzzy Index Matrix (IFIM) in [3].

Initially, in Section 2, we give some elements of Intuitionistic Fuzzy Logic (IFL, see, e.g., [4]). In Section 3, following [3], we give definition of IFIM and some operations over them.

Eight new operations over IFIMs are introduced in Section 4 and some of their properties are ddiscussed.

2 Short remarks on intuitionistic fuzzy logic

In IFL, the truth-value function V assigns to each proposition p two real numbers $\mu(p), \nu(p) \in [0, 1]$, called "degree of validity" and "degree of non-validity", which satisfy condition (see [**?**]):

$$\mu(p) + \nu(p) \leq 1.$$

If for propositions x and y, $V(x) = \langle a, b\rangle$ and $V(y) = \langle c, d\rangle$, where $a, b, c, d, a+b, c+d \in [0, 1]$, then

$$V(x) = V(y) \text{ if and only if } a = c \text{ and } b = d,$$

$$V(x) \leq V(y) \text{ if and only if } a \leq c \text{ and } b \geq d,$$

$$V(x) < V(y) \text{ if and only if } a \leq c \text{ and } b > d, \text{ or } a < c \text{ and } b \geq d,$$

$$V(x) \vee V(y) = V(x \vee y) = \langle \max(a, c), \min(b, d)\rangle,$$

$$V(x) \wedge V(y) = V(x \wedge y) = \langle \min(a, c), \max(b, d)\rangle.$$

When we have n variables $x_1, x_2, ..., x_n$ with truth-values $\langle a_i, b_i\rangle$ for $a_i, b_i, a_i + b_i \in [0, 1]$, where $i = 1, 2, ..., n$, then

$$V(\vee_i x_i) = \langle \max_i a_i, \min_i b_i\rangle,$$

$$V(\wedge_i x_i) = \langle \min_i a_i, \max_i b_i\rangle.$$

For the needs for Section 4, we introduce also operation "average value" for the truth-values of $n \geq 2$ variables, that is analogous of operation @, defined over the intuitionistic fuzzy sets (see [4]):

$$V(@_i x_i) = \langle \frac{1}{n}\sum_i a_i, \frac{1}{n}\sum_i b_i\rangle.$$

3 Short remarks on index matrices and intuitionistic fuzzy index matrices

Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. In [2], by IM with index sets K and L $(K, L \subset I)$, we denoted the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\},$ for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

In [2], different operations, relations and operators are defined over IMs.

In [3], by IFIM with the above mentioned index sets, we denoted the object:

$$[K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1}\rangle & \langle \mu_{k_1,l_2}, \nu_{k_1,l_2}\rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n}\rangle \\ k_2 & \langle \mu_{k_2,l_1}, \nu_{k_2,l_1}\rangle & \langle \mu_{k_2,l_2}, \nu_{k_2,l_2}\rangle & \dots & \langle \mu_{k_2,l_n}, \nu_{k_2,l_n}\rangle \\ \vdots & & & & \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1}\rangle & \langle \mu_{k_m,l_2}, \nu_{k_m,l_2}\rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n}\rangle \end{array},$$

where for every $1 \leq i \leq m, 1 \leq j \leq n$: $0 \leq \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \leq 1$.

For the IMs $A = [K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}], B = [P, Q, \{\langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle\}]$, operations that are analogous of the usual matrix operations of addition and multiplication are defined, as well as other specific ones.

(a) addition $A \oplus B = [K \cup P, L \cup Q, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}]$, where

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \\ \langle \max(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \min(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \text{and } v_w = l_j = q_s \in L \cap Q \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(b) termwise multiplication $A \otimes B = [K \cap P, L \cap Q, \langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}]$, where

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \min(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \max(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \text{and } v_w = l_j = q_s \in L \cap Q \end{cases}$$

(c) multiplication $A \odot B = [K \cup (P - L), Q \cup (L - P), \langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}]$, where

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - P \\ \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P - L \text{ and } v_w = q_s \in Q \\ \\ \langle \max\limits_{l_j = p_r \in L \cap P} (\min(\mu_{k_i,l_j}, \rho_{p_r,q_s})), & \text{if } t_u = k_i \in K \text{ and } v_w = q_s \in Q \\ \\ \min\limits_{l_j = p_r \in L \cap P} (\max(\nu_{k_i,l_j}, \sigma_{p_r,q_s})) \rangle, & \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(d) structural subtraction $A \ominus B = [K - P, L - Q, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}]$, where "–" is the set–theoretic difference operation and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, \text{ for } t_u = k_i \in K - P \text{ and } v_w = l_j \in L - Q.$$

(e) negation of an IFIM $\neg A = [K, L, \{\neg \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$, where $\neg$ is one of the above (or another) negations.

(f) termwise subtraction $A - B = A \oplus \neg B$.

Let the two IFIMs $A = [K, L, \{\langle a_{k,l}, b_{k,l}\rangle\}]$ and $B = [P, Q, \{\langle c_{p,q}, d_{p,q}\rangle\}]$ be given. We shall introduce the following (new) definitions where $\subset$ and $\subseteq$ denote the relations *"strong inclusion"* and *"weak inclusion"*.

(a) strict relation "inclusion about dimension"

$$A \subset_d B \text{ iff } ((K \subset P)\&(L \subset Q)) \vee (K \subseteq P)\&(L \subset Q) \vee (K \subset P)\&(L \subseteq Q))$$

$$\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle = \langle c_{k,l}, d_{k,l}\rangle).$$

(b) non-strict relation "inclusion about dimension"

$$A \subseteq_d B \text{ iff } (K \subseteq P)\&(L \subseteq Q)\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle = \langle c_{k,l}, d_{k,l}\rangle).$$

(c) strict relation "inclusion about value"

$$A \subset_v B \text{ iff } (K = P)\&(L = Q)\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle < \langle c_{k,l}, d_{k,l}\rangle).$$

(d) non-strict relation "inclusion about value"

$$A \subseteq_v B \text{ iff } (K = P)\&(L = Q)\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle \leq \langle c_{k,l}, d_{k,l}\rangle).$$

(e) strict relation "inclusion"

$$A \subset B \text{ iff } ((K \subset P)\&(L \subset Q)) \vee (K \subseteq P)\&(L \subset Q) \vee (K \subset P)\&(L \subseteq Q))$$

$$\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle < \langle c_{k,l}, d_{k,l}\rangle).$$

(f) non-strict relation "inclusion"

$$A \subseteq B \text{ iff } (K \subseteq P)\&(L \subseteq Q)\&(\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l}\rangle \leq \langle c_{k,l}, d_{k,l}\rangle).$$

4 Main results

Let the IFIM A be given and let $k_0 \notin K$ and $l_0 \notin L$ are two indices. Now, we introduce the following eight operations over it:

(a) max-row-aggregation

$$\rho_{max}(A, k_0) = \begin{array}{c|ccc} & l_1 & \ldots & l_n \\ \hline k_0 & \langle \max\limits_{1\leq i\leq m}(a_{k_i,l_1}), \min\limits_{1\leq i\leq m}(b_{k_i,l_1})\rangle & \ldots & \max\limits_{1\leq i\leq m}(a_{k_i,l_n}), \min\limits_{1\leq i\leq m}(b_{k_i,l_n})\rangle \end{array},$$

(b) min-row-aggregation

$$\rho_{min}(A, k_0) = \begin{array}{c|ccc} & l_1 & \ldots & l_n \\ \hline k_0 & \langle \min\limits_{1\leq i\leq m}(a_{k_i,l_1}), \max\limits_{1\leq i\leq m}(b_{k_i,l_1})\rangle & \ldots & \langle \min\limits_{1\leq i\leq m}(a_{k_i,l_n}), \max\limits_{1\leq i\leq m}(b_{k_i,l_n})\rangle \end{array},$$

(c) average-row-aggregation

$$\rho_{av}(A,k_0) = \begin{array}{c|ccc} & l_1 & \dots & l_n \\ \hline k_0 & \langle \frac{1}{m}\sum_{i=1}^{m} a_{k_i,l_1}, \frac{1}{m}\sum_{i=1}^{m} b_{k_i,l_1}\rangle & \dots & \langle \frac{1}{m}\sum_{i=1}^{m} a_{k_i,l_n}, \frac{1}{m}\sum_{i=1}^{m} b_{k_i,l_n}\rangle\rangle \end{array},$$

(d) max-column-aggregation

$$\sigma_{max}(A,l_0) = \begin{array}{c|c} & l_0 \\ \hline k_1 & \langle \max_{1\le j\le n}(a_{k_1,l_j}), \min_{1\le j\le n}(b_{k_1,l_j})\rangle \\ \vdots & \vdots \\ k_m & \langle \max_{1\le j\le n}(a_{k_m,l_j}), \min_{1\le j\le n}(b_{k_m,l_j})\rangle \end{array},$$

(e) min-column-aggregation

$$\sigma_{min}(A,l_0) = \begin{array}{c|c} & l_0 \\ \hline k_1 & \langle \min_{1\le j\le n}(a_{k_1,l_j}), \max_{1\le j\le n}(b_{k_1,l_j})\rangle \\ \vdots & \vdots \\ k_m & \langle \min_{1\le j\le n}(a_{k_m,l_j}), \max_{1\le j\le n}(b_{k_m,l_j})\rangle \end{array},$$

(f) average-column-aggregation

$$\sigma_{av}(A,l_0) = \begin{array}{c|c} & l_0 \\ \hline k_1 & \langle \frac{1}{n}\sum_{j=1}^{n} a_{k_1,l_j}, \frac{1}{n}\sum_{j=1}^{n} b_{k_1,l_j}\rangle \\ \vdots & \\ k_m & \langle \frac{1}{n}\sum_{j=1}^{n} a_{k_m,l_j}, \frac{1}{n}\sum_{j=1}^{n} b_{k_m,l_j}\rangle \end{array}.$$

We can see immediately, that for every IFIM A and for every index i:

(1) $\rho_{max}(\rho_{max}(A,i),i) = \rho_{max}(A,i)$,

(2) $\rho_{min}(\rho_{min}(A,i),i) = \rho_{min}(A,i)$,

(3) $\rho_{av}(\rho_{av}(A,i),i) = \rho_{av}(A,i)$,

(4) $\sigma_{max}(\sigma_{max}(A,i),i) = \sigma_{max}(A,i)$,

(5) $\sigma_{min}(\sigma_{min}(A,i),i) = \sigma_{min}(A,i)$,

(6) $\sigma_{av}(\sigma_{av}(A,i),i) = \sigma_{av}(A,i)$.

and for every two indices i and j:

(1) $\rho_{max}(\sigma_{max}(A,j),i) = \sigma_{max}(\rho_{max}(A,i),j)$,

(2) $\rho_{min}(\sigma_{min}(A,j),i) = \sigma_{min}(\rho_{min}(A,i),j)$,

(3) $\rho_{av}(\sigma_{av}(A, j), i) = \sigma_{av}(\rho_{av}(A, i), j)$.

The following assertion is valid.

Theorem. For every two IFIMs A and B and for every index i:

(1) $\rho_{max}(A \oplus B, i) \subseteq_v \rho_{max}(A, i) \oplus \rho_{max}(B, i)$,

(2) $\rho_{min}(A \oplus B, i) \supseteq_v \rho_{min}(A, i) \oplus \rho_{min}(B, i)$,

(3) $\rho_{av}(A \oplus B, i) = \rho_{av}(A, i) \oplus \rho_{av}(B, i)$,

(4) $\rho_{max}(A \otimes B, i) \supseteq_v \rho_{max}(A, i) \oplus \rho_{max}(B, i)$,

(5) $\rho_{min}(A \otimes B, i) \subseteq_v \rho_{min}(A, i) \oplus \rho_{min}(B, i)$,

(6) $\rho_{max}(A \ominus B, i) = \rho_{max}(A, i) \ominus \rho_{max}(B, i)$,

(7) $\rho_{min}(A \ominus B, i) = \rho_{min}(A, i) \ominus \rho_{min}(B, i)$,

(8) $\rho_{av}(A \ominus B, i) = \rho_{av}(A, i) \ominus \rho_{av}(B, i)$,

(9) $\sigma_{max}(A \oplus B, i) \subseteq_v \sigma_{max}(A, i) \oplus \sigma_{max}(B, i)$,

(10) $\sigma_{min}(A \oplus B, i) \supseteq_v \sigma_{min}(A, i) \oplus \sigma_{min}(B, i)$,

(11) $\sigma_{av}(A \oplus B, i) = \sigma_{av}(A, i) \oplus \sigma_{av}(B, i)$,

(12) $\sigma_{max}(A \otimes B, i) \supseteq_v \sigma_{max}(A, i) \oplus \sigma_{max}(B, i)$,

(14) $\sigma_{min}(A \otimes B, i) \subseteq_v \sigma_{min}(A, i) \oplus \sigma_{min}(B, i)$,

(15) $\sigma_{max}(A \ominus B, i) = \sigma_{max}(A, i) \ominus \sigma_{max}(B, i)$,

(16) $\sigma_{min}(A \ominus B, i) = \sigma_{min}(A, i) \ominus \sigma_{min}(B, i)$,

(17) $\sigma_{av}(A \ominus B, i) = \sigma_{av}(A, i) \ominus \sigma_{av}(B, i)$.

The operation $\odot$ does not enter relation with the new eight operations.

5 Conclusion

In future, we will study the connectins between the new eight operations and the rest operations and relations defined over IFIMs.

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