

# Cartesian composition of intuitionistic fuzzy finite automata with unique membership transition on an input symbol

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**Abstract:** This paper presents a study on Cartesian composition of two intuitionistic fuzzy finite automata with unique membership transition on an input symbol (IFAUM). The condition for intuitionistic retrievable and intuitionistic connectedness of two IFAUM's  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are analyzed.

**Keywords:** Intuitionistic fuzzy finite Automata, Intuitionistic retrievable, Intuitionistic connectedness.

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## 1 Introduction

The evolution of fuzzy sets by Zadeh [18] is a milestone in the theory of formal languages. Fuzziness reduces a gap between formal language and natural language in terms of precision, leading to describe fuzzy language by Lee and Zadeh [12]. In a fuzzy finite state automaton, there may be more than one fuzzy transition from a state on an input symbol with a given membership value given by Santos, Wee and Fu [15, 17]. This development was followed by the postulation called deterministic fuzzy finite state automaton as in Malik and Mordeson [13] in which there can be at most one transition on an input, which can be constructed equivalently from a fuzzy finite state automaton. However, it only acts as a deterministic fuzzy recognizer, and the fuzzy regular languages accepted by the fuzzy finite state automaton and deterministic fuzzy finite state automaton need not necessarily be equal (i.e., the degree of a string need not be the same). Rajaretnam and Ayyaswamy [14] introduced fuzzy finite state automaton with unique membership

transition on an input symbol, one kind of determinism of a given fuzzy finite automaton in which the membership value of any recognized string in both the systems are the same.

Using the notion of intuitionistic fuzzy sets by Atanassov [1, 2], it is possible to obtain intuitionistic fuzzy language, by introducing non-membership value to the strings of fuzzy language. This is a natural generalisation of a fuzzy language characterised by two functions expressing the degree of belongingness and non-belongingness. Out of several higher order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [1, 3, 4, 5] have been found to be highly useful to deal with vagueness. Gau and Buehrer [7] presented the concepts of vague sets. But, Burillo and Bustince [6] showed the notion of vague sets coincides with that of intuitionistic fuzzy sets. Jun [10, 11] introduced the concepts of intuitionistic fuzzy finite state machines (iffsm). The use of algebraic techniques in determining the structure of automata and solving practical problems has been significant proved by Holcombe [8]. A study on intuitionistic fuzzy finite automata with unique membership transition on an input symbol (IFAUM) is introduced in [9, 16].

In this paper, the definition of Cartesian composition of an IFAUM is given. It is shown that the cartesian composition of intuitionistic fuzzy finite automata with unique membership transition on an input symbol preserve the properties of intuitionistic retrievability and intuitionistic connectedness.

## 2 Basic definitions

**Definition 2.1** ([1]). Given a nonempty set  $\Sigma$ . Intuitionistic fuzzy sets (IFS) in  $\Sigma$  is an object having the form  $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in \Sigma\}$  where the functions  $\mu_A : \Sigma \rightarrow [0, 1]$  and  $\nu_A : \Sigma \rightarrow [0, 1]$  denote the degree of membership and the degree of non-membership of each element  $x \in \Sigma$  to the set  $A$  respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in \Sigma$ . For the sake of simplicity, we shall use the notation  $A = (\mu_A, \nu_A)$  instead of  $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in \Sigma\}$ .

We define the following.

**Definition 2.2** ([16]). An Intuitionistic fuzzy finite automaton with unique membership transition on an input symbol is an ordered 5-tuple (IFAUM)  $\mathcal{A} = (Q, \Sigma, A, i, f)$ , where

- (i)  $Q$  is a finite non-empty set of states.
- (ii)  $\Sigma$  is a finite non-empty set of input symbols.
- (iii)  $A = (\mu_A, \nu_A)$ , each is an intuitionistic fuzzy subset of  $Q \times \Sigma \times Q$ . The fuzzy subset  $\mu_A : Q \times \Sigma \times Q \rightarrow [0, 1]$  denotes the degree of membership value such that  $\mu_A(p, a, q) = \mu_A(p, a, q')$  for some  $q, q' \in Q$  then  $q = q'$  and  $\nu_A : Q \times \Sigma \times Q \rightarrow [0, 1]$  denotes the degree of non-membership value of every element in  $Q \times \Sigma \times Q$ .
- (iv)  $i = (i_{\mu_A}, i_{\nu_A})$ , each is an intuitionistic fuzzy subset of  $Q$ , i.e.,  $i_{\mu_A} : Q \rightarrow [0, 1]$  and  $i_{\nu_A} : Q \rightarrow [0, 1]$  called the intuitionistic fuzzy initial state.
- (v)  $f = (f_{\mu_A}, f_{\nu_A})$ , each is an intuitionistic fuzzy subset of  $Q$ , i.e.,  $f_{\mu_A} : Q \rightarrow [0, 1]$  and  $f_{\nu_A} : Q \rightarrow [0, 1]$  called the intuitionistic fuzzy subsets of final states.

**Definition 2.3.** Let  $\mathcal{A} = (Q, \Sigma, A, i, f)$  be an IFAUM. Define an ifs  $A^* = (\mu_A^*, \nu_A^*)$  in  $Q \times \Sigma^* \times Q$  as follows:  $\forall p, q \in Q, x \in \Sigma^*, a \in \Sigma$ .

$$\begin{aligned}\mu_A(q, \epsilon, p) &= \begin{cases} 1, & \text{if } p = q \\ 0, & \text{if } p \neq q \end{cases}, \quad \nu_A(q, \epsilon, p) = \begin{cases} 0, & \text{if } p = q \\ 1, & \text{if } p \neq q \end{cases} \\ \mu_A^*(q, xa, p) &= \vee\{\mu_A^*(q, x, r) \wedge \mu_A(r, a, p) | r \in Q\} \\ \text{and } \nu_A^*(q, xa, p) &= \wedge\{\nu_A^*(q, x, r) \vee \nu_A(r, a, p) | r \in Q\}\end{aligned}$$

### 3 Cartesian composition of IFAUM's

A product of two intuitionistic fuzzy finite automata  $\mathcal{A}_1$  and  $\mathcal{A}_2$ , written  $\mathcal{A}_1 \odot \mathcal{A}_2$  called the Cartesian composition of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  is given in this section. It is shown that  $\mathcal{A}_1$  and  $\mathcal{A}_2$  preserve the structural properties of intuitionistic retrievable and intuitionistic connectedness.

**Definition 3.1.** Let  $\mathcal{A}_1 = (Q_1, \Sigma_1, A_1, i_1, f_1)$  and  $\mathcal{A}_2 = (Q_2, \Sigma_2, A_2, i_2, f_2)$  be IFAUM's with  $\Sigma_1 \cap \Sigma_2 = \emptyset$ . Then the Cartesian composition of  $\mathcal{A}_1$  and  $\mathcal{A}_2$  is an IFAUM  $\mathcal{A}_1 \odot \mathcal{A}_2 = (Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, A_1 \odot A_2, i_1 \times i_2, f_1 \times f_2)$ , where

(i)  $(\mu_{A_1} \odot \mu_{A_2}), (\nu_{A_1} \odot \nu_{A_2}) : Q_1 \times Q_2 \times (\Sigma_1 \cup \Sigma_2) \times Q_1 \times Q_2 \rightarrow [0, 1]$  are defined by

$$(\mu_{A_1} \odot \mu_{A_2})((p_1, p_2), a, (q_1, q_2)) = \begin{cases} \mu_{A_1}(p_1, a, q_1), & \text{if } a \in \Sigma_1 \text{ and } p_2 = q_2 \\ \mu_{A_2}(p_2, a, q_2), & \text{if } a \in \Sigma_2 \text{ and } p_1 = q_1 \\ 0 & \text{, otherwise}\end{cases}$$

and

$$(\nu_{A_1} \odot \nu_{A_2})((p_1, p_2), a, (q_1, q_2)) = \begin{cases} \nu_{A_1}(p_1, a, q_1), & \text{if } a \in \Sigma_1 \text{ and } p_2 = q_2 \\ \nu_{A_2}(p_2, a, q_2), & \text{if } a \in \Sigma_2 \text{ and } p_1 = q_1 \\ 0 & \text{, otherwise.}\end{cases}$$

(ii)  $(i_{\mu_{A_1}} \times i_{\mu_{A_2}}), (i_{\nu_{A_1}} \times i_{\nu_{A_2}}) : Q_1 \times Q_2 \rightarrow [0, 1]$  are defined by

$$\begin{aligned}(i_{\mu_{A_1}} \times i_{\mu_{A_2}})(p_1, p_2) &= \begin{cases} i_{\mu_{A_1}}(p_1), & \text{if } p_1 \in Q_1 \\ i_{\mu_{A_2}}(p_2), & \text{if } p_2 \in Q_2 \end{cases} \\ \text{and } (i_{\nu_{A_1}} \times i_{\nu_{A_2}})(p_1, p_2) &= \begin{cases} i_{\nu_{A_1}}(p_1), & \text{if } p_1 \in Q_1 \\ i_{\nu_{A_2}}(p_2), & \text{if } p_2 \in Q_2. \end{cases}\end{aligned}$$

(iii)  $(f_{\mu_{A_1}} \times f_{\mu_{A_2}}), (f_{\nu_{A_1}} \times f_{\nu_{A_2}}) : Q_1 \times Q_2 \rightarrow [0, 1]$  are defined by

$$\begin{aligned}(f_{\mu_{A_1}} \times f_{\mu_{A_2}})(q_1, q_2) &= \begin{cases} f_{\mu_{A_1}}(q_1), & \text{if } q_1 \in Q_1 \\ f_{\mu_{A_2}}(q_2), & \text{if } q_2 \in Q_2 \end{cases} \\ \text{and } (f_{\nu_{A_1}} \times f_{\nu_{A_2}})(q_1, q_2) &= \begin{cases} f_{\nu_{A_1}}(q_1), & \text{if } q_1 \in Q_1 \\ f_{\nu_{A_2}}(q_2), & \text{if } q_2 \in Q_2 \end{cases}\end{aligned}$$

$$\forall(p_1, p_2), (q_1, q_2) \in Q_1 \times Q_2, a \in \Sigma_1 \cup \Sigma_2.$$

**Theorem 3.1.** Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be IFAUM's and let  $\Sigma_1 \cap \Sigma_2 = \phi$ . Let  $\mathcal{A}_1 \odot \mathcal{A}_2 = (Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, A_1 \odot A_2, i_1 \times i_2, f_1 \times f_2)$  be the Cartesian composition of  $\mathcal{A}_1$  and  $\mathcal{A}_2$ . Then  $\forall x \in \Sigma_1^* \cup \Sigma_2^*$ ,  $x \neq \epsilon$ ,

$$(\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x, (q_1, q_2)) = \begin{cases} \mu_{A_1}^*(p_1, x, q_1), & \text{if } x \in \Sigma_1^* \text{ and } p_2 = q_2 \\ \mu_{A_2}^*(p_2, x, q_2), & \text{if } x \in \Sigma_2^* \text{ and } p_1 = q_1 \\ 0 & , \text{ otherwise} \end{cases}$$

and

$$(\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x, (q_1, q_2)) = \begin{cases} \nu_{A_1}^*(p_1, x, q_1), & \text{if } x \in \Sigma_1^* \text{ and } p_2 = q_2 \\ \nu_{A_2}^*(p_2, x, q_2), & \text{if } x \in \Sigma_2^* \text{ and } p_1 = q_1 \\ 0 & , \text{ otherwise.} \end{cases}$$

*Proof.* Let  $x \in \Sigma_1^* \cup \Sigma_2^*$ ,  $x \neq \epsilon$  and let  $|x| = n$ . Suppose that  $x \in \Sigma_1^*$ . Clearly, the result is true if  $n = 1$ . Suppose the result is true  $\forall y \in \Sigma_1^*$ ,  $|y| = n - 1$ ,  $n > 1$ . Let  $x = ay$  where  $a \in \Sigma_1$  and  $y \in \Sigma_1^*$ . Now

$$\begin{aligned} & (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x, (q_1, q_2)) \\ &= (\mu_{A_1} \odot \mu_{A_2})^*((p_1, p_2), ay, (q_1, q_2)) \\ &= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})((p_1, p_2), a, (r_1, r_2)) \wedge (\mu_{A_1} \odot \mu_{A_2})^*((r_1, r_2), y, (q_1, q_2)) \right. \\ & \quad \left. | (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \vee \left\{ \mu_{A_1}(p_1, a, r_1) \wedge (\mu_{A_1} \odot \mu_{A_2})^*((r_1, p_2), y, (q_1, q_2)) | r_1 \in Q_1 \right\} \\ &= \begin{cases} \vee \{\mu_{A_1}(p_1, a, r_1) \wedge \mu_{A_1}^*(r_1, y, q_1) | r_1 \in Q_1\}, & \text{if } p_2 = q_2 \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \mu_{A_1}^*(p_1, ay, q_1), & \text{if } p_2 = q_2 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

and  $(\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x, (q_1, q_2))$

$$\begin{aligned} &= (\nu_{A_1} \odot \nu_{A_2})^*((p_1, p_2), ay, (q_1, q_2)) \\ &= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})((p_1, p_2), a, (r_1, r_2)) \vee (\nu_{A_1} \odot \nu_{A_2})^*((r_1, r_2), y, (q_1, q_2)) \right. \\ & \quad \left. | (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \wedge \left\{ \nu_{A_1}(p_1, a, r_1) \vee (\nu_{A_1} \odot \nu_{A_2})^*((r_1, p_2), y, (q_1, q_2)) | r_1 \in Q_1 \right\} \\ &= \begin{cases} \wedge \{\nu_{A_1}(p_1, a, r_1) \vee \nu_{A_1}^*(r_1, y, q_1) | r_1 \in Q_1\}, & \text{if } p_2 = q_2 \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \nu_{A_1}^*(p_1, ay, q_1), & \text{if } p_2 = q_2 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

The result now follows by induction. The proof is similar, if  $x \in \Sigma_2^*$ .  $\square$

**Theorem 3.2.** Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be IFAUM's and let  $\Sigma_1 \cap \Sigma_2 = \phi$ . Then  $\forall x \in \Sigma_1^*, \forall y \in \Sigma_2^*$ ,

$$\begin{aligned} (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) &= \mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2) \\ &= (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), yx, (q_1, q_2) \right) \end{aligned}$$

and

$$\begin{aligned} (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) &= \nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2) \\ &= (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), yx, (q_1, q_2) \right). \end{aligned}$$

*Proof.* Let  $x \in \Sigma_1^*, y \in \Sigma_2^*, (p_1, p_2), (q_1, q_2) \in Q_1 \times Q_2$ .

If  $x = \epsilon = y$ , then  $xy = \epsilon$ .

Suppose  $(p_1, p_2) = (q_1, q_2)$ . Then  $p_1 = q_1$  and  $p_2 = q_2$ .

Hence  $(\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) = 1 = 1 \wedge 1 = \mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2)$  and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) = 0 = 0 \vee 0 = \nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2)$ .

If  $(p_1, p_2) \neq (q_1, q_2)$ , then either  $p_1 \neq q_1$  or  $p_2 \neq q_2$ .

Thus  $\mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2) = 0$  and  $\nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2) = 1$ .

Hence  $(\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) = 0 = \mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2)$

and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) = 1 = \nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2)$ .

If  $x = \epsilon$  and  $y \neq \epsilon$  or  $x \neq \epsilon$  and  $y = \epsilon$ , then the result follows by the theorem 3.1.

Suppose  $x \neq \epsilon$  and  $y \neq \epsilon$ . Now

$$\begin{aligned} &(\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right) \\ &= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x, (r_1, r_2) \right) \wedge \right. \\ &\quad \left. (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), y, (q_1, q_2) \right) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \vee \left\{ \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x, (r_1, r_2) \right) \wedge \right. \right. \\ &\quad \left. \left. (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), y, (q_1, q_2) \right) \mid r_2 \in Q_2 \right\} \mid r_1 \in Q_1 \right\} \\ &= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x, (r_1, p_2) \right) \wedge \right. \\ &\quad \left. (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, p_2), y, (q_1, q_2) \right) \mid r_1 \in Q_1 \right\} \\ &= \mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2) \end{aligned}$$

and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), xy, (q_1, q_2) \right)$

$$\begin{aligned} &= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x, (r_1, r_2) \right) \vee \right. \\ &\quad \left. (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), y, (q_1, q_2) \right) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \wedge \left\{ \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x, (r_1, r_2) \right) \vee \right. \right. \\ &\quad \left. \left. (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), y, (q_1, q_2) \right) \mid r_2 \in Q_2 \right\} \mid r_1 \in Q_1 \right\} \\ &= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x, (r_1, p_2) \right) \vee \right. \\ &\quad \left. (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, p_2), y, (q_1, q_2) \right) \mid r_1 \in Q_1 \right\} \\ &= \nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2) \end{aligned}$$

Similarly, we prove that  $(\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, yx, (q_1, q_2)) = \mu_{A_1}^*(p_1, x, q_1) \wedge \mu_{A_2}^*(p_2, y, q_2)$  and  $(\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, yx, (q_1, q_2)) = \nu_{A_1}^*(p_1, x, q_1) \vee \nu_{A_2}^*(p_2, y, q_2)$ . Hence the theorem.  $\square$

**Theorem 3.3.** Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be IFAUM's and let  $\Sigma_1 \cap \Sigma_2 = \emptyset$ .

Then  $\forall w \in (\Sigma_1^* \cup \Sigma_2^*) \exists u \in \Sigma_1^*, v \in \Sigma_2^*$  such that

$$(\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, w, (q_1, q_2)) = (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, uv, (q_1, q_2))$$

and

$$(\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, w, (q_1, q_2)) = (\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, uv, (q_1, q_2))$$

$\forall (p_1, p_2), (q_1, q_2) \in Q_1 \times Q_2$ .

*Proof.* Let  $w \in (\Sigma_1^* \cup \Sigma_2^*)$  and  $(p_1, p_2), (q_1, q_2) \in Q_1 \times Q_2$ .

If  $w = \epsilon$ , then we can choose  $u = \epsilon = v$ . In this case the result is trivially true.

Suppose  $w \neq \epsilon$ . If  $w \in \Sigma_1^*$  or  $w \in \Sigma_2^*$ , then the result is true.

Suppose  $w \notin \Sigma_1^*$  and  $w \in \Sigma_2^*$ .

Case(1): If  $w = xy, x \in \Sigma_1^+, y \in \Sigma_2^+$ , then the result follows by the above theorem 3.2.

Case(2): Suppose  $w = x_1yx_2, x_1, x_2 \in \Sigma_1^*$  and  $y \in \Sigma_2^*$ ,  $x_i$  and  $y$  are non-empty strings,  $i = 1, 2$ .

Let  $u = x_1x_2 \in \Sigma_1^*$  and  $v = y$ . Now,  $\forall (r_1, r_2), (q_1, q_2) \in Q_1 \times Q_2$ ,

$$(\mu_{A_1} \odot \mu_{A_2})^*((r_1, r_2), x_2y, (q_1, q_2)) = (\mu_{A_1} \odot \mu_{A_2})^*((r_1, r_2), yx_2, (q_1, q_2))$$

$$(\nu_{A_1} \odot \nu_{A_2})^*((r_1, r_2), x_2y, (q_1, q_2)) = (\nu_{A_1} \odot \nu_{A_2})^*((r_1, r_2), yx_2, (q_1, q_2)).$$

$$\begin{aligned} \text{Thus } & (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x_1yx_2, (q_1, q_2)) \\ &= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x_1, (r_1, r_2)) \wedge \right. \\ &\quad \left. (\mu_{A_1} \odot \mu_{A_2})^*((r_1, r_2), yx_2, (q_1, q_2)) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x_1, (r_1, r_2)) \wedge \right. \\ &\quad \left. (\mu_{A_1} \odot \mu_{A_2})^*((r_1, r_2), x_2y, (q_1, q_2)) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= (\mu_{A_1} \odot \mu_{A_2})^*(p_1, p_2, x_1x_2y, (q_1, q_2)) \end{aligned}$$

$$\begin{aligned} \text{and } & (\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x_1yx_2, (q_1, q_2)) \\ &= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x_1, (r_1, r_2)) \vee \right. \\ &\quad \left. (\nu_{A_1} \odot \nu_{A_2})^*((r_1, r_2), yx_2, (q_1, q_2)) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x_1, (r_1, r_2)) \vee \right. \\ &\quad \left. (\nu_{A_1} \odot \nu_{A_2})^*((r_1, r_2), x_2y, (q_1, q_2)) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\ &= (\nu_{A_1} \odot \nu_{A_2})^*(p_1, p_2, x_1x_2y, (q_1, q_2)) \end{aligned}$$

Case(3): Suppose  $w = y_1xy_2, y_1, y_2 \in \Sigma_2^*$  and  $x \in \Sigma_1^*$ ,  $y_i$  and  $x$  are nonempty strings,  $i = 1, 2$ .

Let  $v = y_1y_2 \in \Sigma_2^*$  and  $u = x$ . The proof of this case is similar to Case(2).

Case(4): Suppose  $w = x_1y_1x_2y_2$ ,  $x_1, x_2 \in \Sigma_1^*$  and  $y_1, y_2 \in \Sigma_2^*$ ,  $x_i$  and  $y_i$  are nonempty strings,  $i = 1, 2$ . Let  $u = x_1x_2 \in \Sigma_1^*$  and  $v = y_1y_2 \in \Sigma_2^*$ . Then

$$\begin{aligned}
& (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2, (q_1, q_2) \right) \\
&= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1, (r_1, r_2) \right) \wedge (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), y_1x_2y_2, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\
&= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1, (r_1, r_2) \right) \wedge (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), x_2y_1y_2, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} [\text{by Case(3)}] \\
&= (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1x_2y_1y_2, (q_1, q_2) \right) \\
&= (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), uv, (q_1, q_2) \right) \\
&\text{and } (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2, (q_1, q_2) \right) \\
&= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1, (r_1, r_2) \right) \vee (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), y_1x_2y_2, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\
&= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1, (r_1, r_2) \right) \vee (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), x_2y_1y_2, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} [\text{by case(3)}] \\
&= (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1x_2y_1y_2, (q_1, q_2) \right) \\
&= (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), uv, (q_1, q_2) \right)
\end{aligned}$$

Case(5): Suppose  $w = y_1x_1y_2x_2$ ,  $x_1, x_2 \in \Sigma_1^*$ ,  $y_1, y_2 \in \Sigma_2^*$ .

Let  $u = x_1x_2 \in \Sigma^*$  and  $v = y_1y_2 \in \Sigma^*$ . The proof of this case is similar to case (4).

Case(6): Let  $w = (\Sigma_1 \cup \Sigma_2)^*$ .

Then  $w = x_1y_1x_2y_2 \cdots x_ny_n$  or  $w = y_1x_1y_2x_2 \cdots y_nx_n$ ,  $x_i \in \Sigma_1^*$ ,  $y_i \in \Sigma_2^*$ ,  $x_i$  and  $y_i$  are non-empty strings,  $i = 1, 2, \dots, n - 2$ . To be specific, let  $w = x_1y_1x_2y_2 \cdots x_ny_n$ . The proof of the second case is similar. If  $n = 0, 1$  or  $2$ , then the result is true by the previous cases.

Suppose the result for all  $z = x_1y_1x_2y_2 \cdots x_{n-1}y_{n-1} \in (\Sigma_1 \cup \Sigma_2)^*$ ,  $n \geq 2$ .

Let  $u_1 = x_1x_2 \cdots x_{n-1}$ ,  $v_1 = y_1y_2 \cdots y_{n-1}$ ,  $u = u_1x_n$  and  $v = v_1y_n$ . Now

$$\begin{aligned}
& (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2 \cdots x_ny_n, (q_1, q_2) \right) \\
&= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2 \cdots x_{n-1}y_{n-1}, (r_1, r_2) \right) \wedge \right. \\
&\quad \left. (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), x_ny_n, (q_1, q_2) \right) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\
&= \vee \left\{ (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), u_1v_1, (r_1, r_2) \right) \wedge (\mu_{A_1} \odot \mu_{A_2})^* \left( (r_1, r_2), x_ny_n, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\
&= (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), u_1v_1x_ny_n, (q_1, q_2) \right) \\
&= (\mu_{A_1} \odot \mu_{A_2})^* \left( (p_1, p_2), uv, (q_1, q_2) \right)
\end{aligned}$$

and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2 \cdots x_ny_n, (q_1, q_2) \right)$

$$\begin{aligned}
&= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), x_1y_1x_2y_2 \cdots x_{n-1}y_{n-1}, (r_1, r_2) \right) \vee \right. \\
&\quad \left. (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), x_ny_n, (q_1, q_2) \right) \mid (r_1, r_2) \in Q_1 \times Q_2 \right\}
\end{aligned}$$

$$\begin{aligned}
&= \wedge \left\{ (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), u_1 v_1, (r_1, r_2) \right) \vee (\nu_{A_1} \odot \nu_{A_2})^* \left( (r_1, r_2), x_n y_n, (q_1, q_2) \right) \right. \\
&\quad \left. \mid (r_1, r_2) \in Q_1 \times Q_2 \right\} \\
&= (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), u_1 v_1 x_n y_n, (q_1, q_2) \right) \\
&= (\nu_{A_1} \odot \nu_{A_2})^* \left( (p_1, p_2), u v, (q_1, q_2) \right).
\end{aligned}$$

The result now follows by induction.  $\square$

**Definition 3.2.** Let  $\mathcal{A} = (Q, \Sigma, A, i, f)$  be an IFAUM.  $\mathcal{A}$  is said to be intuitionistic retrievable if  $q \in Q, \forall y \in \Sigma^*$  if  $t \in Q$  such that  $\mu_A^*(q, y, t) > 0$  and  $\nu_A^*(q, y, t) < 1$ , then  $\exists x \in \Sigma^*$  such that  $\mu_A^*(t, x, q) > 0$  and  $\nu_A^*(t, x, q) < 1$ .

**Theorem 3.4.** Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be IFAUM's and let  $\Sigma_1 \cap \Sigma_2 = \emptyset$ . Then the Cartesian composition  $\mathcal{A}_1 \odot \mathcal{A}_2$  is intuitionistic retrievable if and only if  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic retrievable.

*Proof.* Suppose that  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic retrievable.

Let  $(q, p), (t, s) \in Q_1 \times Q_2$  and  $w \in (\Sigma_1^* \cup \Sigma_2^*)$  be such that  $(\mu_{A_1} \odot \mu_{A_2})^* \left( (q, p), w, (t, s) \right) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (q, p), w, (t, s) \right) < 1$ .

Let  $w^* = uv$  be the standard form of  $w$ ,  $u \in \Sigma_1^*$ ,  $v \in \Sigma_2^*$ . Then

$$\begin{aligned}
(\mu_{A_1} \odot \mu_{A_2})^* \left( (q, p), w, (t, s) \right) &= (\mu_{A_1} \odot \mu_{A_2})^* \left( (q, p), u v, (t, s) \right) \\
&= \mu_{A_1}^*(q, u, t) \wedge \mu_{A_2}^*(p, v, s) \\
\text{and } (\nu_{A_1} \odot \nu_{A_2})^* \left( (q, p), w, (t, s) \right) &= (\nu_{A_1} \odot \nu_{A_2})^* \left( (q, p), u v, (t, s) \right) \\
&= \nu_{A_1}^*(q, u, t) \vee \nu_{A_2}^*(p, v, s).
\end{aligned}$$

Thus  $\mu_{A_1}^*(q, u, t) > 0$ ,  $\nu_{A_1}^*(q, u, t) < 1$  and  $\mu_{A_2}^*(p, v, s) > 0$ ,  $\nu_{A_2}^*(p, v, s) < 1$ .

Since  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic retrievable,  $\exists u' \in \Sigma_1^*$ ,  $v' \in \Sigma_2^*$

such that  $\mu_{A_1}^*(t, u', q) > 0$ ,  $\nu_{A_1}^*(t, u', q) < 1$  and  $\mu_{A_2}^*(s, v' p) > 0$ ,  $\nu_{A_2}^*(s, v' p) < 1$ .

Thus  $(\mu_{A_1} \odot \mu_{A_2})^* \left( (t, s), u' v', (q, p) \right) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (t, s), u' v', (q, p) \right) < 1$ .

Hence  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic retrievable.

Conversely, suppose that  $\mathcal{A} \odot \mathcal{A}$  is intuitionistic retrievable.

Let  $q, t \in Q_1$  and  $y \in \Sigma_1^*$  be such that  $\mu_{A_1}^*(q, y, t) > 0$  and  $\nu_{A_1}^*(q, y, t) < 1$ .

Then  $\forall s \in Q_2$ ,

$$(\mu_{A_1} \odot \mu_{A_2})^* \left( (q, s), y, (t, s) \right) = \mu_{A_1}^*(q, y, t) > 0$$

$$\text{and } (\nu_{A_1} \odot \nu_{A_2})^* \left( (q, s), y, (t, s) \right) = \nu_{A_1}^*(q, y, t) < 1.$$

Thus  $\exists w \in (\Sigma_1^* \cup \Sigma_2^*)$

such that  $(\mu_{A_1} \odot \mu_{A_2})^* \left( (t, s), w, (q, s) \right) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})^* \left( (t, s), w, (q, s) \right) < 1$ .

Let  $w^* = uv$  be the standard form of  $w$ ,  $u \in \Sigma_1^*$ ,  $v \in \Sigma_2^*$ .

$$\text{Then } 0 < (\mu_{A_1} \odot \mu_{A_2})^* \left( (t, s), w, (q, s) \right) = \mu_{A_1}^*(t, u, q) \wedge \mu_{A_2}^*(s, v, s)$$

$$\text{and } 1 > (\nu_{A_1} \odot \nu_{A_2})^* \left( (t, s), w, (q, s) \right) = \nu_{A_1}^*(t, u, q) \vee \nu_{A_2}^*(s, v, s).$$

Thus  $\mu_{A_1}^*(t, u, q) > 0$  and  $\nu_{A_1}^*(t, u, q) < 1$ .

Hence  $\mathcal{A}_1$  is intuitionistic retrievable. Similarly  $\mathcal{A}_2$  is intuitionistic retrievable.  $\square$

**Definition 3.3.** Let  $\mathcal{A} = (Q, \Sigma, A, i, f)$  be an IFAUM. let  $p, q \in Q$ . Then  $p$  and  $q$  are said to be intuitionistic connected if either  $q = p$  or there exists  $q_0, q_1, \dots, q_k \in Q, p = q_0, q = q_k$  and there exists  $a_1, a_2, a_3, \dots, a_k \in \Sigma$  such that  $\forall i = 1, 2, \dots, k$ , either  $\mu_A(q_{i-1}, a_i, q_i) > 0$  and  $\nu_A(q_{i-1}, a_i, q_i) < 1$  or  $\mu_A(q_i, a_i, q_{i-1}) > 0$  and  $\nu_A(q_i, a_i, q_{i-1}) < 1$ .

**Theorem 3.5.** Let  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be IFAUM's and let  $\Sigma_1 \cap \Sigma_2 = \emptyset$ . Then the Cartesian composition  $\mathcal{A}_1 \odot \mathcal{A}_2$  is intuitionistic connected if and only if  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic connected.

*Proof.* Suppose that  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are intuitionistic connected.

Let  $(p, p')(q, q') \in Q_1 \times Q_2$ . Now  $\exists p_0, p_1, \dots, p_n \in Q_1, p = p_0, q = p_n$  and  $\exists a_1, a_2, \dots, a_n \in \Sigma_1$  such that  $\forall i = 1, 2, \dots, n$  either  $\mu_{A_1}(p_{i-1}, a_i, p_i) > 0$  and  $\nu_{A_1}(p_{i-1}, a_i, p_i) < 1$  or  $\mu_{A_1}(p_i, a_i, p_{i-1}) > 0$  and  $\nu_{A_1}(p_i, a_i, p_{i-1}) < 1$  and  $\exists p'_0, p'_1, \dots, p'_m \in Q_2, p' = p'_0, q' = p'_m$  and  $\exists b_1, b_2, \dots, b_m \in \Sigma_2$  such that  $\forall i = 1, 2, \dots, m$  either  $\mu_{A_2}(p_{i-1}, b_i, p_i) > 0$  and  $\nu_{A_2}(p_{i-1}, b_i, p_i) < 1$  or  $\mu_{A_2}(p'_i, b_i, p'_{i-1}) > 0$  and  $\nu_{A_2}(p'_i, b_i, p'_{i-1}) < 1$ .

Consider the sequence of states

$$(p, p') = (p_0, p'_0), (p_1, p'_0), \dots, (p_n, p'_0), (p_n, p'_1), \dots, (p_n, p'_m) = (q, q') \in Q_1 \times Q_2$$

and the sequence  $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_m \in \Sigma_1 \cup \Sigma_2$ .

Then  $\forall i = 1, 2, \dots, n$  either  $(\mu_{A_1} \odot \mu_{A_2})(p_{i-1}, p'_0, a_i, (p_i, p'_0)) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_{i-1}, p'_0, a_i, (p_i, p'_0)) < 1$  or  $(\mu_{A_1} \odot \mu_{A_2})(p_i, p'_0, a_i, (p_{i-1}, p'_0)) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_i, p'_0, a_i, (p_{i-1}, p'_0)) < 1 \forall j = 1, 2, \dots, m$  either  $(\mu_{A_1} \odot \mu_{A_2})(p_n, p'_{j-1}, b_j, (p_n, p'_j)) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_n, p'_{j-1}, b_j, (p_n, p'_j)) < 1$  or  $(\mu_{A_1} \odot \mu_{A_2})(p_n, p'_j, b_j, (p_n, p'_{j-1})) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_n, p'_j, b_j, (p_n, p'_{j-1})) < 0$ .

Hence  $(p, p')$  and  $(q, q')$  are intuitionistic connected.

Conversely, Suppose that  $\mathcal{A}_1 \odot \mathcal{A}_2$  is intuitionistic connected.

Let  $p, q \in Q_1$  and let  $r \in Q_2$ .

If  $p = q$  then  $p$  and  $q$  are intuitionistic connected.

Suppose  $p \neq q$  then  $\exists (p, r) = (p_0, q_0), (p_1, q_1), \dots, (p_n, q_n) = (q, r) \in Q_1 \times Q_2$  and  $a_1, a_2, \dots, a_n \in \Sigma_1 \cup \Sigma_2$  such that  $\forall i = 1, 2, \dots, n$  either  $(\mu_{A_1} \odot \mu_{A_2})(p_{i-1}, q_{i-1}, a_i, (p_i, q_i)) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_{i-1}, q_{i-1}, a_i, (p_i, q_i)) < 1$  or  $(\mu_{A_1} \odot \mu_{A_2})(p_i, q_i, a_i, (p_{i-1}, q_{i-1})) > 0$  and  $(\nu_{A_1} \odot \nu_{A_2})(p_i, q_i, a_i, (p_{i-1}, q_{i-1})) < 1$ .

Clearly, if  $p_{i-1} \neq p_i$  then  $q_{i-1} = q_i$  and if  $q_{i-1} \neq q_i$  then  $p_{i-1} = p_i \forall i = 1, 2, \dots, n$ .

Let  $\{p = p_{i_1}, p_{i_2}, \dots, p_{i_k} = q\}$  be the set of all distinct  $p_i \in \{p_0, p_1, \dots, p_n\}$  and let  $a_{i_1}, a_{i_2}, \dots, a_{i_k}$  be the corresponding  $a'_i$ 's.

Then  $a_{i_1}, a_{i_2}, \dots, a_{i_k} \in \Sigma_1$  and  $\forall j = 1, 2, \dots, k$

either  $\mu_{A_1}(p_{i_{j-1}}, a_{i_j}, p_{i_{j-1}}) > 0$  and  $\nu_{A_1}(p_{i_{j-1}}, a_{i_j}, p_{i_{j-1}}) < 1$  or  $\mu_{A_1}(p_{i_j}, a_{i_j}, p_{i_{j-1}}) > 0$  and  $\nu_{A_1}(p_{i_j}, a_{i_j}, p_{i_{j-1}}) < 1$ .

Thus,  $p$  and  $q$  are intuitionistic connected and hence  $\mathcal{A}_1$  is intuitionistic connected.

Similarly  $\mathcal{A}_2$  is intuitionistic connected. □

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