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A Generalized Net Model for Estimation of Decisions for PIM-Algorithm in Crossbar Switch Node*

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Abstract: The present paper concerns the modelling of switching processes in the communication node. Investigations on PIM-algorithm (Parallel Iterative Matching) for non-conflict schedule computing in packet crossbar switch communication node are considered. The PIM-algorithm is specified by means of Generalized Nets and some possibilities of improvement of the algorithm characteristics are discussed. It is shown that the Generalized nets are an effective formal apparatus to deal with this kind of tasks.

Keywords: Modelling, Generalized nets, Communication node, Crossbar switch

1 Introduction

Generalized nets (GN) [1, 2, 3] are a contemporary formal tool created to make detailed representation of connections between the structure and timing correspondence in parallel processes. They are applied to different fields, including telecommunications [3, 4]. The apparatus of GN in this research is used to synthesize a model of the well known PIM-algorithm (Parallel Iterative Matching) [5], which main feature is specifying of parallel processes while sending in switch node. Obtaining of GN-model is going to provide the possibility for analysis of the algorithm performance. If the results of its computer simulation are satisfactory, this will allow us to, with great confidence, apply the apparatus of GN to modelling and analyzing of recent algorithms of switch node, as a well as to synthesis of better modifications and new algorithms.

2 PIM Algorithm for Elimination of Conflicts during Commutation

Requests for transmission through switching n x n line switch node is presented by a $n \times n$ matrix T, named traffic matrix (n is integer). Every element t_{ij} ($t_{ij} \in$ [0, 1, 2, …]) of the traffic matrix represents a request for packet from input i to output j. For example t_{ij} = 2 means that two packets from the $i^{\text{-th}}$ input line have to be send to $j^{\text{-th}}$ output line of the switch node, etc.

It is assumed that a conflict situation is created when in any row of the T matrix the number of requests is more than 1 – this corresponds to the case when one source declares connection with more than one receiver. The any column of the T matrix hosts more than one digit 1, this also indicates a conflict situation. Avoiding conflicts is related to the switch node efficiency.

In our previous investigations algorithms for computing of non-conflict schedule are modeled by GN based on the principle of sequent-random choice, [6]. PIM-algorithm is based on distributive-random choice. Here are clearly defined stages of parallel processing of information. By means of GN we can effectively model those processes. For this aim we have to synthesize GN model of the PIM-algorithm.

We will give a succinct description of PIM-algorithm. It has three phases, [5].

1. **Request:** Every input sends request to every output for which is has a packet for transmission.
2. **Grant:** Every output chooses randomly one of the received requests and grants permission for sending to corresponding input.
3. **Accept:** Every input received grant chooses randomly one of them. That packet will be accepted for commutation.

Inputs execute parallel first phase. Outputs execute parallel second phase. Inputs are working parallel in the third phase. This parallelism is suitable for applying of GN apparatus.

3 GN Model of PIM-Algorithm

The described three phases of algorithm lead to three transitions in the GN model. The model is developed for switch node with n inputs and n outputs. Its graphic form is shown on Figure 1.

The particular places in the model are following signification:

- *in*_1, …, *in*_*n* – modelling inputs of the switch node in the initial moment.
- *p*_1, …, *p*_*n* – modelling outputs of the switch node in phase two.
- *q*_1, ..., *q*_*n* – modelling inputs of the switch node in phase three.
- *r*_1, ..., *r*_*n* – modelling non-chosen requests after phase two.
- *start* – place for initiating of schedule computing.
- *q* – place for starting of phase two.
- *o* – place for starting of phase three.
- *out* – place for current solution.
- s – non-chosen requests after phase three.
- *schedule* – place for solutions.
- *stop* – place for the end of schedule computing.

Every token in places *in_1, in_2, ..., in_n* represents a request for sending a packet and it has an initial characteristic: orderly couple of numbers including the number of the input (signed by i) and the number of output (signed by j) for which the packet is directed:

$$\text{“ } ch_0 = \langle\, pr_1\, ch_0, pr_2\, ch_0 \,\rangle = \langle\, i, j \,\rangle \text{ ”}$$

The token in place *start* has initial characteristic: $ch_0 = 0$.

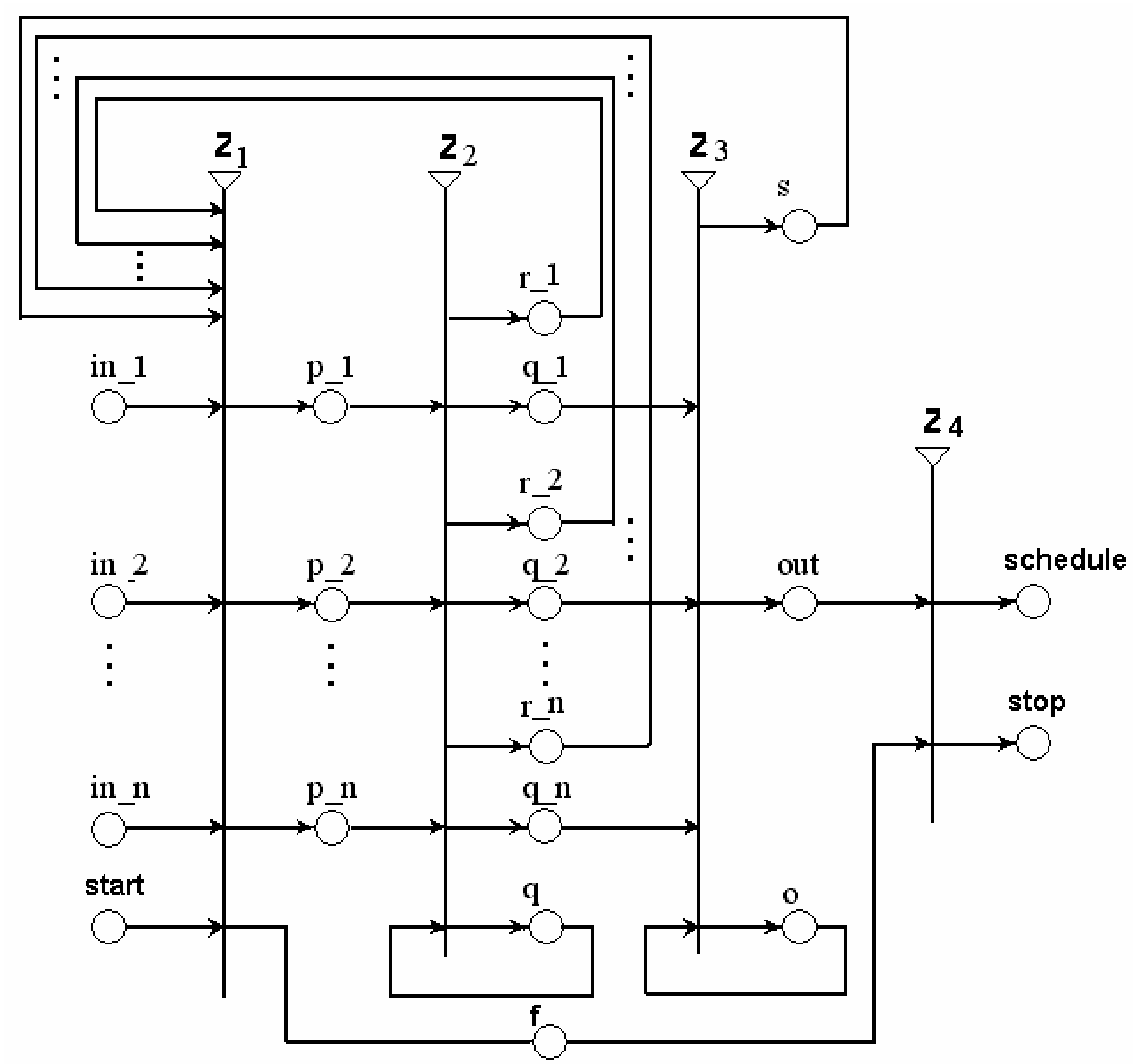

Figure 1: Graphical form of GN-model of PIM algorithm.

At the beginning of the GN functioning, the tokens in places *in_1, in_2, ..., in_n* are distributed by the first projection of their initial characteristics – in place *in*_1 there are tokens, representing packets received on the first input, in place *in_2* there are tokens, representing packets received on the second input, etc. This corresponds to the Virtual Output Queues of the PIM-algorithm (representing the traffic matrix *T*), from which the algorithm starts functioning [5].

Transition *Tr*_1 models the requests according to the outputs of switch node, for which the modeled packets are directed – Phase 1. As a result of transfer through transition *Tr*_1, tokens are distributed in places *p*_1, *p*_2, ..., *p_n* according to the second projection of their initial characteristics – in place *p*_1 are presented all requests for packets proposed for the first output, etc.

Every token that enters place *p_i* ($i = 1, \ldots, n$) obtains the following characteristics:

$$\text{“ } ch_{last} = br(\{p\_i\}) + 1 \text{ ”}$$

where $br(A)$ is the total number (sum) of the tokens in the set of places A. In this way, the necessary numeration of tokens in places *p_i* is achieved.

Transition *Tr_2* models the equipossible choice of a request for each of outputs (if there are requests for corresponding outputs) – Phase 2. This is realized by the transfer of a single token from each of place *p_i* *(i =1, ..., n)* to one ot the places *q_*1*, q_*2*, ..., q_n* (according to the input and in case that *p_i* is not empty).

As a starting place of Phase 2, place q is introduced and in the initial moment n this place is occupied by one token with initial characteristic:

$$\text{“ } ch_0{}^q = \langle\, pr_1\, ch_0{}^q, pr_2\, ch_0{}^q, \ldots, pr_n\, ch_0{}^q, pr_{n+1}\, ch_0{}^q \,\rangle \text{ ”}$$

where

- $pr_i\, ch_0^q = 0$ ($i = 1, \ldots, n$)
- $pr_{n+1} ch_0^q = -1$.

During its transfer through transition *Tr_*2, the token looping in q receives the characteristic:

$$\text{“ } ch_{last}{}^q = \langle\, pr_1\, ch_{last}{}^q, pr_2\, ch_{last}{}^q, \ldots, pr_n\, ch_{last}{}^q, pr_{n+1}\, ch_{last}{}^q \,\rangle \text{ ”}$$

where:

- $pr_i\, ch_{last}^q = random(br(\{p\_i\}))$ for $i = 1, \ldots, n$;
- $pr_{n+1} ch_{last}^q = t_{cur}$,
- $random(k)$ is a function defined in {0, 1, 2, …} returning as a result a random integer from 1 to k if the argument is larger than 0, and 0 if the argument is 0;
- t_{cur} is the current modelling time.

Every token, that enters places *q_i* ($i = 1, \ldots, n$), receives the following characteristic:

$$\text{“}ch_{last} = br(\{q\_i\}) + 1\text{”}$$

and this way the necessary numeration of tokens in *q_i* is confirmed.

By analogy, for each of the inputs, transition *Tr_*3 models the equipossible choice of a request among those already chosen (if there are requests for corresponding inputs) – Phase 3. This is realized by the transfer of the single token from each of places *q_i* to place *out* (in case that *q_i* is not empty).

To trigger the beginning of Phase 3, place o is introduced; in the first moment of the GN functioning it contains one token with initial characteristic:

$$\text{“}ch_0{}^{\mathrm{o}} = \langle\, pr_1\, ch_0{}^{\mathrm{o}}, pr_2\, ch_0{}^{\mathrm{o}}, \ldots, pr_n\, ch_0{}^{\mathrm{o}}, pr_{n+1}\, ch_0{}^{\mathrm{o}} \,\rangle\text{”}$$

where

- $pr_i\, ch_0^{\mathrm{o}} = 0$ for $i = 1, 2, \ldots, n$
- $pr_{n+1} ch_0^{\mathrm{o}} = -1$.

During the transfer through *Tr_*3 the token lopping in o obtains the characteristic:

$$\text{“}ch_{last}{}^{\mathrm{o}} = \langle\, pr_1\, ch_{last}{}^{\mathrm{o}}, pr_2\, ch_{last}{}^{\mathrm{o}}, \ldots, pr_n\, ch_{last}{}^{\mathrm{o}}, pr_{n+1}\, ch_{last}{}^{\mathrm{o}} \,\rangle\text{”}$$

where

- $pr_i\, ch_{last}{}^{\mathrm{o}} = random(br(\{q\_i\}))$ for $i = 1, \ldots, n$
- $pr_{n+1}\, ch_{last}^{\mathrm{o}} = t_{cur.}$

Having in mind to mark formation of the current solution of non-conflict schedule tokens received in place *out* become characteristics :

$$\text{“}ch_{last} = t_{cur}\text{”}.$$

The predicates associated with the transitions form corresponding index matrices, as shown in the following section.

4 Formal Description of the GN Model

The form of the first transition of GN model is :

$$Z_1 = \langle L'_1, L''_1, R_1, \vee(L'_1)\rangle$$

where

- $L'_1 = \{in\_1, \ldots, in\_n, r\_1, \ldots, r\_n, s, start\}$
- $L''_1 = \{p\_1, \ldots, p\_n, f\}$

and the index matrix is:

$$R_1 = \begin{array}{c|ccccc}
 & p_1 & p_2 & \ldots & p_n & f \\ \hline
start & false & false & \ldots & false & u_f \\
s & u_1 & u_2 & \ldots & u_n & false \\
in_1 & u_1 & u_2 & \ldots & u_n & false \\
in_2 & u_1 & u_2 & \ldots & u_n & false \\
\vdots & \vdots & \vdots & \ddots & \vdots & false \\
in_n & u_1 & u_2 & \ldots & u_n & false \\
r_1 & u_1 & u_2 & \ldots & u_n & false \\
r_2 & u_1 & u_2 & \ldots & u_n & false \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
r_n & u_1 & u_2 & \ldots & u_n & false
\end{array}$$

The predicates in R_1 have the following forms:

- $u_f =$ “$pr_{n+1}ch_{last}{}^q = -1$”
- $u_j =$ “$pr_2ch_0 = j$” $\wedge$ “$pr_{n+1}ch_{last}{}^q < t_{cur}$” for $j = 1, 2, \ldots, n$

The characteristic functions are:

- $\Phi_f =$ “$ch_{last} = br(\{in\_1, \ldots, in\_n\})$”
- $\Phi_{p-i} =$ “$ch_{last} = br(\{p\_i\}) + 1$” for $i = 1, 2, \ldots, n$

The form of the second transition of GN model is :

$$Z_2 = \langle L'_2, L''_2, R_2, \vee(L'_2)\rangle$$

where

- $L'_2 = \{p\_1, \ldots, p\_n, q\}$
- $L''_2 = \{q\_1, \ldots, q\_n, r\_1, \ldots, r\_n, q\}$

and the index matrix is:

$$R_2 = \begin{array}{c|ccccccccc} & q\_1 & q\_2 & \dots & q\_n & q & r\_1 & r\_2 & \dots & r\_n \\ \hline q & false & false & \dots & false & v_q & false & false & \dots & false \\ p\_1 & v_1 & v_2 & \dots & v_n & false & g_1 & g_2 & \dots & g_n \\ p\_2 & v_1 & v_2 & \dots & v_n & false & g_1 & g_2 & \dots & g_n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p\_n & v_1 & v_2 & \dots & v_n & false & g_1 & g_2 & \dots & g_n \end{array}$$

The predicates in R_2 have the following forms:

- v_q = "$br(\{p\_1, \dots, p\_n\}) > 0$" $\wedge$ "$br(\{in\_1, \dots, in\_n, r\_1, \dots, r\_n, s\}) = 0$"
- v_j = "$pr_{n+1}\, ch_{last}{}^{q} = t_{cur}$" $\wedge$ "$ch_{last} = pr_j\, ch_{last}{}^{q}$" $\wedge$ "$pr_1 ch_0 = j$" for j = 1, 2, …, n
- g_j = "$pr_{n+1}\, ch_{last}{}^{q} = t_{cur}$" $\wedge$ "$ch_{last} \neq pr_j\, ch_{last}{}^{q}$" $\wedge$ "$pr_1 ch_0 = j$" for j = 1, 2, …, n

The characteristic functions are:

- Φ_q = "$ch_{last} = \{random(br(\{p\_i\}))\}, t_{cur}$" for i=1, 2, …, n
- Φ_{q-i} = "$ch_{last} = br(\{p\_i\})+1$"
- Φ_{r-i} = " * "

The form of the third transition of GN model is :

$$Z_3 < L'_3, L''_3, R_3, \vee(L'_3) >$$

where

- $L'_3 = \{q\_1, \dots, q\_n, o\}$
- $L''_3 = \{out, s, o\}$

and the index matrix is

$$R_3 = \begin{array}{c|ccc} & out & o & s \\ \hline o & false & w_o & false \\ q\_1 & w_i & false & \neg w_i \\ q\_2 & w_i & false & \neg w_i \\ \dots & \dots & \dots & \dots \\ q\_n & w_i & false & \neg w_i \end{array}$$

The predicates in R_3 have the following forms:

- w_o= "$br(\{q\_1, \dots, q\_n\}) > 0$" $\wedge$ "$br(\{p\_1, \dots, p\_n\}) = 0$"
- w_i = "$pr_{n+1}\, ch_{last}{}^{o} = t_{cur}$" $\wedge$ "$ch_{last} = pr_i\, ch_{last}{}^{o}$" for i = 1, 2, …, n

The characteristic functions are:

- Φ_o = "$ch_{last} = \{random(br(\{q\_i\}))\}, t_{cur}$" for i = 1, 2, …, n
- Φ_{out} = "$ch_{last} = t_{cur}$"
- Φ_s = " * "

The form of the fourth transition of GN model is :

$$Z_4 = < L'_4, L''_3, R_4, \vee(L'_4) >$$

where

- $L'_3 = \{out, f\}$
- $L''_3 = \{schedule, stop\}$

and the index matrix is:

$$R_4 = \begin{array}{c|cc} & schedule & stop \\ \hline f & false & m_f \\ out & true & false \end{array}$$

The predicate in R_4 has the following form:

- m_f = "$ch_{last}{}^f = br(\{schedule\})$".

The characteristic functions are:

- $\Phi_{schedule}$ = " * ",
- Φ_{stop} = "End".

5 Properties of the GN Model

The capacity of input places of the first transition depends on the size of the input lines buffer. Without loss of integrity, we can assume that it equals the rank of the switch node (n). Then, the capacity of output places $p\_1, p\_2, ..., p\_n$ of the first transition is equal to n. The capacity of places $q\_1, q\_2, ..., q\_n$ is also n. The capacity of the exit place *out* is equal to n. The capacity of the additional places q and o is equal to 1. Each of the transitions has one and the same priority. The same refers to the tokens.

Analysis of the model proves receiving of a non-conflict schedule. Owing to the characteristic functions in the model, numerical values for the throughput can be obtained in the course of the model simulation. That must be done as well for balanced, as for non-balanced traffic of uniform and non-uniform kind. The model has possibilities to provide information about the number of switching of crossbar matrix, as well as about the average number of packets transmitted by one switch.

Concerning timing characteristics such as time for delay of packets, etc., this model needs further adjustment. For this purpose, the model must be complemented by more characteristic functions. Transition Z_3 (and eventually Z_4) shall be decomposed. It is a task for further investigations.

Conclusion

With the help of GN apparatus a model of PIM-algorithm for computing of non-conflict schedule of packet crossbar switch node was synthesized.

GN-model clearly presents parallelism of the processes within the algorithm. The formal apparatus used gives possibility to obtain quantity characteristics as a result of work of modelling algorithm. GN-model is a well suited for numerical simulations.

In perspective applying of GN for synthesizing models for computing of non-conflict schedule in packet switch node promises the possibility synthesizing of new algorithms.

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