**NIFS 16 (2010), 3, 28-39**

# Intuitionistic fuzzy generalized semi-pre closed mappings

**R. Santhi and D. Jayanthi***
Department of Mathematics,
NGM College, Pollachi, Tamil Nadu, India.
E-mails: *santhifuzzy@yahoo.co.in*, *jayanthimaths@rediffmail.com*

**Abstract:**
In this paper we introduce intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. We investigate some of their properties. We also introduce intuitionistic fuzzy *M*-generalized semi-pre closed mappings as well as intuitionistic fuzzy *M*-generalized semi-pre open mappings. We provide the relation between intuitionistic fuzzy *M*-generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre closed mappings.

**Keywords:** Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings.

**Mathematics Subject Classification:** 03E72.

## 1 Introduction

Zadeh [14] introduced the notion of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings and study some of their properties. We also introduce intuitionistic fuzzy M-generalized semi-pre closed mappings as well as intuitionistic fuzzy *M*-generalized semi-pre open mappings. We provide the relation between intuitionistic fuzzy *M*-generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre closed mappings.

## 2 Preliminaries

**Definition 2.1:** [1] An *intuitionistic fuzzy set* (IFS in short) $A$ in $X$ is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS($x$), the set of all intuitionistic fuzzy sets in $X$.

**Definition 2.2:** [1] Let $A$ and $B$ be IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\} \text{ and } B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}.$$

Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$;
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle \mid x \in X\}$ are respectively the empty set and the whole set of $X$. For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$.

**Definition 2.3:** [3] An *intuitionistic fuzzy topology* (IFT for short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in $\tau$ is known as an *intuitionistic fuzzy open set* (IFOS in short) in $X$. The complement $A^c$ of an IFOS A in IFTS $(X, \tau)$ is called an *intuitionistic fuzzy closed set* (IFCS in short) in $X$.

**Definition 2.4:**[3] Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in $X$. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* are defined by

$\text{int}(A) = \cup \{G \mid G$ is an IFOS in $X$ and $G \subseteq A\}$.
$\text{cl}(A) = \cap \{K \mid K$ is an IFCS in $X$ and $A \subseteq K\}$

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$ [13].

**Definition 2.5:**[5] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS $(X, \tau)$ is said to be an

(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$
(iii) *intuitionistic fuzzy α closed set* (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A)) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

**Definition 2.6:**[13] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS $(X, \tau)$ is said to be

(i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS for short) if there exists an IFPCS $B$ such that $\text{int}(B) \subseteq A \subseteq B$.
(ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) $B$ such that $B \subseteq A \subseteq \text{cl}(B)$.

The family of all IFSPCSs (respectively, IFSPOSs) of an IFTS $(X,\tau)$ is denoted by IFSPC($X$) (respectively IFSPO($X$)). Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general [13].

Note that an IFS $A$ is an IFSPCS if and only if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ [7].

**Definition 2.7:**[7] Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the semi-pre interior and the semi-pre closure of $A$ are defined as
$\text{spint}(A) = \cup \{G \mid G$ is an IFSPOS in $X$ and $G \subseteq A\}$.
$\text{spcl}(A) = \cap \{K \mid K$ is an IFSPCS in $X$ and $A \subseteq K\}$.

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{spcl}(A^c) = [\text{spint}(A)]^c$ and $\text{spint}(A^c) = [\text{spcl}(A)]^c$ [7]

**Definition 2.8:**[10] An IFS $A$ is an

(i) *intuitionistic fuzzy regular closed set* (IFRCS for short) if $A = \text{cl int}(A)$
(ii) *intuitionistic fuzzy generalized closed set* (IFGCS for short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS.

**Definition 2.9:**[7] An IFS $A$ in an IFTS $(X,\tau)$ is said to be an *intuitionistic fuzzy generalized semi-pre closed set* (IFGSPCS for short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $(X, \tau)$.

Every IFCS, IFSCS, IFPCS, IFRCS, IF$\alpha$CS, IFSPCS is an IFGSPCS but the separate converses may not be true in general.[7] The family of all IFGSPCSs of an IFTS $(X,\tau)$ is denoted by IFGSPC(X).

**Definition 2.10:**[7] The complement $A^c$ of an IFGSPCS $A$ in an IFTS $(X,\tau)$ is called an *intuitionistic fuzzy generalized semi-pre open set* (IFGSPOS for short) in $X$.

Every IFOS, IFSOS, IFPOS, IFROS, IF$\alpha$OS, IFSPOS is an IFGSPOS but the separate converses may not be true in general.[7] The family of all IFGSPOSs of an IFTS $(X,\tau)$ is denoted by IFGSPO(X).

**Definition 2.11:**[5] Let f be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be *intuitionistic fuzzy continuous* (IF continuous for short) mapping if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

**Definition 2.12:**[7] If every IFGSPCS in $(X, \tau)$ is an IFSPCS in $(X, \tau)$, then the space can be Called as an *intuitionistic fuzzy semi- pre $T_{1/2}$ space* (IFSP$T_{1/2}$ space for short).

**Definition 2.13:**[8]A mapping $f : (X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy generalized semi-pre continuous* (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in $(X, \tau)$ for every IFCS V of $(Y, \sigma)$.

**Definition 2.14:**[9] A map $f : X \to Y$ is called an *intuitionistic fuzzy closed mapping* (IFCM for short) if $f(A)$ is an IFCS in $Y$ for each IFCS $A$ in $X$.

**Definition 2.15:**[5] A map $f: X \rightarrow Y$ is called an

(i) *intuitionistic fuzzy semi-open mapping* (IFSOM for short) if $f(A)$ is an IFSOS in $Y$ for each IFOS $A$ in $X$.

(ii) *intuitionistic fuzzy $\alpha$-open mapping* (IF$\alpha$OM for short) if $f(A)$ is an IF$\alpha$OS in $Y$ for each IFOS $A$ in $X$.

(iii) *intuitionistic fuzzy preopen mapping* (IFPOM for short) if $f(A)$ is an IFPOS in $Y$ for each IFOS $A$ in $X$.

**Definition 2.16:**[13] A mapping $f: (X, \tau) \rightarrow (Y,\sigma)$ is called an *intuitionistic fuzzy pre regular closed mapping* (IFPRCM for short) if $f(V)$ is an IFRCS in $(Y, \sigma)$ for every IFRCS $V$ of $(X, \tau)$.

**Definition 2.17:** [10] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an *intuitionistic fuzzy point* (IFP for short) in $X$.

Note that an IFP $c(\alpha, \beta)$ is said to belong to an IFS $A = \langle x, \mu_A, \gamma_A \rangle$ of $X$ denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \gamma_A$.

**Definition 2.18:**[9] Let $c(\alpha,\beta)$ be an IFP of an IFTS $(X, \tau)$. An IFS $A$ of $X$ is called an *intuitionistic fuzzy neighborhood* (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS $B$ in $X$ such that $c(\alpha, \beta) \in B \subseteq A$.

**Definition 2.19:**[8] Let $c(\alpha,\beta)$ be an IFP in $(X, \tau)$. An IFS $A$ of $X$ is called an *intuitionistic fuzzy semi neighborhood* (IFSN for short) of $c(\alpha,\beta)$ if there is an IFSPOS $B$ in $X$ such that $c(\alpha,\beta) \in B \subseteq A$.

**Theorem 2.20:** Let $(X, \tau)$ be an IFTS where $X$ is an IFSPT$_{1/2}$ space. An IFS $A$ is an IFGSPOS in
$X$ if and only if $A$ is an IFSN of $c(\alpha,\beta)$ for each IFP $c(\alpha,\beta) \in A$.

**Proof:** *Necessity:* Let $c(\alpha, \beta) \in A$. Let $A$ be an IFGSPOS in $X$. Since $X$ is an IFSPT$_{1/2}$ space, $A$ is an IFSPOS in $X$. Then clearly $A$ is an IFSN of $c(\alpha, \beta)$.

*Sufficiency:* Let $c(\alpha, \beta) \in A$. Since $A$ is an IFSN of $c(\alpha, \beta)$, there is an IFSPOS $B$ in $X$ such that $c(\alpha, \beta) \in B \subseteq A$. Now

$$A = \cup\{ c(\alpha, \beta) \mid c(\alpha, \beta) \in A\} \subseteq \cup \{B_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in A\} \subseteq A.$$

This implies $A = \cup\{B_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in A\}$. Since each B is an IFSPOS, A is an IFSPOS and hence an IFGSPOS in $X$.

**Theorem 2.21:** For any IFS $A$ in an IFTS $(X, \tau)$ where $X$ is an IFSPT$_{1/2}$ space, $A \in$IFGSPO($X$) if and only if for every IFP $c(\alpha, \beta) \in A$, there exists an IFGSPOS $B$ in $X$ such that

$$c(\alpha, \beta) \in B \subseteq A.$$

**Proof:** *Necessity:* If $A \in$ IFGSPO($X$), then we can take $B = A$ so that $c(\alpha, \beta) \in B \subseteq A$ for every IFP $c(\alpha, \beta) \in A$.

*Sufficiency:* Let $A$ be an IFS in $X$ and assume that there exists $B \in$ IFGSPO($X$) such that $c(\alpha, \beta) \in B \subseteq A$. Since $X$ is an IFSPT$_{1/2}$ space, $B$ is an IFSPOS of $X$. Then

$$A = \cup_{c(\alpha, \beta) \in A}\{c(\alpha, \beta)\} \subseteq \cup_{c(\alpha, \beta) \in A} B \subseteq A.$$

Therefore $A = \cup_{c(\alpha,\ \beta) \in A} B$ is an IFSPOS [13] and hence $A$ is an IFGSPOS in $X$. Thus $A \in$ IFGSPO($X$).

## 3 Intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings.

In this section we introduce intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. We study some of their properties

**Definition 3.1:** A map $f: X \to Y$ is called an *intuitionistic fuzzy generalized semi-pre closed mapping* (IFGSPCM for short) if $f(A)$ is an IFGSPCS in $Y$ for each IFCS $A$ in $X$.
For the sake of simplicity, we shall use the notation $A = \langle x,(\mu_a,\mu_b), (\nu_a,\nu_b)\rangle$ instead of $A = \langle\ x,\ (a/\mu_a,\ b/\mu_b),\ (a/\nu_a,\ b/\nu_b)\ \rangle$ in the following examples. Similarly we shall use the notation
$B = \langle y,(\mu_u,\mu_v), (\nu_u,\nu_v)\rangle$ instead of $B = \langle\ y,\ (u/\mu_u,\ v/\mu_v),\ (u/\nu_u,\ v/\nu_v)\ \rangle$ in the following examples.

**Example 3.2:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle,\ G_2 = \langle\ y, (0.3_u, 0.4_v), (0.7_u, 0.6_v) \rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGSPCM.

**Theorem 3.3:** Every IFCM is an IFGSPCM but not conversely.
**Proof:** Let $f: X \to Y$ be an IFCM. Let $A$ be an IFCS in $X$. Then $f(A)$ is an IFCS in $Y$. Since every IFCS is an IFGSPCS, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.

**Example 3.4:** In Example 3.2 $f$ is an IFGSPCM but not an IFCM, since $G_1^c = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ is an IFCS in $X$, but $f(G_1^c) = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is not an IFCS in $Y$, since $cl(f(G_1^c)) = G_2^c = f(G_1^c)$

**Theorem 3.5:** Every IF$\alpha$CM is an IFGSPCM but not conversely.

**Proof:** Let $f: X \to Y$ be an IF$\alpha$CM. Let $A$ be an IFCS in $X$. Then $f(A)$ is an IF$\alpha$CS in $Y$. Since every IF$\alpha$CS is an IFGSPCS, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.
**Example 3.6:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle\ x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle,\ G_2 = \langle\ y, (0.8_u, 0.7_v), (0.2_u, 0.3_v) \rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGSPCM but not an IF$\alpha$CM. Since $G_1^c$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is not an IF$\alpha$CS in $Y$, since $cl(int(cl(f(G_1^c)))) = 1_\sim \subseteq f(G_1^c)$

**Theorem 3.7:** Every IFSCM is an IFGSPCM but not conversely.

**Proof:** Let $f : X \to Y$ be an IFSCM. Let $A$ be an IFCS in $X$. Then $f(A)$ is an IFSCS in $Y$. Since every IFSCS is an IFGSPCS, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.

**Example 3.8:** In Example 3.6, $f$ is an IFGSPCM but not an IFSCM, since Since $G_1^c$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ is not an IFSCS in $Y$, since

$$\text{int}(\text{cl}(f(G_1^c))) = 1_\sim \subseteq f(G_1^c).$$

**Theorem 3.9:** Every IFPCM is an IFGSPCM but not conversely.
**Proof:** Let $f : X \to Y$ be an IFPCM. Let $A$ be an IFCS in $X$. Then $f(A)$ is an IFPCS in $Y$. Since every IFPCS is an IFGSPCS, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.

**Example 3.10:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle, G_2 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\}.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGSPCM but not an IFPCM, since $f(G_1^c)$ is an IFCS in $Y$ but not an IFPCS in $Y$, since $\text{cl}(\text{int}(f(G_1^c))) \subseteq G_2^c \subseteq f(G_1^c)$.

**Definition 3.11:** A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy M-generalized semi-pre closed mapping* (IFMGSPCM, for short) if $f(A)$ is an IFGSPCS in $Y$ for every IFGSPCS $A$ in $X$.
**Example 3.12:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle, G_2 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v) \rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFMGSPCM.

**Theorem 3.13 :** Every IFMGSPCM is an IFGSPCM but not conversely.
**Proof:** Let $f : X \to Y$ be an IFMGSPCM. Let $A$ be an IFCS in $X$. Then $A$ is an IFGSPCS in $X$. By hypothesis $f(A)$ is an IFGSPCS in $Y$. Therefore $f$ is an IFGSPCM.

**Example 3.14:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and

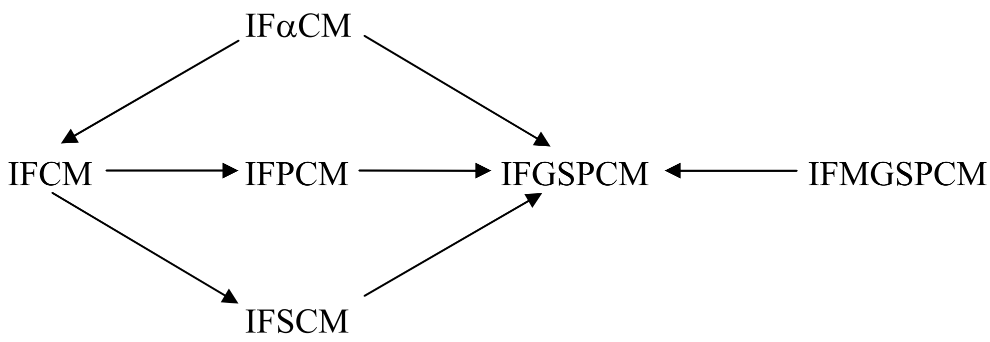
$$G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle,$$
$$G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$$

and

$$G_3 = \langle y, (0.7_u, 08_v), (0.3_u, 0.2_v) \rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IFGSPCM but not an IFMGSPCM. Since $A = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is IFGSPCS in $X$ but $f(A) = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)\rangle$ is not an IFGSPCS in $Y$, since $f(A) \subseteq G_3$ but $\text{spcl}(f(A)) = 1_\sim \subseteq G_3$.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.




The reverse implications are not true in general in the above diagram.

**Theorem 3.15:** Let $f : X \rightarrow Y$ be a mapping. Then the following are equivalent if $Y$ is an IFSP$T_{1/2}$ space

(i) $f$ is an IFGSPCM

(ii) $\text{spcl}(f(A)) \subseteq f(\text{cl}(A))$ for each IFS $A$ of $X$.

**Proof:** *(i) ⇒ (ii)* Let $A$ be an IFS in $X$. Then $\text{cl}(A)$ is an IFCS in $X$. (i) implies that $f(\text{cl}(A))$ is an IFGSPCS in $Y$. Since $Y$ is an IFSP$T_{1/2}$ space, $f(\text{cl}(A))$ is an IFSPCS in $Y$. Therefore $\text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Now $\text{spcl}(f(A)) \subseteq \text{spcl}(f(\text{cl}(A))) = f(\text{cl}(A))$. Hence $\text{spcl}(f(A)) \subseteq f(cl(A))$ for each IFS $A$ of $X$.

*(ii) ⇒ (i)* Let $A$ be any IFCS in $X$. Then $\text{cl}(A) = A$. *(ii)* implies that

$$\text{spcl}(f(A)) \subseteq f(\text{cl}(A)) = f(A).$$

But $f(A) \subseteq \text{spcl}(f(A))$. Therefore $\text{spcl}(f(A)) = f(A)$. This implies $f(A)$ is an IFSPCS in $Y$. Since every IFSPCS is an IFGSPCS, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.

**Theorem 3.16:** Let $f : X \rightarrow Y$ be a bijection. Then the following are equivalent if $Y$ is an IFSP$T_{1/2}$ space

(i) $f$ is an IFGSPCM

(ii) $\text{spcl}(f(A)) \subseteq f(\text{cl}(A))$ for each IFS $A$ of $X$

(iii) $f^{-1}(\text{spcl}(B)) \subseteq \text{cl}(f^{-1}(B))$ for every IFS $B$ of $Y$.

**Proof:** *(i) ⇔ (ii)* is obvious from Theorem 3.15.

*(ii) ⇒ (iii)* Let $B$ be an IFS in $Y$. Then $f^{-1}(B)$ is an IFS in $X$. Since $f$ is onto, $\text{spcl}(B) = \text{spcl}(f(f^{-1}(B)))$ and (ii) implies $\text{spcl}(f(f^{-1}(B))) \subseteq f(\text{cl}(f^{-1}(B)))$. Therefore $\text{spcl}(B) \subseteq f(\text{cl}(f^{-1}(B)))$. Now $f^{-1}(\text{spcl}(\text{B})) \subseteq f^{-1}(f(\text{cl}(f^{-1}(B)))$. Since $f$ is one to one, $f^{-1}(\text{spcl}(B)) \subseteq \text{cl}(f^{-1}(B))$.

*(iii) ⇒ (ii)* Let $A$ be any IFS of $X$. Then $f(A)$ is an IFS of $Y$. Since $f$ is one to one, *(iii)* implies that $f^{-1}(\text{spcl}(f(A)) \subseteq \text{cl}(f^{-1}f(A)) = \text{cl}(A)$. Therefore $f(f^{-1}(\text{spcl}(f(A)))) \subseteq f(\text{cl}(A))$. Since $f$ is onto $\text{spcl}(f(A)) = f(f^{-1}(\text{spcl}(f(A)))) \subseteq f(\text{cl}(A))$.

**Theorem 3.17:** Let $f : X \rightarrow Y$ be an IFGSPCM. Then for every IFS $A$ of $X$, $f(\text{cl}(A))$ is an IFGSPCS in $Y$.

**Proof:** Let $A$ be any IFS in $X$. Then $\text{cl}(A)$ is an IFCS in $X$. By hypothesis $f(\text{cl}(A))$ is an IFGSPCS in $X$.

**Theorem 3.18:** Let $f : X \to Y$ be an IFGSPCM where $Y$ is an IFSPT$_{1/2}$space, then $f$ is an IFCM if every IFSPCS is an IFCS in $Y$.
**Proof:** Let $f$ be an IFGSPCM. Then for every IFCS $A$ in $X$, $f(A)$ is an IFGSPCS in Y. Since Y is an IFSPT$_{1/2}$ space, f(A) is an IFSPCS in Y and by hypothesis f(A) is an IFCS in Y. Hence f is an IFCM.

**Theorem 3.19:** Let $f : X \to Y$ be an IFGSPCM where $Y$ is an IFSPT$_{1/2}$ space. Then $f$ is an IFPRCM if every IFSPCS is an IFRCS in $Y$.
**Proof:** Let $A$ be an IFRCS in $X$. since every IFRCS is an IFCS, $A$ is an IFCS in $X$. By hypothesis $f(A)$ is an IFGSPCS in $Y$. Since $Y$ is an IFSPT$_{1/2}$ space, $f(A)$ is an IFSPCS in $Y$ and hence is an IFRCS in $Y$, by hypothesis. This implies that $f(A)$ is an IFPRCM.

**Theorem 3.20:** If every IFS is an IFCS, then an IFGSPCM is an IFGSP continuous mapping.
**Proof:** Let $A$ be an IFCS in $Y$. Then $f^{-1}(A)$ is an IFS in $X$. Therefore $f^{1}(A)$ is an IFCS in $X$. Since every IFCS is an IFGSPCS, $f^{-1}(A)$ is an IFGSPCS in $X$. This implies that $f$ is an IFGSP continuous mapping.

**Theorem 3.21:** Let $A$ be an IFGCS in $X$. An onto mapping $f : X \to Y$ is both IF continuous mapping and IFGSPCM, then $f(A)$ is an IFGSPCS in $Y$.
**Proof:** Let $f(A) \subseteq U$ where $U$ is an IFOS in $Y$, then $A \subseteq f^{-1}(U)$ where $f^{-1}(U)$ is an IFOS in $X$, by hypothesis. Since $A$ is an IFGCS, $cl(A) \subseteq f^{-1}(U)$ in $X$. Hence, $f(cl(A)) \subseteq f(f^{-1}(U)) = U$. But $f(cl(A))$ is an IFGSPCS in $Y$, since $cl(A)$ is an IFCS in $X$ and $f$ is an IFGSPCM. We have therefore $spcl(f(cl(A))) \subseteq U$. Now $spcl(f(A)) \subseteq spcl(f(cl(A))) \subseteq U$. Hence $f(A)$ is an IFGSPCS in $Y$.

**Theorem 3.22:** A mapping $f : X \to Y$ is an IFGSPCM if and only if for every IFS $B$ of $Y$ and for every IFOS $U$ containing $f^{-1}(B)$, there is an IFGSPOS $A$ of $Y$ such that $B \subset A$ and $f^{-1}(A) \subset U$.
**Proof:** *Necessity:* Let $B$ be any IFS in $Y$. Let $U$ be an IFOS in $X$ such that $f^{-1}(B) \subset U$, then $U^c$ is an IFCS in $X$. By hypothesis $f(U^c)$ is an IFGSPCS in $Y$. Let $A = (f(U^c))^c$, then $A$ is an IFGSPOS in $Y$ and $B \subset A$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subset U$.
*Sufficiency:* Let $A$ be any IFCS in $X$, then $A^c$ is an IFOS in $X$ and $f^{-1}(f(A^c))^c \subset A^c$. By hypothesis there exists an IFGSPOS $B$ in $Y$ such that $f(A^c) \subset B$ and $f^{-1}(B) \subset A^c$. therefore $A \subset (f^{-1}(B))^c$. Hence $B^c \subset f(A) \subset f(f^{-1}(B))^c \subset B^c$. This implies that $f(A) = B^c$. Since $B^c$ is an IFGSPCS in $Y$, $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFGSPCM.

**Theorem 3.23:** If $f : X \to Y$ is an IFCM and $g$: $Y \to Z$ is an IFGSPCM, then $g \circ f$ is an IFGSPCM.
**Proof:** Let $A$ be an IFCS in $X$, then $f(A)$ is an IFCS in $Y$, Since $f$ is an IFCM. Since $g$ is an IFGSPCM, $g(f(A))$ is an IFGSPCS in $Z$. Therefore $g \circ f$ is an IFGSPCM.

**Theorem 3.24:** Let $f : X \to Y$ be a bijective map where $Y$ is an IFSPT$_{1/2}$ space. Then the following are equivalent.
(i) $f$ is an IFGSPCM
(ii) $f(B)$ is an IFGSPOS in $Y$ for every IFOS $B$ in $X$.
(iii) $f(int(B)) \subseteq cl(int(cl(f(B))))$ for every IFS $B$ in $X$.
**Proof:** *(i) ⇒ (ii)* is obvious.

*(ii) ⇒ (iii)* Let $B$ be an IFS in $X$, then int($B$) is an IFOS in $X$. By hypothesis $f$(int($B$)) is an IFGSPOS in $Y$. Since $Y$ is an IFSPT$_{1/2}$ space, $f$(int($B$)) is an IFSPOS in $Y$. Therefore $f(\text{int}(B)) = \text{spint}(f(\text{int}(B))) = f(\text{int}(B)) \cap \text{cl}(\text{int}(\text{cl}(f(\text{int}(B))))) \subseteq \text{cl}(\text{int}(\text{cl}(f(\text{int}(B))))) \subseteq \text{cl}(\text{int}(\text{cl}(f(B))))$.

*(iii) ⇒ (i)* let $A$ be an IFCS in $X$. Then $A^c$ is an IFOS in $X$. By hypothesis, $f(\text{int}(A^c)) = f(A^c) \subseteq \text{cl}(\text{int}(\text{cl}(f(A^c))))$. That is $\text{int}(\text{cl}(\text{int}(f(A)))) \subseteq f(A)$. This implies $f(A)$ is an IFSPCS in $Y$ and hence an IFGSPCS in $Y$. Therefore $f$ is an IFGSPCM.

**Theorem 3.25:** Let $f : X \to Y$ be a bijective map where $Y$ is an IFSPT$_{1/2}$ space. Then the following are equivalent.

(i) $f$ is an IFGSPCM

(ii) $f(B)$ is an IFGSPCS in $Y$ for every IFCS $B$ in $X$.

(iii) $\text{int}(\text{cl}(\text{int}(f(B)))) \subseteq f(\text{cl}(B))$ for every IFS $B$ in $X$.

**Proof:** *(i) ⇒ (ii)* is obvious.

*(ii) ⇒ (iii)* Let $B$ be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFGSPCS in $Y$. Since Y is an IFSPT$_{1/2}$ space, f(cl(B)) is an IFSPCS in Y. Therefore $\text{f(cl(B))} = \text{spcl(f(cl(B)))} = \text{f(cl(B))} \cup \text{int(cl(int(f(cl(B)))))} \supseteq \text{int(cl(int(f(cl(B)))))} \supseteq \text{int(cl(int(f(B))))}$.

*(iii) ⇒ (i)* let A be an IFCS in X. By hypothesis, $\text{f(cl(A))} = \text{f(A)} \subseteq \text{int(cl(int(f(A))))}$. This implies f(A) is an IFSPCS in Y and hence an IFGSPCS in Y. Therefore f is an IFGSPCM.

**Definition 3.26:** A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy open mapping* (IFOM for short) if f(A) is an IFOS in Y for each IFOS A in X.

**Definition 3.27:** A mapping $f : X \to Y$ is said to be an *intuitionistic fuzzy generalized semi-pre open mapping* (IFGSPOM for short) if f(A) is an IFGSPOS in Y for each IFOS in X.

**Theorem 3.28:** If $f : X \to Y$ is a mapping. Then the following are equivalent if Y is an IFSPT$_{1/2}$ space

(i) f is an IFGSPOM

(ii) $\text{f(int(A))} \subseteq \text{spint(f(A))}$ for each IFS A of X

(iii) $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$ for every IFS B of Y.

**Proof:** *(i) ⇒ (ii)* Let f be an IFGSPOM. Let A be any IFS in X. Then int(A) is an IFOS in X. (i) implies that f(int(A)) is an IFGSPOS in Y. Since Y is an IFSPT$_{1/2}$ space, f(intA)) is an IFSPOS in Y. Therefore $\text{spint(f(int(A)))} = \text{f(int(A))} \subseteq \text{f(A)}$. Now $\text{f(int(A))} = \text{spint(f(int(A)))} \subseteq \text{spint(f(A))}$

*(ii) ⇒ (iii)* Let B be any IFS in Y. Then $f^{-1}(B)$ is an IFS in X. (ii) implies that

$$f(\text{int}(f^{-1}(B))) \subseteq \text{spint}(f(f^{-1}(B))) = \text{spint}(B).$$

Now $\text{int}(f^{-1}(B)) \subseteq f^{-1}(f(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(\text{spint}(B))$

*(iii) ⇒ (i)* Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. *(iii)* implies that $\text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. Now $A = \text{int}(A) \subseteq \text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. Therefore $f(A) \subseteq f(f^{-1}(\text{spint}(f(A)))) = \text{spint}(f(A)) \subseteq f(A)$. This implies spint(f(A)) = $f(A)$. Hence $f(A)$ is an IFSPOS in $Y$. Since every IFSPOS is an IFGSPOS, $f(A)$ is an IFGSPOS in $Y$. Thus $f$ is an IFGSPOM.

**Theorem 3.29:** A mapping $f : X \to Y$ is an IFGSPOM if $f(\text{spint}(A)) \subseteq \text{spint}(f(A))$ for every $A \subseteq X$

**Proof:** Let $A$ be an IFOS in $X$. Then $\text{int}(A) = A$. Now $f(A) = f(\text{int}(A)) \subseteq f(\text{spint}(A)) \subseteq \text{spint}(f(A))$, by hypothesis. But $\text{spint}(f(A)) \subseteq f(A)$. Therefore $f(A)$ is an IFSPOS in $X$. That is $f(A)$ is an IFGSPOS in $X$. Hence $f$ is an IFGSPOM.

**Theorem 3.30:** A mapping $f : X \to Y$ is an IFGSPOM if and only if $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$ for every $B \subseteq Y$, where $Y$ is an $\text{IFSPT}_{1/2}$ space.

**Proof:** *Necessity:* Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$ and $\text{int}(f^{-1}(B))$ is an IFOS in $X$. By hypothesis, $f(\text{int}(f^{-1}(B)))$ is an IFGSPOS in $Y$. Since $Y$ is an $\text{IFSPT}_{1/2}$ space, $f(\text{int}(f^{-1}(B)))$ is an IFSPOS in $Y$. Therefore $f(\text{int}(f^{-1}(B))) = \text{spint}(f(\text{int}(f^{-1}(B)))) \subseteq \text{spint}(B)$. This implies $\text{int}(f^{-1}(B)) \subseteq f^{-1}(\text{spint}(B))$.

*Sufficiency:* Let $A$ be an IFOS in $X$. Therefore $\text{int}(A) = A$. Then $f(A) \subseteq Y$. By hypothesis $\text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A)))$. That is

$$\text{int}(A) \subseteq \text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A))).$$

Therefore $A \subseteq f^{-1}(\text{spint}(f(A)))$. This implies $f(A) \subseteq \text{spint}(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFSPOS in $Y$ and hence an IFGSPOS in $Y$. Thus $f$ is an IFGSPOM.

**Theorem 3.31:** Let $f : X \to Y$ be an onto mapping where $Y$ is an $\text{IFSPT}_{1/2}$ space. Then $f$ is an IFGSPOM if and only if for any IFP $c(\alpha, \beta) \in Y$ and for any IFN $B$ of $f^{-1}(c(\alpha, \beta))$, there is an IFSN $A$ of $c(\alpha, \beta)$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$.

**Proof:** *Necessity:* Let $c(\alpha, \beta) \in Y$ and let $B$ be an IFN of $f^{-1}(c(\alpha, \beta))$. Then there is an IFOS $C$ in $X$ such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq B$. Since $f$ is an IFGSPCM, $f(C)$ is an IFGSPOS in $Y$. Since $Y$ is an $\text{IFSPT}_{1/2}$ space, $f(C)$ is an IFSPOS in $Y$ and

$$c(\alpha, \beta) \in f(f^{-1}(c(\alpha, \beta))) \subseteq f(C) \subseteq f(B).$$

Put $A = f(C)$. Then $A$ is an IFSN of $c(\alpha, \beta)$ and $c(\alpha, \beta) \in A \subseteq f(B)$. Thus $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$. That is $f^{-1}(A) \subseteq B$.

*Sufficiency:* Let $B \subseteq X$ be an IFOS. If $f(B) = 0_\sim$ then there is nothing to prove. Suppose that $c(\alpha, \beta) \in f(B)$. This implies $f^{-1}(c(\alpha, \beta)) \in B$. Then $B$ is an IFN of $f^{-1}(c(\alpha, \beta))$. By hypothesis there is an IFSN $A$ of $c(\alpha, \beta)$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFSPOS $C$ in $Y$ such that

$$c(\alpha, \beta) \in C \subseteq A = = f(f^{-1}(A)) \subseteq f(B).$$

Hence $f(B) = \cup\{c(\alpha, \beta) \mid c(\alpha, \beta) \in f(B\} \subseteq \cup\{C_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in f(B)\} \subseteq f(B)$. Thus $f(B) = \cup\{C_{c(\alpha, \beta)} \mid c(\alpha, \beta) \in f(B)\}$. Since each $C$ is an IFSPOS, $f(B)$ is also an IFSPOS and hence is an IFGSPOS in $Y$. Therefore $f$ is an IFGSPOM.

**Theorem 3.32:** If $f : X \to Y$ is a mapping, then the following are equivalent.

(i) $f$ is an IFMGSPCM

(ii) $f(A)$ is an IFGSPCS in $Y$ for every IFGSPCS A in $X$

(iii) $f(A)$ is an IFGSPOS in $Y$ for every IFGSPOS A in $X$

**Proof:** *(i) $\Leftrightarrow$ (ii)* is obvious from the Definition 3.11.

*(ii) $\Rightarrow$ (iii)* Let $A$ be an IFGSPOS in $X$ Then $A^c$ is an IFGSPCS in $X$. By hypothesis, $f(A^c)$ is an IFGSPCS in $Y$. That is $f(A)^c$ is an IFGSPCS in $Y$ and hence $f(A)$ is an IFGSPOS in $Y$.

*(iii) ⇒ (i)* Let $A$ be an IFGSPCS in $X$ Then $A^c$ is an IFGSPOS in $X$. By hypothesis, $f(A^c)$ is an IFGSPOS in $Y$. That is $f(A)^c$ is an IFGSPOS in $Y$ and hence $f(A)$ is an IFGSPCS in $Y$. Hence $f$ is an IFMGSPCM.

**Theorem 3.33:** Let $f : X \to Y$ be a bijective mapping, where $X$ is an $IFSPT_{1/2}$ space. Then the following are equivalent.

(i) $f$ is an IFMGSPCM

(ii) for each IFP $c(\alpha, \beta) \in Y$ and every IFSN $A$ of $f^{-1}(c(\alpha, \beta))$, there exists an IFGSPOS $B$ in $Y$ such that $c(\alpha, \beta) \in B \subseteq f(A)$.

(iii) for each IFP $c(\alpha, \beta) \in Y$ and every IFSN $A$ of $f^{-1}(c(\alpha, \beta))$, there exists an IFGSPOS $B$ in $Y$ such that $c(\alpha, \beta) \in B$ and $f^{-1}(B) \subseteq A$.

**Proof:** *(i) ⇒ (ii)* Let $c(\alpha, \beta) \in Y$ and $A$ the IFSN of $f^{-1}(c(\alpha, \beta))$. Then there exists an IFSPOS $C$ in $X$ such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq A$. Since every IFSPOS is an IFGSPOS, $C$ is an IFGSPOS in $X$. Then by hypothesis, $f(C)$ is an IFGSPOS in $Y$. Now $c(\alpha,\beta) \in f(C) \subseteq f(A)$. Put $B = f(C)$. This implies $c(\alpha, \beta) \in B \subseteq f(A)$.

*(ii) ⇒ (iii)* Let $c(\alpha, \beta) \in Y$ and $A$ the IFSN of $f^{-1}(c(\alpha, \beta))$. Then there exists an IFSPOS $C$ in $X$ such that $f^{-1}(c(\alpha, \beta)) \in C \subseteq A$. Since every IFSPOS is an IFGSPOS, $C$ is an IFGSPOS in $X$. Then by hypothesis, $f(C)$ is an IFGSPOS in $Y$. Now

$$c(\alpha, \beta) \in f(C) \subseteq f(A).$$

Put $B = f(C)$. This implies $c(\alpha, \beta) \in B \subseteq f(A)$. Now $f^{-1}(B) \subseteq f^{-1}(f(A)) \subseteq A$. That is $f^{-1}(B) \subseteq A$.

*(iii) ⇒ (i)* Let $A$ be an IFGSPOS in $X$. Since $X$ is an $IFSPT_{1/2}$ space, $A$ is an IFSPOS in $X$. Let $c(\alpha, \beta) \in Y$ and $f^{-1}(c(\alpha, \beta)) \in A$. That is $c(\alpha, \beta) \in f(A)$. This implies $A$ is an IFSN of $f^{-1}(c(\alpha, \beta))$. Then by hypothesis, there exists an IFGSPOS $B$ in $Y$ such that $c(\alpha, \beta) \in B$ and $f^{-1}(B) \subseteq A$. Hence by Theorem 2.21, $f(A)$ is an IFGSPOS in $Y$. Therefore $f$ is an IFMGSPCM

**Theorem 3.34:** If $f : X \to Y$ *is* a bijective mapping, then the following are equivalent.

(i) $f$ is an IFMGSPCM

(ii) $f(A)$ is an IFGSPOS in $Y$ for every IFGSPOS $A$ in $X$

(iii) for every IFP $c(\alpha, \beta) \in Y$ and for every IFGSPOS $B$ in $X$ such that $f^{-1}(c(\alpha, \beta)) \in B$, there exists an IFGSPOS $A$ in $Y$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B$.

**Proof:** *(i) ⇒ (ii)* is obvious by Theorem 3.32.

*(ii) ⇒ (iii)* Let $c(\alpha, \beta) \in Y$ and let $B$ be an IFGSPOS in $X$ such that $f^{-1}(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f(B)$. By hypothesis, $f(B)$ is an IFGSPOS in $Y$. Let $A = f(B)$. Therefore $c(\alpha, \beta) \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

*(iii) ⇒ (i)* Let $B$ be an IFGSPCS in $X$. Then $B^c$ is an IFGSPOS in $X$. Let $c(\alpha, \beta) \in Y$ and $f^{1}(c(\alpha, \beta)) \in B^c$. This implies $c(\alpha, \beta) \in f(B^c)$. By hypothesis there exists an IFGSPOS $A$ in $Y$ such that $c(\alpha, \beta) \in A$ and $f^{-1}(A) \subseteq B^c$. Put $A = f(B^c)$. Then $c(\alpha, \beta) \in f(B^c)$ and $A = f(f^{1}(B^c)) \subseteq f(B^c)$. Hence by Theorem 2.21, $f(B^c)$ is an IFGSPOS in $Y$. Therefore $f(B)$ is an IFGSPCS in $Y$. Thus $f$ is an IFMGSPCM.

**Theorem 3.35:** If $f : X \to Y$ is a bijective mapping, then the following are equivalent.

(i) $f$ is an IFMGSPCM

(ii) $f(A)$ is an IFGSPOS in $Y$ for every IFGSPOS $A$ in $X$

(iii) $f(\text{spint}(B)) \subseteq \text{spint}(f(B))$ for every IFS $B$ in $X$

(iv) $\text{spcl}(f(B)) \subseteq f(\text{spcl}(B))$ for every IFS $B$ in $X$.

**Proof:** *(i) ⇒ (ii)* is obvious.

*(ii) ⇒ (iii)* Let $B$ be any IFS in $X$. Since spint($B$) is an IFSPOS, it is an IFGSPOS in $X$. Then by hypothesis, $f(\text{spint}(B))$ is an IFGSPOS in $Y$. Since $Y$ is an $\text{IFSPT}_{1/2}$ space, f(spint(B)) is an IFSPOS in $Y$. Therefore $f(\text{spint}(B)) = \text{spint}(f(\text{spint}(B))) \subseteq \text{spint}(f(B))$.

*(iii) ⇒ (iv)* can easily proved by taking complement in *(iii)*.

*(iv) ⇒ (i)* Let $A$ be an IFGSPCS in $X$. By hypothesis, $\text{spcl}(f(A)) \subseteq f(\text{spcl}(A))$. Since $X$ is an $\text{IFSPT}_{1/2}$ space, $A$ is an IFSPCS in $X$. Therefore,

$$\text{spcl}(f(A)) \subseteq f(\text{spcl}(A)) = f(A) \subseteq \text{spcl}(f(A)).$$

Hence $f(A)$ is an IFSPCS in $Y$ and hence an IFGSPCS in $Y$. Thus $f$ is an IFMGSPCM.

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