

A NEW GEOMETRICAL INTERPRETATION OF SOME CONCEPTS IN THE INTUITIONISTIC FUZZY LOGICS

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The Intuitionistic Fuzzy (IF) Logics are introduced in [1]. Some geometrical interpretations of the truth function, operations and operators over truth values in these logics are given in [2], [3], [4], [5] and [6]. In the present work we shall describe a new geometrical interpretation of these concepts.

Let S is a set of propositions and the functions μ , ν and π are respectively the degrees of validity, non-validity and uncertainty of a given proposition from S , i.e.

$$\mu, \nu, \pi: S \rightarrow [0,1]$$

$$\forall p \in S: \mu(p) + \nu(p) + \pi(p) = 1.$$

Let us consider the regular triangle MNP with side's length equal to $2/\sqrt{3}$. Let us denote by XX_M , XX_N and XX_P the perpendiculars from an arbitrary inside MNP point X to NP , MP and MN respectively. It is known from elementary geometry that

$$XX_M + XX_N + XX_P = 1.$$

Hence we can define a function $GI: S \rightarrow \{Z: Z \text{ is a point inside } MNP\}$ as

$$GI(p) = X \quad \text{iff} \quad \mu(p) = XX_M \wedge \nu(p) = XX_N \wedge \pi(p) = XX_P.$$

This function GI defines our geometric interpretation shown in **Fig 1**.

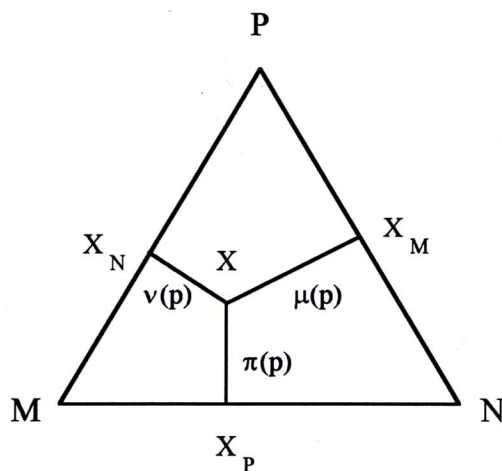


Fig 1

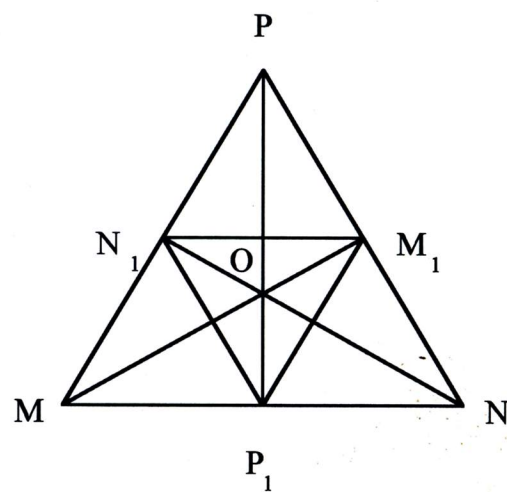


Fig 2

Some concepts in the IF Logics are defined in [1]. For example p is called IF true (false) iff $\mu(p) \geq 1/2$ ($\nu(p) \geq 1/2$); p is called IF tautology iff $\mu(p) \geq \nu(p)$. In our interpretation p is IF true, false or tautology iff $GI(p)$ belongs to triangles MP_1N_1 , P_1NM_1 and MP_1P respectively (see Fig 2).

We shall give geometrical interpretation of some operations defined over the truth values of propositions. In [1] the operations \neg , \wedge , \vee and \supset ("not", "and", "or" and "implies") are defined as follows: if $\mu(p) = a$, $\nu(p) = b$ and $\mu(q) = c$, $\nu(q) = d$ for some $p, q \in S$ then

$$\begin{aligned} \mu(\neg p) &= b & \nu(\neg p) &= a \\ \mu(p \wedge q) &= \min(a, c) & \nu(p \wedge q) &= \max(b, d) \\ \mu(p \vee q) &= \max(a, c) & \nu(p \vee q) &= \min(b, d) \\ \mu(p \supset q) &= \max(b, c) & \nu(p \supset q) &= \min(a, d). \end{aligned}$$

The interpretation of \neg is given in Fig 3 - $GI(p) = X$ and $GI(\neg p) = Y$ are symmetrical with respect to the median PP_1 . The interpretation of \wedge and \vee can be shown in Fig 4 - here the lines X_1X_4 and X_2X_3 are parallels to the side PN and the lines X_1X_3 and X_2X_4 - to the side MP . It is easy to see that $GI(p \vee q) = X_1$ and $GI(p \wedge q) = X_2$ in the cases $GI(p) = X_1$, $GI(q) = X_2$ and $GI(p) = X_3$, $GI(q) = X_4$. The interpretation of \supset can be presented by some combination of Fig 3 and Fig 4 because $p \supset q = (\neg p) \vee q$ - see Fig 5 ($GI(p) = X$, $GI(q) = Y$ and $GI(p \supset q) = Z$).

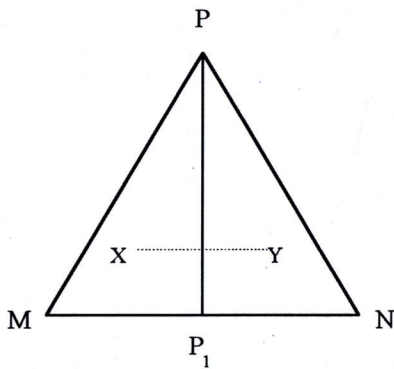


Fig 3

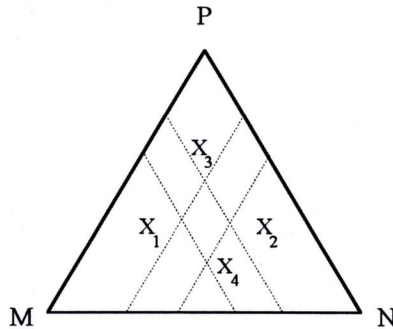


Fig 4

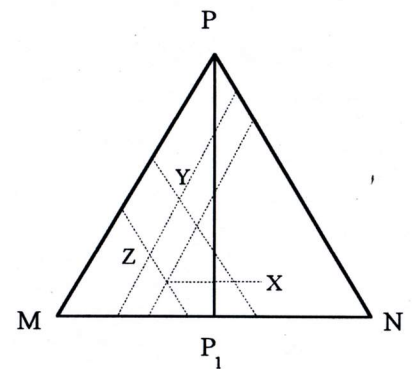


Fig 5

It can be described the geometrical interpretation of some other operations and of some other possible definitions of the above operations. It is interesting to obtain also geometrical

interpretations of the operators defined over truth values (for example see [7]). These problems can be objects of further works.

References

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