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Generalized net model of the Firefly algorithm

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Abstract: The apparatus of generalized nets is herewith applied to describe the Firefly algorithm (FA). The FA is very efficient and can outperform other meta-heuristics, such as genetic algorithms, for solving many optimization problems. Although the FA has many similarities with other swarm intelligence based algorithms, such as Particle Swarm Optimization, Artificial Bee Colony Algorithm, and Bacterial Foraging Algorithm, it is indeed much simpler both in concept and implementation. The proposed generalized net model provides the opportunity to describe the logic of FA.
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1 Introduction

Although a lot of different global optimization methods exist, the efficacy of an optimization method is always problem-specific.

While searching for new, more adequate modeling metaphors and concepts, methods which draw their initial inspiration from nature have received the early attention. During the last decade, a broad class of meta-heuristics has been developed and applied to a variety of areas. The three most well-known heuristics are the iterative improvement algorithms, the probabilistic optimization algorithms, and the constructive heuristics. Recently, a new meta-heuristics called Firefly Algorithm (FA) algorithm has emerged. This algorithm was proposed by Xin-She Yang [13].

There are already several applications of FA for different optimization problems [1, 5, 7, 10, 12, 17]. Based on bibliography results, it is evident that the FA is a powerful novel population-based method for solving optimization problems.

The use of Generalized Nets (GNs) [2-4] affords the opportunities for different on-line applications; searching of optimal conditions; learning on the basis of experimental data; control on the basis of expert systems, etc. Until now the apparatus of GNs has been used as a tool for a description of parallel processes in several areas – economics, transport, medicine, computer technologies, etc. [6]. The facility of obtaining GN-models demonstrates the flexibility and the efficiency of generalized nets as modelling tools in different fields [2-4].

This fact provokes the idea of developing a GN-model of FA.

So far, GNs have been used as a tool for modelling of various Genetic algorithms – standard, multi-population and different modifications of the algorithm [8, 9, 11]. Developed GN-models execute the genetic algorithm procedure performing basic genetic operators and realizes an optimal search in the parameter space. The apparatus of GN allows describing the genetic algorithm performance for various algorithm modifications in only one GN-model. The aim of this study is to describe the algorithm of FA with a GN-model as a premise for it qualitative learning.

2 Firefly algorithm

The Firefly Algorithm is a new meta-heuristic algorithm which is inspired from flashing light behaviour of fireflies in nature. The pattern of flashes is often unique for a particular species of fireflies. Two basic functions of such flashes are to attract mating partners or communicate with them, and to attract potential victim. Additionally, flashing may also serve as a protective warning mechanism.

The flashing light can be formulated in such a way that it is associated with the objective function to be optimized. This makes it possible to formulate new meta-heuristic algorithms idealizing some of the flashing characteristics of fireflies. According to [13], the FA uses three idealized rules for simplification the algorithm:

1) All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
2) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness and brightness both decrease as the distance between them increases. If there is no brighter one than a particular firefly, it will move randomly.
3) The brightness of a firefly is affected or determined by the landscape of the objective function. For a maximization or minimization problem, the brightness can simply be proportional to the value of the objective function.

Based on these three idealized rules [13], the basic steps of the FA can be summarized as the pseudo code, presented in Fig. 1.

In the FA, there are two important issues of the variation of light intensity and the formulation of the attractiveness. For simplicity, it is assumed that the attractiveness of a firefly is determined by its brightness, which in turn is associated with the encoded objective function of the optimization problems.

In this algorithm, each firefly has a location $x = (x_1, ..., x_d)^T$ in a d-dimensional space and light intensity $I(x)$ or attractiveness $\beta(x)$, which are proportional to an objective function $f(x)$. Attractiveness $\beta(x)$ and light intensity $I(x)$ are relative and these should be judged by the rest fireflies. Thus, attractiveness will vary with the distance r_{ij} between firefly i and firefly j. So, the attractiveness β of a firefly can be defined by Eq. (1) [14-16]:

$$\beta(r) = \beta_0 e^{-\gamma r^2}, \tag{1}$$

where r or r_{ij} is the distance between the i-th and j-th of two fireflies. β_0 is the initial attractiveness at $r = 0$ and γ is a fixed light absorption coefficient that controls the decrease of the light intensity.

```
begin
Define
 light absorption coefficient γ
 initial attractiveness β0
 randomization parameter α
 objective function f(x), where
 x = (x1, ..., xd)^T
Generate initial population of fireflies
xi (i = 1, 2, ..., n)
Determine light intensity Ii at xi via
f(xi)
  while (t < MaxGeneration) do
    for i = 1 : n all n fireflies do
      for j = 1 : i all n fireflies do
        if (Ij > Ii) then
        Move firefly i towards j
        based on Eq. (3)
        end if
        Attractiveness varies with
        distance r via exp[-γr^2]
        Evaluate new solutions and
        update light intensity
      end for j
    end for i
    Rank the fireflies and find
    the current best
  end while
  Postprocess results and visualization
end begin
```

Figure 1. Pseudo code of FA

The initial solution is generated based on:

$$x_j = rand(Ub - Lb) + Lb, \tag{2}$$

where *rand* is a random number generator uniformly distributed in the space [0, 1]; *Ub* and *Lb* are the upper range and lower range of the j-th firefly, respectively.

When firefly i is attracted to another more attractive firefly j, its movement is determined by:

$$x_{i+1} = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j) + \alpha(rand - \frac{1}{2}), \tag{3}$$

where the first term is the current position of a firefly, the second term is used for considering a firefly attractiveness to light intensity seen by adjacent fireflies $\beta(r)$ (Eq. (1)), and the third term is used to describe the random movement of a firefly in case there are no brighter ones. The coefficient α is a randomization parameter determined by the problem of interest. The distance $r_{i,j}$ between any two fireflies i and j at x_i and x_j, respectively, is defined as a Cartesian or Euclidean distance, according to [14–16]:

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2}, \tag{4}$$

where $x_{i,k}$ is the k-th component of the spatial coordinate x_i of the i-th firefly.

3 Generalized net model of Firefly algorithm

The GN-model, describing the Firefly algorithm, is presented in Figure 2.

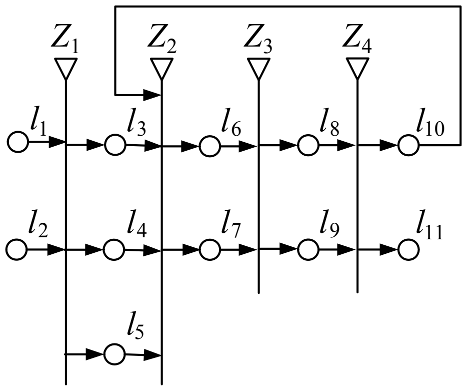

Figure 2. GN-model of Firefly algorithm

The token χ enters GN through place l_1 with an initial characteristic "FA parameters: α, β_0, γ". The token δ enters GN through place l_2 with an initial characteristic "Objective function $f(x)$". The form of the first transition of the GN-model is

$$Z_1 = \langle \{l_1, l_2\}, \{l_3, l_4, l_5\}, r_1, \vee(l_1, l_2)\rangle,$$

$$r_1 = \begin{array}{c|ccc} & l_3 & l_4 & l_5 \\ \hline l_1 & true & false & true \\ l_2 & false & true & false \end{array} .$$

The tokens χ and δ are combined in a new token ε. In place l_3, based on Eq. (2), the token ε obtains the characteristic "Initial solution". The token δ (from place l_2) keeps the same characteristic in place l_4. The token χ (from place l_1) keeps the same characteristic in place l_5.

The form of the second transition of the GN-model is

$$Z_2 = \langle \{l_3, l_4, l_5, l_{10}\}, \{l_6, l_7\}, r_2, \vee(l_3, l_4, l_5)\rangle,$$

$$r_2 = \begin{array}{c|cc} & l_6 & l_7 \\ \hline l_3 & true & false \\ l_4 & false & true \\ l_5 & true & false \\ l_{10} & false & true \end{array} .$$

In place l_6, based on Eq. (3), the token ε obtains a characteristic "New firefly position". The token δ (from place l_4) keeps the same characteristic in place l_7.

The form of the third transition of the GN-model is

$$Z_3 = \langle \{l_6, l_7\}, \{l_8, l_9\}, r_3, \vee(l_6, l_7)\rangle,$$

$$r_3 = \begin{array}{c|cc} & l_8 & l_9 \\ \hline l_6 & true & false \\ l_7 & true & true \end{array}.$$

In place l_8 the token ε obtains a characteristic "Ranked fireflies". The token δ (from place l_7) keeps the same characteristic in place l_9.

Here, based on each firefly performance, all fireflies are ranked.

The form of the fourth transition of the GN-model is

$$Z_4 = \langle \{l_{12}, l_{15}\}, \{l_{14}, l_{15}\}, r_4, \vee(l_{12}, l_{15})\rangle,$$

$$r_4 = \begin{array}{c|cc} & l_{10} & l_{11} \\ \hline l_8 & W_{8,10} & W_{8,11} \\ l_9 & W_{9,10} & W_{9,11} \end{array},$$

where:

- $W_{8,10} = W_{9,10} =$ "*End of the firefly algorithm is not reached*";
- $W_{8,11} = W_{9,11} = \neg\ W_{8,10}$.

The token δ keeps the characteristic "Objective function $f(x)$" in place l_{10} and the token ε obtains a characteristic "Best firefly" in place l_{11}.

4 Conclusion

In this paper the theory of generalized nets is used to describe the Firefly algorithm (FA). Generalized nets are preliminary proved to be an appropriate tool for description of the logics of different optimization techniques. Here, a GN-model of the optimization process based on firefly behavior is considered. Developed model executes the algorithm procedure performing basic steps and realizes an optimal search.

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