Notes on Intuitionistic Fuzzy Sets
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Properties of some operations defined over intuitionistic fuzzy sets

Rangasamy Parvathi[1], **Beloslav Riečan**[2], **Krassimir Atanassov**[3]

[1] Department of Mathematics, Vellalar College for Women
Erode - 638 012, Tamilnadu, India
e-mail: paarvathis@rediffmail.com

[2] Department of Mathematics, Faculty of Natural Sciences, Matej Bel University
Tajovského 40, 974 01 Banská Bystrica, Slovakia
Mathematical Institute of Slovak Academy of Sciences
Štefánikova 49, SK–81473 Bratislava, Slovakia
e-mails: riecan@mat.savba.sk, riecan@fpv.umb.sk

[3] Bioinformatics and Mathematical Modelling Department
IBPhBME - Bulgarian Academy of Sciences
Acad. Georgi Bonchev Str., block 105, Sofia 1113, Bulgaria
e-mail: krat@bas.bg

Abstract: A lot of operations are defined oved the intuitionistic fuzzy sets. Here, five equalities with participation of some of these operations are introduced and proved. The paper is a continuation of [4, 5], where two new operations over Intuitionistic Fuzzy Sets (IFSs, see [1, 2]) are introduced. The operations and their properties may find applications as the operations A^n, $n.A$ and $\frac{1}{n}.A$ find their places and play role in contrast enhancement and defining statistical tools in intuitionistic fuzzy environment.

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1 Introduction

Let a set E be fixed. The IFS A in E is defined by (see, e.g., [1, 2]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A Ex) \leq 1.$$

Different relations and operations are introduced over the IFSs. Some of them are the following:

$$A = B \text{ iff } (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)),$$

$$\neg A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$n.A = \{\langle x, 1 - (1 - \mu_A(x))^n, (\nu_A(x))^n\rangle \mid x \in E\},$$

$$A^n = \{\langle x, (\mu_A(x))^n, 1 - (1 - \nu_A(x))^n\rangle \mid x \in E\},$$

$$\sqrt[n]{A} = \{\langle \sqrt[n]{\mu_A(x)}, 1 - \sqrt[n]{1 - \nu_A(x)}, \rangle | x \in E\},$$

$$\frac{1}{n}.A = \{\langle x, 1 - \sqrt[n]{1 - \mu_A(x)}, \sqrt[n]{\nu_A(x)}\rangle | x \in E\},$$

where n is a natural number.

Operations $n.A$ and A^n are introduced for a first time in [3] by Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy, while operations $\sqrt[n]{A}$ and $\frac{1}{n}.A$ – in [4] and [5], respectively, by the first two authors.

2 Main results

The four new operations possess some interesting properties, as given in the following theorems. We start with the following:

Theorem 1: For every IFS A and for every natural number $n \geq 1$:

(a) $\neg\frac{1}{n}.\neg A = \sqrt[n]{A}$,

(b) $\neg\sqrt[n]{\neg A} = \frac{1}{n}.A$,

(c) $\neg n.\neg A = A^n$,

(d) $\neg(\neg A)^n = n.A$.

Proof. (a)

$$\neg\frac{1}{n}.\neg A$$

$$= \neg\frac{1}{n}.\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}$$

$$= \neg\{\langle x, 1 - \sqrt[n]{1 - \nu_A(x)}, \sqrt[n]{\mu_A(x)}\rangle | x \in E\}$$

$$= \{\langle x, \sqrt[n]{\mu_A(x)}, 1 - \sqrt[n]{1 - \nu_A(x)}\rangle | x \in E\}$$

$$= \sqrt[n]{A}.$$

(b), (c) and (d) are proved by analogy. $\square$

Theorem 2: For every IFS A and for every natural number n

$$\neg n.(\neg\frac{1}{n}.\sqrt[n]{A})^n = A = \neg\frac{1}{n}.\sqrt[n]{\neg n.A^n}.$$

Proof.

$$\neg n.(\neg\frac{1}{n}.\sqrt[n]{A})^n$$
$$= \neg n.(\neg\frac{1}{n}.\{\langle x, \sqrt[n]{\mu_A(x)}, 1-\sqrt[n]{1-\nu_A(x)}\rangle | x \in E\})^n$$
$$= \neg n.(\neg\{\langle x, 1-\sqrt[n]{1-\sqrt[n]{\mu_A(x)}}, \sqrt[n]{1-\sqrt[n]{1-\nu_A(x)}}\rangle | x \in E\})^n$$
$$= \neg n.(\{\langle x, \sqrt[n]{1-\sqrt[n]{1-\nu_A(x)}}, 1-\sqrt[n]{1-\sqrt[n]{\mu_A(x)}}\rangle | x \in E\})^n$$
$$= \neg n.\{\langle x, 1-\sqrt[n]{1-\nu_A(x)}, 1-(1-1+\sqrt[n]{1-\sqrt[n]{\mu_A(x)}})^n\rangle | x \in E\}$$
$$= \neg n.\{\langle x, 1-\sqrt[n]{1-\nu_A(x)}, 1-1+\sqrt[n]{\mu_A(x)}\rangle | x \in E\}$$
$$= \neg n.\{\langle x, 1-\sqrt[n]{1-\nu_A(x)}, \sqrt[n]{\mu_A(x)}\rangle | x \in E\}$$
$$= \neg\{\langle x, 1-(1-1+\sqrt[n]{1-\nu_A(x)})^n, (\sqrt[n]{\mu_A(x)})^n\rangle | x \in E\}$$
$$= \neg\{\langle x, 1-(\sqrt[n]{1-\nu_A(x)})^n, \mu_A(x)\rangle | x \in E\}$$
$$= \neg\{\langle x, 1-1+\nu_A(x), \mu_A(x)\rangle | x \in E\}$$
$$= \neg\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}$$
$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$
$$= A$$
$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$
$$= \neg\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}$$
$$= \neg\frac{1}{n}.\{\langle x, 1-1+\nu_A(x), \mu_A(x)\rangle \mid x \in E\}$$
$$= \neg\frac{1}{n}.\{\langle x, 1-\sqrt[n]{1-1+(1-\nu_A(x))^n}, \sqrt[n]{(\mu_A(x))^n}\rangle \mid x \in E\}$$
$$= \neg\frac{1}{n}.\{\langle x, 1-(1-\nu_A(x))^n, \mu_A(x))^n\rangle \mid x \in E\}$$
$$= \neg\frac{1}{n}.\{\langle x, 1-(1-\nu_A(x))^n, 1-1+(\mu_A(x))^n\rangle \mid x \in E\}$$
$$= \neg\frac{1}{n}.\{\langle x, \sqrt[n]{(1-(1-\nu_A(x))^n)^n}, 1-\sqrt[n]{1-1+(1-(\mu_A(x))^n)^n}\rangle \mid x \in E\}$$
$$= \neg\frac{1}{n}.\sqrt[n]{\{\langle x, (1-(1-\nu_A(x))^n)^n, 1-(1-(\mu_A(x))^n)^n\rangle \mid x \in E\}}$$
$$= \neg\frac{1}{n}.\sqrt[n]{\neg\{\langle x, 1-(1-(\mu_A(x))^n)^n, (1-(1-\nu_A(x))^n)^n\rangle \mid x \in E\}}$$
$$= \neg\frac{1}{n}.\sqrt[n]{\neg n.\{\langle x, (\mu_A(x))^n, 1-(1-\nu_A(x))^n\rangle \mid x \in E\}}$$
$$= \neg\frac{1}{n}.\sqrt[n]{\neg n.A^n}.$$
$\square$

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