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A distance based correction of the unconscientious experts' evaluations

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Abstract: In the intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. In this paper, new ways of correction of the unconscientious experts' evaluations are proposed.[1]

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AMS Classification: 03E72.

1 Introduction

Intuitionistic fuzzy sets (IFS) are first introduced by Krassimir T. Atanassov in 1983. The latest developments of the theory are presented in the monograph [1]. In one of the subsections are discussed the issues regarding the use of experts' opinions to determination of the membership degree and the non-membership degree, with which the evaluated variant belong/not belong to the IF set of variants satisfying certain criterion.

The problem arises if an expert is *more than 100% sure* that the variant belongs either to the set or to the complement of this set. More precisely, we can describe this fact in terms of membership, and non-membership functions as follows.

Let E_i, $I = 1, \ldots, n$, be an i-th expert from the group of n experts. Following Atanassov ([1], p.12) we call the expert E_i *unconscientious*, if among his estimations $\{\langle \mu_{i,j} , \nu_{i,j}\rangle; j \in J_i \}$, where J

[1] In the *Notes on Intuitionistic Fuzzy Sets*, Vol. 18, 2012, No. 3 and Vol. 19, 2013, No. 1 are included papers on the unconscientious experts' evaluations. I have presented there previously given ways of correction of the unconscientious intuitionistic fuzzy evaluations. Additionally, I have proposed some new ways of correction. The present paper contains further remarks on this subject. The introduction here is very similar to that given in [2, 3], but I have decided to leave it for consistency of this paper.

$= \bigcup_{i=1}^{n} J_i$ is an index set (related to the evaluated variants), there exists an estimation for which $\mu_{i,j} \leq 1$ and $\nu_{i,j} \leq 1$, but $\mu_{i,j} + \nu_{i,j} > 1$.

Let us call the IF value $\langle \mu_{i,j}, \nu_{i,j} \rangle$ for which $\mu_{i,j} \leq 1$ and $\nu_{i,j} \leq 1$, but $\mu_{i,j} + \nu_{i,j} > 1$ an *unconscientious evaluation* (UE) of *j*-th variant (feature, event) by the *i*-th expert.

From now on, the UE $\langle \mu_{i,j}, \nu_{i,j} \rangle$ we denote, for shortly, as UE $\langle \mu, \nu \rangle$.

To apply the intuitionistic fuzzy sets theory to the processing of evaluations, the UE $\langle \mu, \nu \rangle$ must be adjusted (convert) to the correct IF value $\langle \bar{\mu}, \bar{\nu} \rangle$ where $\bar{\mu}, \bar{\nu}, \bar{\pi} \in [0, 1]$ and $\bar{\mu} + \bar{\nu} \in [0, 1]$, with hesitation margin $\bar{\pi} = 1 - \bar{\mu} - \bar{\nu}$.

Atanassov notes that the fact of existence of this kind of problems by the evaluation of events distinguishes the decision aid in the intuitionistic fuzzy environment from the decision aid in the (classical) fuzzy environment, where such unconscientious evaluations do not exist (or are easy to correction).

In the literature (except [2, 3]) no general condition has been given that should be fulfilled in order to consider the conversion as being proper. In my opinion the conversion should fulfill at least the properties (P1, P2, P3) given below.

Property (P1)

a) if $\mu \geq \nu$ the $\bar{\mu} \geq \bar{\nu}$;

b) if $\mu \leq \nu$ then $\bar{\mu} \leq \bar{\nu}$.

In the case of unconscientious experts' evaluations the sum $\mu + \nu$ is too large. The reduction of the sum can be done in three ways:

a) we reduce both of the degrees μ and ν,

b) we reduce the membership degree μ leaving the non-membership degree ν,

c) we reduce the non-membership degree ν leaving the membership degree μ, so as to obtain $\mu + \nu \leq 1$.

Based on above reasoning the conversion's mapping should fulfill, in addition to the property (P1) also the property (P2).

Property (P2)

a) $\bar{\mu} \leq \mu$;

b) $\bar{\nu} \leq \nu$.

This property specifies that we should not increase any of the μ and ν values.
In the case of unconscientious experts' evaluations another problem should be considered.

Problem 1

If $\langle \mu, \nu \rangle$ is an UE, then, for the corrected value $\langle \bar{\mu}, \bar{\nu} \rangle$, should be:

a) $\bar{\pi} = 0$;

b) $\bar{\pi} > 0$;

I am not able to solve the Problem 1.

On the one hand, it can be concluded that an *expert* is a serious man, and he does not specify that is more than 100% sure. The *expert* is, at most, 100% sure of his opinion, and the surplus of more than 100% is irrelevant. It seems to be rational because this type of *expert's* mistake can happen just by accident.

On the other hand, it is reasonable that the *unconscientious* expert is, in fact, *unsure* and his estimation should be considered as *uncertain* with the hesitation degree greater than 0. In this case the degree $\bar{\pi}$ should be an increasing (non-decreasing?) function of the sum $\mu + \nu$. It seems to be rational too, because the greater sum $\mu + \nu$ means the greater *un-precision* of the evaluation of the variant by the expert.

The problem 1 can be described also in terms of *accuracy* of the IF value $\langle \bar{\mu}, \bar{\nu} \rangle$. The accuracy is defined as: *accuracy* $(\langle \bar{\mu}, \bar{\nu} \rangle) = \bar{\mu} + \bar{\nu}$. In the problem 1 we would consider the question: should *accuracy* $(\langle \bar{\mu}, \bar{\nu} \rangle)$ be equivalent to 1, or should it be less than 1, or whether does not have to meet any conditions.

Let we denote, for the UE $\langle \mu, \nu \rangle$, by π^0 the value $\mu + \nu - 1$. It can be called *the unconscientious degree*. It is some kind of the hesitance or uncertainty of the expert's assessments, analogous to the typical hesitance degree understood for the IF value $\langle \mu, \nu \rangle$ as $1-\mu-\nu$.

I think that the 'measure of uncertainty' should not be increased by the correction of the UE value. Therefore I suggest the third property of the correction mapping.

Property (P3)

If $\langle \mu, \nu \rangle$ is an UE, then, for the corrected value $\langle \bar{\mu}, \bar{\nu} \rangle$, should be $\bar{\pi} \leq \pi^0$.

In the cited monograph [1], Atanassov proposed some ways for the adjustment of the values in the *unconscientious experts' case*. In [2] and [3] new ways are proposed.

2 Some ways of the correction of unconscientious evaluations

Let $\langle \mu, \nu \rangle$ be an UE. It means that it is $\mu, \nu \in [0, 1]$ and $\mu + \nu > 1$. Similarly to the basic geometric interpretation of IF value (see [1] p. 39), we consider the UE value as the point $\langle \mu, \nu \rangle$ located in the triangle *ABC* marked in Figure 1 (and subsequent). The idea for obtaining the values in the ways given below, of correction of the UE lies in the following reasoning: the expert is a serious, sober, conscious person and, if he/she has made a mistake, then when correcting the opinion, we should give the IF-value *closest* to the presented by the expert. For the measure of closeness we use distance measures on $\boldsymbol{R}^2$. Because they are the known and most frequently used measures, the formal definitions of these measures will not be given.

For the computing of the first measure of closeness we take the *Euclidean distance* on the plane.

Way 1: We calculate the corrected degrees as

$$\bar{\mu}_1 = \frac{\mu - \nu + 1}{2},$$

$$\bar{\nu}_1 = \frac{\nu - \mu + 1}{2}.$$

The correction is well-defined. The sum $\bar{\mu} + \bar{\nu}$ equals to 1, and $\bar{\pi} = 1 - \bar{\mu} - \bar{\nu} = 0 < \pi^0$. Properties (P1, P2, P3) are fulfilled.

In fact, it is a modification of Way 2, presented in [1], p.14 as well as Way 7 in [2] p. 26, but the formulas in the above sources are not directly associated with any distance.

The idea for obtaining the values using Way 2 lies in the following reasoning: the expert has made a mistake, and we leave the measure of unconscientiousness of his opinion (in the sense of equality of the unconscientious degree and hesitancy degree). Therefore, correcting the UE we should leave $\bar{\pi} = \pi^0$, taking the IF-value closest to UE. As the measure of closeness we take, same as above, the Euclidean distance.

Way 2: We calculate the corrected degrees as

$$\bar{\mu}_2 = 1 - \nu,$$

$$\bar{\nu}_2 = 1 - \mu.$$

The correction is well-defined. The sum $\bar{\mu} + \bar{\nu}$ equals to $2 - \mu - \nu \leq 1$, and $\bar{\pi} = \pi^0 = \mu + \nu - 1 > 0$, Properties (P1, P2, P3) are fulfilled.

The Ways 1 and 2 are illustrated in the Figure 1.

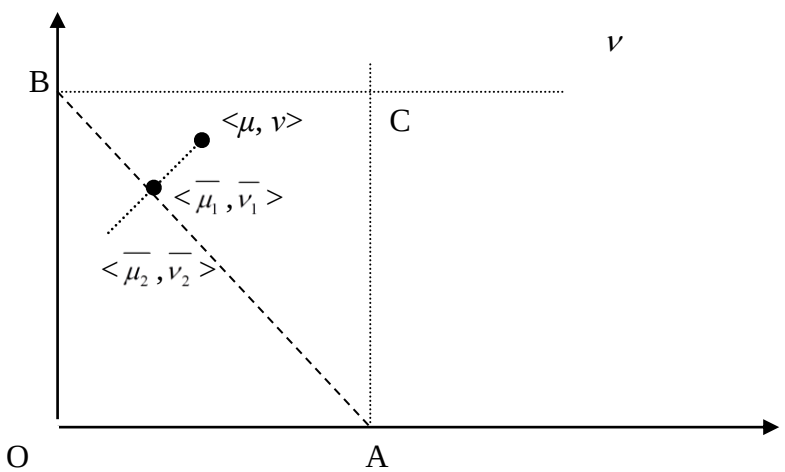

Figure 1. The proposition of UE correction based on *Euclidean distance*

In fact, it is Way 6 presented in [2], p.25 and as a special case of Way 1 and Way 2 in [3], p. 30, but the formulas in the above sources are not associated with any distance.

In Way 1 the closest (Euclidean) distance $d(\langle \mu, \nu \rangle, \langle \bar{\mu}_1, \bar{\nu}_1 \rangle)$ is equal to $\frac{\sqrt{2}}{2}(\mu + \nu - 1)$, while in Way 2, it is $d(\langle \mu, \nu \rangle, \langle \bar{\mu}_2, \bar{\nu}_2 \rangle) = \sqrt{2}(\mu + \nu - 1)$.

The idea for obtaining the values in Way 3 lies in the reasoning presented in Way 1. But, as the measure of closeness we take the *NSCF rail distance (the British Rail distance, the Post Office distance)*, with center point (0, 0).

Way 3: We calculate the corrected degrees as

$$\bar{\mu}_3 = \frac{\mu}{\mu + \nu},$$

$$\bar{\nu}_3 = \frac{\nu}{\mu + \nu}.$$

The correction is well-defined. The sum $\bar{\mu} + \bar{\nu}$ equals to 1, and $\bar{\pi} = 1 - \bar{\mu} - \bar{\nu} = 0 < \pi^0$. Properties (P1, P2, P3) are fulfilled.

In fact, it is Way 1 presented in [1], p.13, as well as a modification of Way 8, presented in [2], p.20, but the formulas in the above sources are not associated with any distance.

The idea for obtaining the corrected values in Way 4 lies in the reasoning presented in Way 2. As the measure of closeness we use again the NSCF rail distance.

Way 4: We calculate the corrected degrees as

$$\bar{\mu}_4 = \frac{\mu(2 - \mu - \nu)}{\mu + \nu},$$

$$\bar{\nu}_4 = \frac{\nu(2 - \mu - \nu)}{\mu + \nu}.$$

The correction is well-defined.

The sum $\bar{\mu} + \bar{\nu}$ equals to $2 - \mu - \nu \leq 1$, and $\bar{\pi} = \pi^0 = \mu + \nu - 1 > 0$, Properties (P1, P2, P3) are fulfilled. In Way 3 the closest (SNCF rail) distance $d(\langle \mu, \nu \rangle, \langle \bar{\mu}_3, \bar{\nu}_3 \rangle)$ is equal to $\frac{\mu + \nu - 1}{\mu + \nu}\sqrt{\mu^2 + \nu^2}$, while in Way 4, it is $d(\langle \mu, \nu > , \langle \bar{\mu}_4, \bar{\nu}_4 >) = 2 \cdot \frac{\mu + \nu - 1}{\mu + \nu}\sqrt{\mu^2 + \nu^2}$.

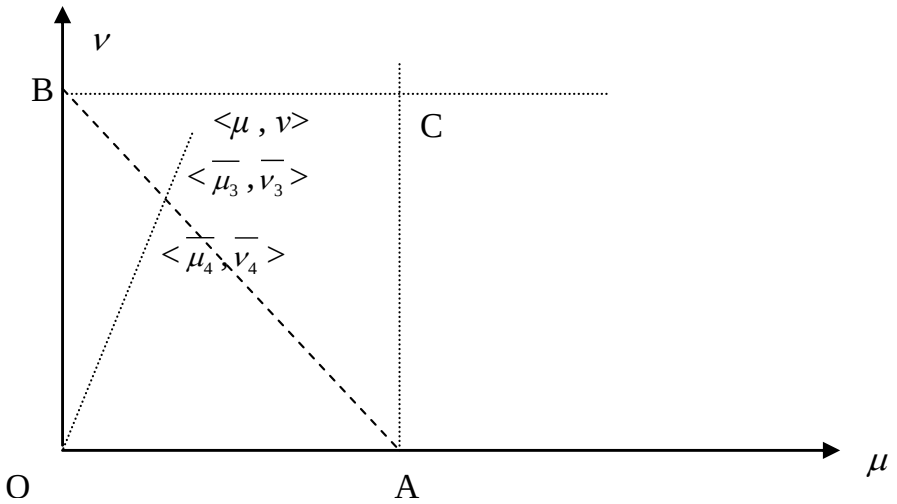

Figure 2. The proposition of UE correction based on *SNCF rail distance*

The idea for obtaining the values in Way 5 lies in the reasoning presented above. In this way we compute the corrected value based on the *jungle river metric*. Due to the properties (P1) and (P2), this case is slightly more complicated.

We assume that the 'river' is straight line $\nu = a \cdot \mu$ where $a < 0$. Based on simple formulas of analytic geometry we can compute the coordinates of the point closest to the given UE $<\mu, \nu>$ in the form $\left\langle \frac{1}{1-a}(\mu + a\nu - a), \frac{1}{1-a}(1-\mu-a\nu) \right\rangle$.

The condition $a < 0$ causes the property $\frac{1}{1-a}(\mu + a\nu - a) + \frac{1}{1-a}(1-\mu-a\nu) \le 1$ holds, which means that the point is an IF value. If we take $a > 0$, this condition may not be satisfied. The point $\left\langle \frac{1}{1-a}(\mu + a\nu - a), \frac{1}{1-a}(1-\mu-a\nu) \right\rangle$, as the corrected evaluation of the UE $\langle \mu, \nu \rangle$, may not fulfill the property (P1). To ensure this property we should take the next way of correction of the UE as follows.

Way 5: We calculate the corrected degrees as

a) for UE $\langle \mu, \nu \rangle$ with $\mu \le \nu$:

$$\bar{\mu}_5 = \min\left\{ \frac{1}{1-a}(\mu + a\nu - a), \frac{1}{2} \right\},$$

$$\bar{\nu}_5 = 1 - \bar{\mu}_5 = \max\left\{ \frac{1}{1-a}(1-\mu-a\nu), \frac{1}{2} \right\},$$

b) for UE $\langle \mu, \nu \rangle$ with $\mu \ge \nu$:

$$\bar{\mu}_5 = \max\left\{\frac{1}{1-a}(\mu + a\nu - a)\,,\,\frac{1}{2}\right\},$$

$$\bar{\nu}_5 = 1 - \bar{\mu}_5 = \min\left\{\frac{1}{1-a}(1 - \mu - a\nu)\,,\,\frac{1}{2}\right\}.$$

The sum $\bar{\mu} + \bar{\nu}$ equals to 1, and $\bar{\pi} = 0$. Properties (P1, P2, P3) are fulfilled.

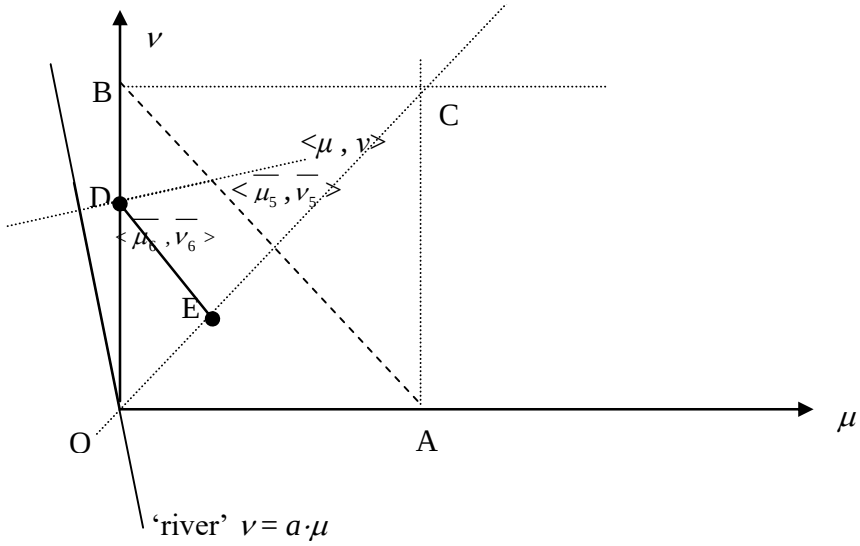

Figure 3. The proposition of UE correction based on *jungle river distance*

As the special case (limit case) of Way 5 we can consider the 'rivers' $\nu = 0$ and $\mu = 0$. For $\nu = 0$ (it is Way 5 by $a \to 0$) we obtain Way 5'.

Way 5': We calculate the corrected degrees as

a) for UE $\langle \mu , \nu \rangle$ with $\mu \le \nu$:

$$\bar{\mu}_{5'} = \min\left\{\mu\,,\,\frac{1}{2}\right\},$$

$$\bar{\nu}_{5'} = 1 - \bar{\mu}_{5'} = \max\left\{1 - \mu\,,\,\frac{1}{2}\right\},$$

b) for UE $\langle \mu , \nu \rangle$ with $\mu \ge \nu$:

$$\bar{\mu}_{5'} = \max\left\{\mu\,,\,\frac{1}{2}\right\} = \mu,$$

$$\bar{\nu}_{5'} = 1 - \bar{\mu}_{5'} = \min\left\{1 - \mu\,,\,\frac{1}{2}\right\} = 1 - \mu.$$

For $\mu = 0$ (it is Way 5 by $a \to -\infty$) we obtain Way 5''.

Way 5'' : We calculate the corrected degrees as

a) for UE $\langle \mu , \nu \rangle$ with $\mu \le \nu$:

$$\bar{\mu}_{5''} = \max\left\{1-\nu\,,\,\frac{1}{2}\right\} = 1-\nu,$$

$$\bar{\nu}_{5''} = 1-\bar{\mu}_{5''} = \min\left\{\nu\,,\,\frac{1}{2}\right\} = \nu,$$

b) for UE $\langle\mu\,,\nu\rangle$ with $\mu \geq \nu$:

$$\bar{\mu}_{5''} = \max\left\{1-\nu\,,\,\frac{1}{2}\right\}$$

$$\bar{\nu}_{5''} = 1-\bar{\mu}_{5''} = \min\left\{\nu\,,\,\frac{1}{2}\right\}.$$

In the case of Ways 5' and 5'' the sum $\bar{\mu}+\bar{\nu}$ equals to 1 and $\bar{\pi}=0$. Properties (P1, P2, P3) are fulfilled.

There remains the question, if there exists some coefficient a that should be preferred in Way 5? I am not able to answer this question.

It is easy to show that for $a = -1$, we obtain, in both cases a) and b), the corrected IF value in the form $\langle\,\frac{\mu-\nu+1}{2}\,,\,\frac{\nu-\mu+1}{2}\,\rangle$, what is Way 1, given earlier. In the second special case, if for the given UE $\langle\mu\,,\nu\rangle$ we take $a = -\frac{\mu}{\nu}$, we obtain $\langle\,\bar{\mu}_5\,,\bar{\nu}_5\,\rangle = \langle\,\frac{\mu}{\mu+\nu}\,,\,\frac{\nu}{\mu+\nu}\,\rangle$, presented as Way 3.

The Way 6 is analogous to Way 5, but with $\bar{\pi} = \pi^0$.

Let us consider the situation $\mu \leq \nu$ (the case for $\mu \geq \nu$ is analogous). In this case, because of the property (P1), the point $\langle\,\bar{\mu}\,,\,\bar{\nu}\,\rangle$ must be situated on the section DE (parallel to AB), Figure 3. For the given UE $\langle\mu\,,\nu\rangle$, the point $\langle\,\bar{\mu}\,,\,\bar{\nu}\,\rangle$ can be equal to the point D or to the point E or to a point from the interior of section DE, according to the slope of the 'river' and the value of π^0 .

Based on, for example, the equality of suitable vectors we obtain, as the correction of the UE $\langle\mu\,,\nu\rangle$, the point $\left\langle\frac{2}{1-a}(\mu+a\nu-a)-\mu\;,\;\frac{2}{1-a}(1-\mu-a\nu)-\nu\right\rangle$. However, we should note, that this point may not be the IF value or may not fulfill the property (P1). Hence we obtain Way 6 as follows.

Way 6: We calculate the corrected degrees as

a) for UE $\langle\mu\,,\nu\rangle$ with $\mu \leq \nu$:

$$\bar{\mu}_6 = \frac{2}{1-a}(\mu+a\nu-a)-\mu\;,$$

$$\bar{\nu}_6 = \frac{2}{1-a}(1-\mu-a\nu)-\nu\,,$$

but

if $\bar{\mu}_6 \leq 0$, then we take

$$\bar{\mu}_6 = 0\,,$$
$$\bar{\nu}_6 = 2-\mu-\nu\,,$$

if $\bar{\mu}_6 \geq \bar{\nu}_6$, then we take

$$\bar{\mu}_6 = 1-\frac{\mu+\nu}{2}\,,$$
$$\bar{\nu}_6 = 1-\frac{\mu+\nu}{2}\,,$$

b) for UE $\langle\mu,\nu\rangle$ with $\mu \geq \nu$:

$$\bar{\mu}_6 = \frac{2}{1-a}(\mu+a\nu-a)-\mu\,,$$
$$\bar{\nu}_6 = \frac{2}{1-a}(1-\mu-a\nu)-\nu\,,$$

but

if $\bar{\nu}_6 \leq 0$, then we take

$$\bar{\mu}_6 = 2-\mu-\nu\,,$$
$$\bar{\nu}_6 = 0,$$

if $\bar{\mu}_6 \leq \bar{\nu}_6$, then we take

$$\bar{\mu}_6 = 1-\frac{\mu+\nu}{2}\,,$$
$$\bar{\nu}_6 = 1-\frac{\mu+\nu}{2}\,.$$

In Way 6 it is $\bar{\pi} = \pi^0 > 0$. Properties (P1, P2, P3) are fulfilled.

In my opinion, from a practical point of view, the Way 6 is (in general) far from encouraging.

For the application more important seem to be, the special cases (limited cases) of Way 6, the cases of the 'rivers' given by formulas $\nu = 0$ and $\mu = 0$.

When the 'river' is $\nu = 0$ (it is Way 6 by $a \to 0$) we obtain Way 6'.

Way 6': We calculate the corrected degrees as

a) for UE $\langle\mu,\nu\rangle$ with $\mu \leq \nu$:

$$\bar{\mu}_{6'} = \mu\,,$$
$$\bar{\nu}_{6'} = 2-2\mu-\nu\,,$$

but

if $\bar{\mu}_{6'} \geq \bar{\nu}_{6'}$, then we take

$$\bar{\mu}_{6'} = 1-\frac{\mu+\nu}{2}\,,$$
$$\bar{\nu}_{6'} = 1-\frac{\mu+\nu}{2}\,,$$

b) for UE $<\mu, \nu>$ with $\mu \geq \nu$:

$$\bar{\mu}_{6'} = \mu\,,$$

$$\bar{\nu}_{6'} = 2 - 2\mu - \nu ,$$

but

if $\bar{\nu}_{6'} \le 0$, then we take

$$\bar{\mu}_{6'} = 2 - \mu - \nu ,$$
$$\bar{\nu}_{6'} = 0 .$$

When the 'river' is $\mu = 0$ (it is Way 6 by $a \to -\infty$) we obtain Way 6''.

Way 6'': We calculate the corrected degrees as

a) for UE $\langle \mu , \nu \rangle$ with $\mu \le \nu$:

$$\bar{\mu}_{6''} = 2 - 2\nu - \mu ,$$
$$\bar{\nu}_{6''} = \nu ,$$

but

if $\bar{\mu}_{6''} \le 0$, then we take

$$\bar{\mu}_{6''} = 0 ,$$
$$\bar{\nu}_{6''} = 2 - \mu - \nu ,$$

b) for UE $\langle \mu , \nu \rangle$ with $\mu \ge \nu$:

$$\bar{\mu}_{6''} = 2 - 2\nu - \mu ,$$
$$\bar{\nu}_{6''} = \nu ,$$

but

if $\bar{\mu}_{6''} \le \bar{\nu}_{6''}$, then we take

$$\bar{\mu}_{6''} = 1 - \frac{\mu + \nu}{2} ,$$
$$\bar{\nu}_{6''} = 1 - \frac{\mu + \nu}{2} .$$

The conditions given in the Way 6, Way 6', and Way 6'' can be written in the form of the relationship between μ, ν, and a, but the notation used above seems to be simpler.

For the particular 'river', when $a = -1$, the corrected IF value $\langle \bar{\mu}_6 , \bar{\nu}_6 \rangle$ is equal to $\langle 1-\nu , 1-\mu \rangle$, what is described as Way 2.

In the monograph [1, p.53] Atanassov gives the modal operators (introduced earlier by him): the necessity operator $\Box$, and the possibility operator $\Diamond$. They are, of course, defined for IF sets (or IF values). But we can define the same operators for the UE. It is namely

$$\Box <\mu, \nu> = <\mu, 1-\mu> ,$$

$$\Diamond <\mu, \nu> = <1-\nu, \nu> .$$

Based on the above operators we can write the results presented as Way 5' and Way 5'' in the form of the $\Box \langle \mu , \nu \rangle$ or $\Diamond \langle \mu , \nu \rangle$, corrected by the value 0.5, to fulfill the property (P1).

The necessity and possibility operators on UE are convenient in the next way of correction of the UE.

As the measure of closeness we take in the ways 7 and 8, the next typical distance measure – the *taxicab metric* (*rectilinear distance, Manhattan distance*).

For any point of the section FG (Figure 4), the distance to the point $\langle \mu , \nu \rangle$ is the same and equal to $\mu + \nu - 1$. At the same time it is the minimal distance between UE $\langle \mu , \nu \rangle$ and any of IF values. The corrected value $\langle \bar{\mu}_7 , \bar{\nu}_7 \rangle$ is a convex combination of $\Box \langle \mu , \nu \rangle$ and $\Diamond \langle \mu , \nu \rangle$, adjusted to satisfy the property (P1).

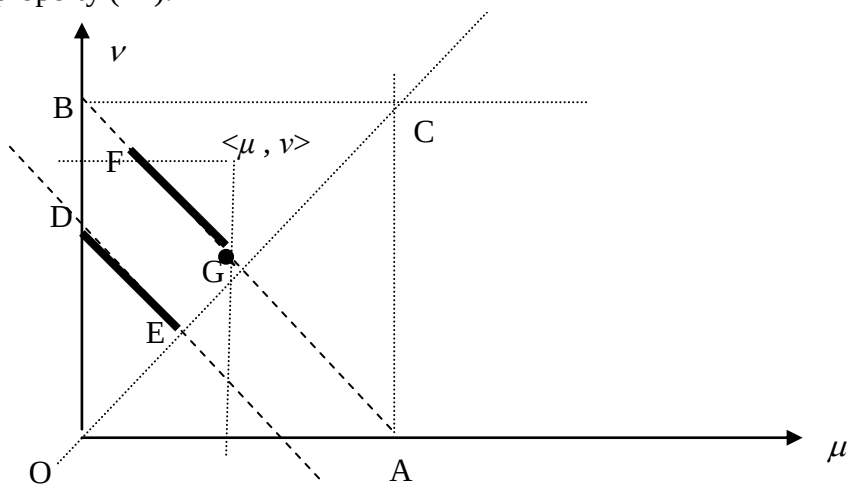

Figure 4. The proposition of UE correction based on *rectilinear distance*

Way 7: We calculate the corrected degrees as

a) for UE $\langle \mu , \nu \rangle$ with $\mu \le \nu$ and $\mu \le 0.5$ or $\mu \ge \nu$ and $\nu \le 0.5$:

$$\bar{\mu}_7 = \alpha\mu + (1-\alpha)(1-\nu),$$

$$\bar{\nu}_7 = \alpha(1-\mu) + (1-\alpha)\nu,$$

b) for UE $\langle \mu , \nu \rangle$ with $\mu \le \nu$ and $\mu \ge 0.5$:

$$\bar{\mu}_7 = \alpha(1-\nu) + 0.5(1-\alpha),$$

$$\bar{\nu}_7 = \alpha\nu + 0.5(1-\alpha),$$

c) for UE $\langle \mu , \nu \rangle$ with $\mu \ge \nu$ and $\nu \ge 0.5$:

$$\bar{\mu}_7 = \alpha\mu + 0.5(1-\alpha),$$

$$\bar{\nu}_7 = \alpha(1-\mu) + 0.5(1-\alpha),$$

where $\alpha \in [0 , 1]$.

The sum $\bar{\mu} + \bar{\nu}$ equals to 1, and $\bar{\pi} = 0$. Properties (P1, P2, P3) are fulfilled.

The Way 7 can be written using the operator D_α (see [1], p.77), which is exactly the convex combination of the necessity and possibility operators. The value, obtained by using the D_α operator, should be of course adjusted, to fulfill the property (P1).

The Way 8 is analogous to Way 7, but with $\bar{\pi} = \pi^0$. In this case, any point on the straight line passing through points D and E can be written as $\langle 1 - \nu + \beta , 1 - \mu - \beta \rangle$, where

$\beta \in (-\infty, \infty)$, but for fulfilling the properties (P1), (P2), and (P3), not every point may be viewed as correction of the UE $\langle \mu, \nu \rangle$. Therefore the Way 8 is as follows

Way 8: We calculate the corrected degrees as

a) for UE $\langle \mu, \nu \rangle$ with $\mu \leq \nu$:

$$\bar{\mu}_8 = 1 - \nu + \beta,$$
$$\bar{\nu}_8 = 1 - \mu - \beta,$$

where $\nu - 1 \leq \beta \leq \dfrac{\nu - \mu}{2}$,

b) for UE $\langle \mu, \nu \rangle$ with $\mu \geq \nu$:

$$\bar{\mu}_8 = 1 - \nu + \beta,$$
$$\bar{\nu}_8 = 1 - \mu - \beta,$$

where $\dfrac{\nu - \mu}{2} \leq \beta \leq 1 - \mu$.

The range of parameter β is chosen, to fulfill properties (P1, P2, P3).

In my opinion, from a practical point of view, the ways 7 and 8 are far from encouraging, because it seems to be good when for certain UE $\langle \mu, \nu \rangle$ we get one, certain, corrected value $\langle \bar{\mu}, \bar{\nu} \rangle$. It the cases of ways 7 and 8 we obtain, in general, infinite set of corrected values.

3 Conclusion

In the intuitionistic fuzzy environment unconscientious opinions may cause problems in the data processing. In this paper old and new ideas and ways of correction of the unconscientious experts' evaluations are presented. The main idea lies in the reasoning that if the expert has made a mistake, then we should take instead of his opinion the IF value closest to it. Considering the question of the closeness, we have to take into account various metrics. Due to this fact we obtain various conversions. The basic and practically useful metrics are used in the paper. One can ask why we use the metrics on $\boldsymbol{R}^2$ and not the distance of IF values in Atanassov sense or Szmidt and Kacprzyk sense (see [1], pp. 137–139). The answer is formally easy: the UE's are not IF values, therefore the distance between the IF values must not be used.

The basis for the correction of the UE can also be a similarity measure, but this goes beyond the scope of this paper.

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