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ON THE GEOMETRICAL INTERPRETATIONS OF THE OPERATIONS OVER THE INTUITIONISTIC FUZZY SETS

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The concept of the Intuitionistic Fuzzy Set (IFS) is introduced in [1,2]. Some geometrical interpretations of the IFSs are discused in [3]. Here we shall show the basic geometrical interpretations of the operations, defined over the IFSs.

Let a set E be fixed. An IFS A^* in E is an object of the following form:

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

If

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),$$

then $\pi_A(x)$ is the degree of non-determinacy of the element $x \in E$ to the set A. In the case of ordinary fuzzy sets, $\pi_A(x) = 0$ for every $x \in E$.

For simplicity below we will write A instead of A^*.

The ordinary fuzzy sets have only one geometrical interpretation, while in [3] two interpretations of IFSs are given.

The (first) interpretation of the IFSs is similar to this for the ordinary fuzzy sets (see Fig. 1).

The basic (second) geometrical interpretation of the IFS (from [3]) is shown on Fig. 2., where the universe E and the figure F in the Euclidean plane with a Cartesian coordinate system are given.

Let $A \subset E$ be a fixed set. Then we can construct a function f_A from E to F such that if $x \in E$, then

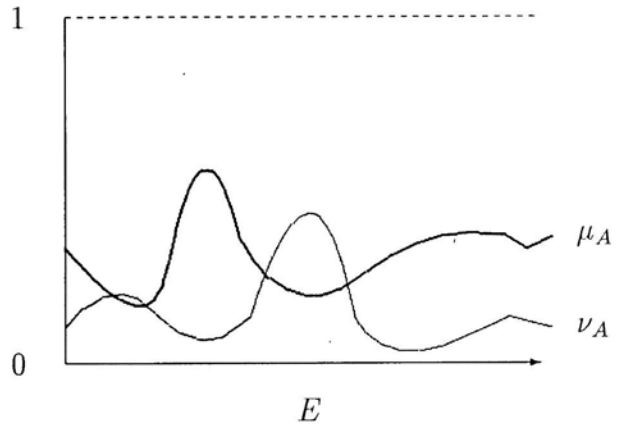


Fig. 1.

$$p = f_A(x) \in F,$$

the point p has coordinates $\langle a, b\rangle$ for which: $0 \leq a + b \leq 1$ and these coordinates are such that $a = \mu_A(x), b = \nu_A(x)$.

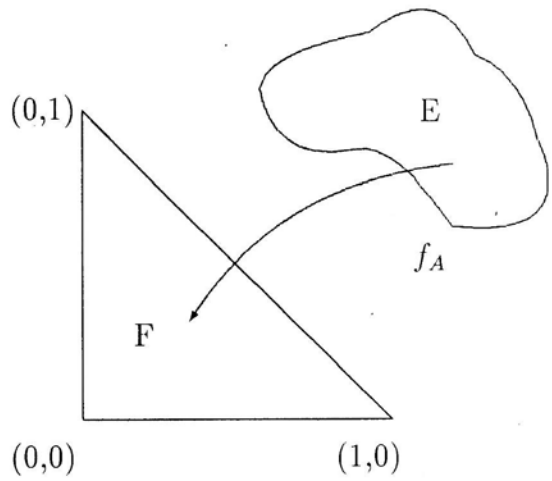


Fig. 2.

Following [1,2], for every two IFSs A and B the following operations can be defined (everywhere below by "iff" we will mean "if and only if"):

$$\begin{aligned}
A \cap B &= \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\
A \cup B &= \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\
A + B &= \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle \mid x \in E\}; \\
A.B &= \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\};
\end{aligned}$$

Geometrical interpretations of the operations in the sense of the interpretation from Fig. 2 are constructed in [3]. Now, we shall show third, fourth and fifth geometrical interpretations of the above operations in the sense of the interpretations from [4-6]. The analogous of the above (second) interpretation are shown on Fig. 3-5.

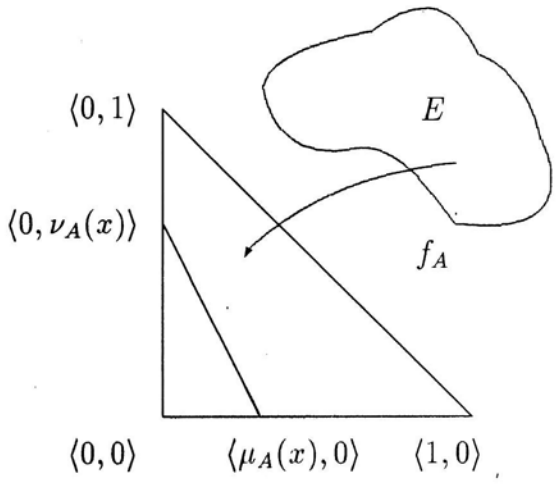
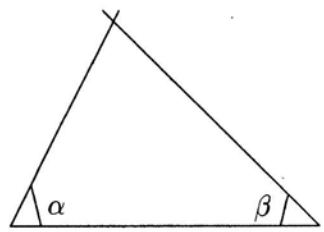


Fig. 3.



where $\alpha = \pi.\mu_A(x), \beta = \pi.\nu_A(x)$ and here $\pi = 3.14...$

Fig. 4.

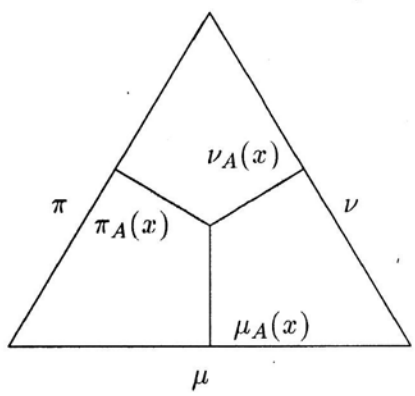


Fig. 5.

The third geometrical interpretations of the operations "$\cap$", '$\cup$", "$+$", "." are shown on Fig. 6-9.

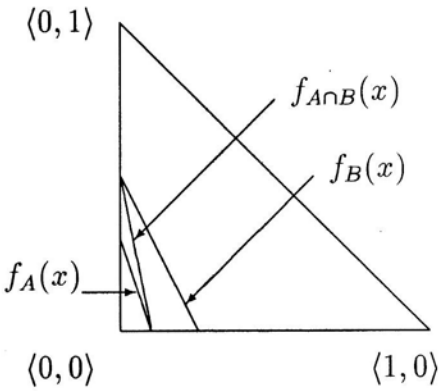


Fig. 6.

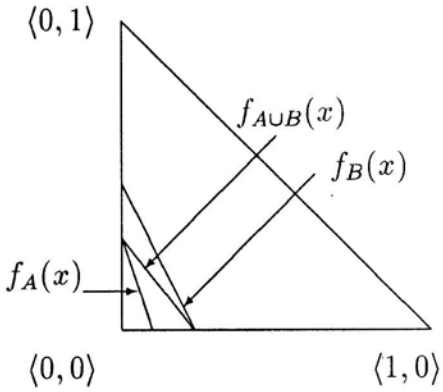


Fig. 7.

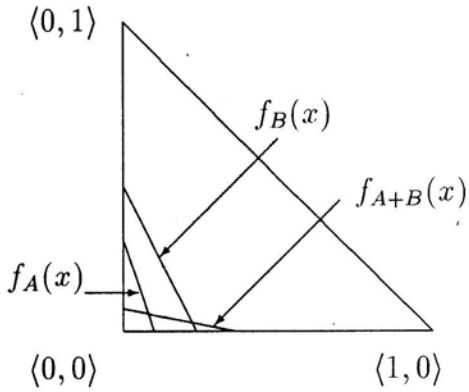


Fig. 8.

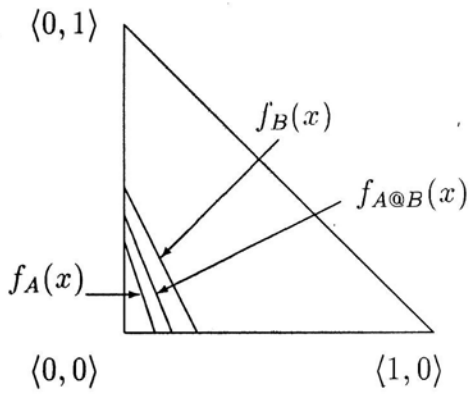


Fig. 9.

Let the fourth geometrical interpretations of the elements $x, y \in E$ about the IFS A be given on Fig. 10 and Fig. 11. Then the fourth geometrical interpretations of the operations "$\cap$", '$\cup$" are shown on Fig. 12 and Fig. 13.

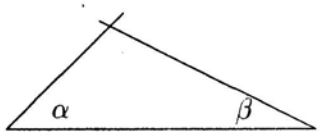


Fig. 10.

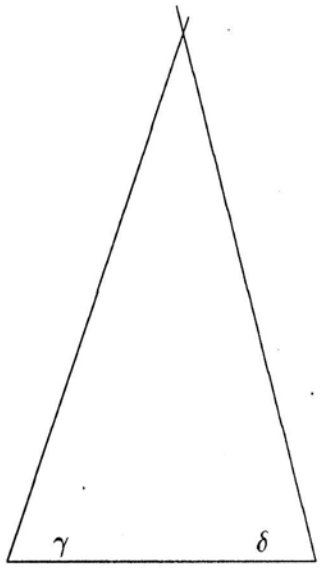


Fig. 11.

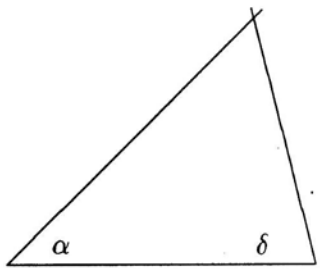


Fig. 12.

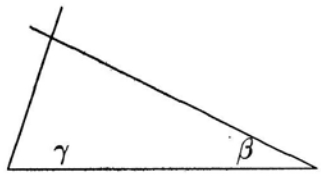


Fig. 13.

Finally, the fifth geometrical interpretations of the operations "$\cap$", '$\cup$', "$+$", "$\cdot$" are shown on Fig. 14-17.

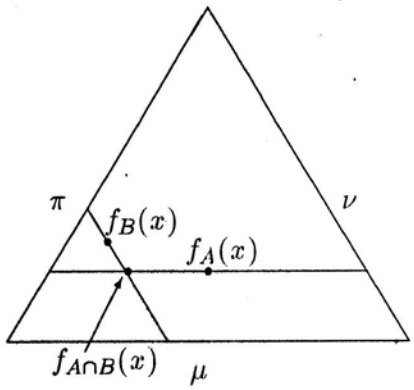


Fig. 14.

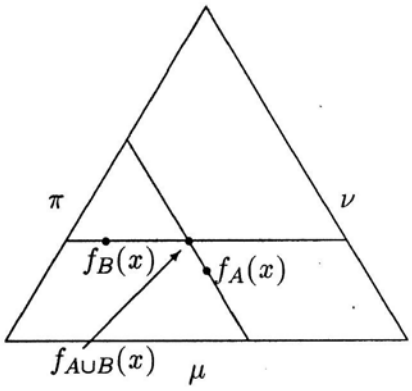


Fig. 15.

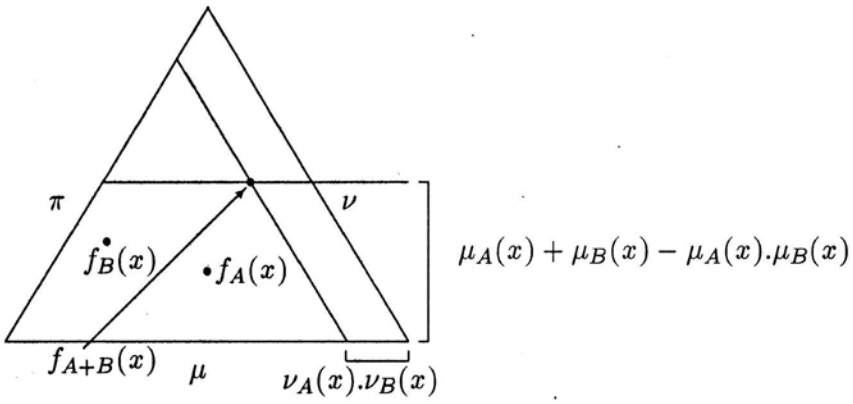


Fig. 16.

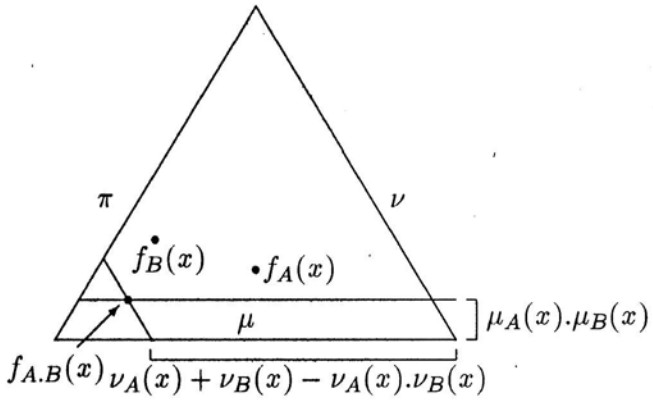


Fig. 17.

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