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Intuitionistic fuzzy perfectly weakly generalized continuous mappings

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Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy perfectly weakly generalized continuous mappings and intuitionistic fuzzy perfectly weakly generalized open mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy weakly generalized closed set, Intuitionistic fuzzy weakly generalized open set, Intuitionistic fuzzy perfectly weakly generalized continuous mappings, Intuitionistic fuzzy perfectly weakly generalized open mappings.

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1 Introduction

Fuzzy set (FS) as proposed by Zadeh [19] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology.

By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

In this paper, we introduce the notion of intuitionistic fuzzy perfectly weakly generalized continuous mappings and intuitionistic fuzzy perfectly weakly generalized open mappings in intuitionistic fuzzy topological space and study some of their properties. We provide some characterizations of intuitionistic fuzzy perfectly weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

2 Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A\rangle$ shall be used instead of the longer $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ are the *empty set* and the *whole set* of X, respectively.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \,/\, i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$$\text{int}(A) = \cup \{ G \,/\, G \text{ is an IFOS in X and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K \,/\, K \text{ is an IFCS in X and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be

(a) *intuitionistic fuzzy semi closed set* [6] (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,

(b) *intuitionistic fuzzy α-closed set* [6] (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,

(c) *intuitionistic fuzzy pre-closed set* [6] (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$,

(d) *intuitionistic fuzzy regular closed set* [6] (IFRCS in short) if $\text{cl}(\text{int}(A)) = A$,

(e) *intuitionistic fuzzy generalized closed set* [17] (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(f) *intuitionistic fuzzy generalized semi closed set* [16] (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(g) *intuitionistic fuzzy α generalized closed set* [14] (IFαGCS in short) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,

(h) *intuitionistic fuzzy γ closed set* [5] (IFγCS in short) if $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy α generalized open set and intuitionistic fuzzy γ open set* (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS, IFαGOS and IFγ *O*S) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS, IFαGCS and IFγCS respectively.

Definition 2.6: [8] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7: [8] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in X.

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Result 2.8: [8] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFWGCS but the converses need not be true in general.

Definition 2.9: [9] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

$$\mathrm{wgint}(A) = \cup \{G \mid G \text{ is an IFWGOS in X and } G \subseteq A\},$$
$$\mathrm{wgcl}(A) = \cap \{K \mid K \text{ is an IFWGCS in X and } A \subseteq K\}.$$

Definition 2.10: [3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \mid y \in Y\}$ is an IFS in Y, then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle \mid x \in X\}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ is an IFS in X, then the *image* of A under f denoted by f(A) is the IFS in Y defined by $f(A) = \{\langle y, f(\mu_A(y)), f\_(\nu_A(y))\rangle \mid y \in Y\}$ where $f\_(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ). Then f is said to be

(a) *intuitionistic fuzzy continuous* [4] (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B\in \sigma$,

(b) *intuitionistic fuzzy α continuous* [6] (IFα continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B\in \sigma$,

(c) *intuitionistic fuzzy pre continuous* [6] (IFP continuous in short) if $f^{-1}(B) \in IFPO(X)$ for every $B\in \sigma$,

(d) *intuitionistic fuzzy generalized continuous* [17] (IFG continuous in short) if $f^{-1}(B) \in IFGO(X)$ for every $B\in\sigma$,

(e) *intuitionistic fuzzy α generalized continuous* [15] (IFαG continuous in short) if $f^{-1}(B) \in IF\alpha GO(X)$ for every $B\in \sigma$,

(f) *intuitionistic fuzzy weakly generalized continuous* [10] (IFWG continuous in short) if $f^{-1}(B) \in IFWGO(X)$ for every $B\in \sigma$,

(g) *intuitionistic fuzzy almost continuous* [18] (IFA continuous in short) if $f^{-1}(B) \in IFO(X)$ for every IFROS $B\in \sigma$,

(h) *intuitionistic fuzzy almost weakly generalized continuous* [11] (IFAWG continuous in short) if $f^{-1}(B) \in IFWGO(X)$ for every IFROS $B\in \sigma$,

(i) *intuitionistic fuzzy quasi weakly generalized continuous* [13] if $f^{-1}(B) \in IFO(X)$ for every IFWGOS $B\in \sigma$,

(j) *intuitionistic fuzzy weakly generalized irresolute* [9] (IFWG irresolute in short) if $f^{-1}(B) \in IFWGO(X)$ for every IFWGOS $B\in \sigma$,

(k) *intuitionistic fuzzy totally continuous mapping* [7] if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen subset in X,

(l) *intuitionistic fuzzy weakly generalized * open mapping* [12] if f(A) is an IFWGOS in Y for every IFWGOS A in X.

Definition 2.12: [8] An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_wT_{1/2}$ *space* ($IF_wT_{1/2}$ space in short) if every IFWGCS in X is an IFCS in X.

Definition 2.13: **[8]** An IFTS (X, τ) is said to be an *intuitionistic fuzzy* $_{wg}T_q$ *space* ($IF_{wg}T_q$ space in short) if every IFWGCS in X is an IFPCS in X.

3 Intuitionistic fuzzy perfectly weakly generalized continuous mappings

In this section, we introduce intuitionistic fuzzy perfectly weakly generalized continuous mappings and study some of their properties.

Definition 3.1: A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an *intuitionistic fuzzy perfectly weakly generalized continuous* (IF perfectly WG continuous in short) *mapping* if the inverse image of every IFWGCS of Y is intuitionistic fuzzy clopen in X.

Theorem 3.2: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy quasi weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Therefore f is an intuitionistic fuzzy quasi weakly generalized continuous mapping. □

Example 3.3: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.7, 0.3), (0.2, 0.5)\rangle$, $T_2 = \langle y, (0.7, 0.3), (0.2, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y, respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuittionistic fuzzy quasi weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $T_2^C = \langle y, (0.2, 0.5), (0.7, 0.3)\rangle$ is an IFWGCS in Y but $f^{-1}(T_2^C) = \langle x, (0.2, 0.5), (0.7, 0.3)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.4: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y,\sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Therefore f is an intuitionistic fuzzy continuous mapping. □

Example 3.5: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.3, 0.4), (0.4, 0.5)\rangle$, $T_2 = \langle y, (0.3, 0.4), (0.4, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.6, 0.7), (0.2, 0.1)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.7), (0.2, 0.1)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.6: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy α continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαCS, $f^{-1}(A)$ is an IFαCS in X. Hence f is an intuitionistic fuzzy α continuous mapping. ☐

Example 3.7: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.4), (0.4, 0.6)\rangle$, $T_2 = \langle y, (0.2, 0.4), (0.4, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy α continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.5, 0.6), (0.2, 0.1)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.5, 0.6), (0.2, 0.1) \rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.8: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy pre continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFPCS, $f^{-1}(A)$ is an IFPCS in X. Hence f is an intuitionistic fuzzy pre continuous mapping. ☐

Example 3.9: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.3), (0.3, 0.6)\rangle$, $T_2 = \langle y, (0.2, 0.3), (0.3, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y, respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy pre continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.6, 0.4), (0.2, 0.1)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.4), (0.2, 0.1)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.10: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFGCS, $f^{-1}(A)$ is an IFGCS in X. Hence f is an intuitionistic fuzzy generalized continuous mapping. ☐

Example 3.11: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.2), (0.3, 0.4)\rangle$, $T_2 = \langle y, (0.2, 0.2), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.4, 0.5), (0.2, 0)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.4, 0.5), (0.2, 0)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.12: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy α generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαGCS, $f^{-1}(A)$ is an IFαGCS in X. Hence f is an intuitionistic fuzzy α generalized continuous mapping. ☐

Example 3.13: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.3), (0.4, 0.5)\rangle$, $T_2 = \langle y, (0.2, 0.3), (0.4, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y,\sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy α generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.3, 0.4), (0.2, 0)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.3, 0.4), (0.2, 0)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.14: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy almost weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy almost weakly generalized continuous mapping. ☐

Example 3.15: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.4, 0.5), (0.5, 0.5)\rangle$, $T_2 = \langle y, (0.4, 0.5), (0.5, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuittionistic fuzzy almost weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.16: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy almost continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Hence f is an intuitionistic fuzzy almost continuous mapping. ☐

Example 3.17: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.3, 0.5), (0.4, 0.5)\rangle$, $T_2 = \langle y, (0.3, 0.5), (0.4, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy almost continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.5, 0.6), (0.4, 0.2)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.5, 0.6), (0.4, 0.2)\rangle$ is not intuitionistic fuzzy clopen in X.

Theorem 3.18: Every intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFWGCS, A is an IFWGCS in Y. By hypothesis, $f^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFWGCS, $f^{-1}(A)$ is an IFWGCS in X. Hence f is an intuitionistic fuzzy weakly generalized continuous mapping. ☐

Example 3.19: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.2, 0.1), (0.4, 0.5)\rangle$, $T_2 = \langle y, (0.2, 0.1), (0.4, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y, respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = u and f (b) = v. This f is an intuitionistic fuzzy weakly generalized continuous mapping but not an intuitionistic fuzzy perfectly weakly generalized continuous mapping, since the IFS $B = \langle y, (0.6, 0.7), (0.2, 0.2)\rangle$ is an IFWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.7), (0.2, 0.2)\rangle$ is not intuitionistic fuzzy clopen in X.

The relations among various types of intuitionistic fuzzy continuities are given in the following diagram. In this diagram 'cts' means continuous.

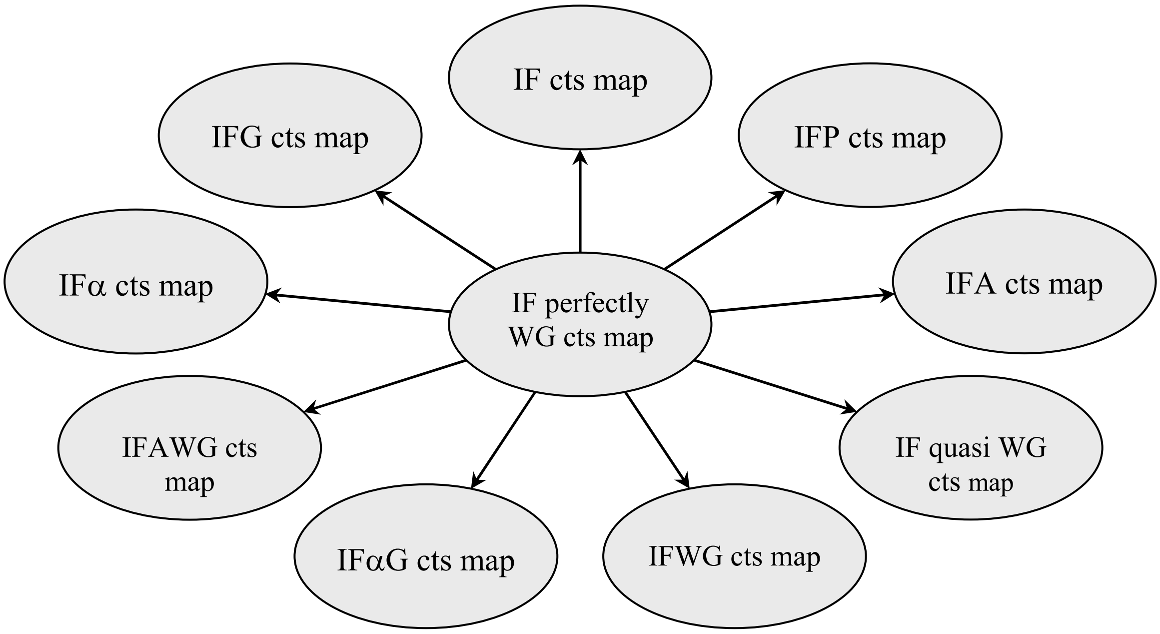

Figure 1: Relation between intuitionistic fuzzy perfectly weakly generalized continuous mappings and other existing intuitionistic fuzzy continuous mappings

The reverse implications are not true in general in the above diagram. In this diagram by "A $\to$ B" we mean A implies B but not conversely.

Theorem 3.20: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the following statements are equivalent.

(a) f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping,

(b) $f^{-1}(B)$ is intuitionistic fuzzy clopen in X for every IFWGOS B in Y.

Proof: (a) ⇒ (b): Let B be an IFWGOS in Y. Then B^c is an IFWGCS in Y. Since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(B^c) = (f^{-1}(B))^c$ is intuitionistic fuzzy clopen in X. This implies $f^{-1}(B)$ is intuitionistic fuzzy clopen in X.

(b) ⇒ (a): Let B be an IFWGCS in Y. Then B^c is an IFWGOS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is intuitionistic fuzzy clopen in X, which implies $f^{-1}(B)$ is intuitionistic fuzzy clopen in X. Therefore f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping. □

Theorem 3.21: If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ), then $f(cl(A)) \subseteq wgcl(f(A))$ for every IFS A in X.

Proof: Let A be an IFS in X. Then $wgcl(f(A))$ is an IFWGCS in Y. Since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(wgcl(f(A)))$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(wgcl(f(A)))$ is an IFCS in X. Clearly $A \subseteq f^{-1}(wgcl(f(A)))$. Therefore, $cl(A) \subseteq cl(f^{-1}(wgcl(f(A)))) = f^{-1}(wgcl(f(A)))$. Hence $f(cl(A)) \subseteq wgcl(f(A))$ for every IFS A in X. □

Theorem 3.22: If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y, σ), then $cl(f^{-1}(B)) \subseteq f^{-1}(wgcl(B))$ for every IFS B in Y.

Proof: Let B be an IFS in Y. Then wgcl(B) is an IFWGCS in Y. By hypothesis, $f^{-1}(wgcl\ (B))$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(wgcl(B))$ is an IFCS in X. Clearly $B \subseteq wgcl(B)$ implies $f^{-1}(B) \subseteq f^{-1}(wgcl(B))$. Therefore $cl(f^{-1}(B)) \subseteq cl(f^{-1}(wgcl\ (B))) = f^{-1}(wgcl(B))$. Hence $cl(f^{-1}(B)) \subseteq f^{-1}(wgcl(B))$ for every IFS B in Y. □

Theorem 3.23: If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping from an IFTS (X, τ) into an IFTS (Y,σ), then $f^{-1}(wgint(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let B be an IFS in Y. Then wgint(B) is an IFWGOS in Y. By hypothesis, $f^{-1}(wgint(B))$ is intuitionistic fuzzy clopen in X. Thus $f^{-1}(wgint(B))$ is an IFOS in X. Clearly $wgint(B) \subseteq B$ implies $f^{-1}(wgint(B)) \subseteq f^{-1}(B)$. Therefore $int(f^{-1}(wgint(B))) \subseteq int(f^{-1}(B))$. Hence, $f^{-1}(wgint(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y. □

Theorem 3.24: The composition of two intuitionistic fuzzy perfectly weakly generalized continuous mapping is an intuitionistic fuzzy perfectly weakly generalized continuous mapping in general.

Proof: Let A be an IFWGCS in Z. By hypothesis, $g^{-1}(A)$ is intuitionistic fuzzy clopen in Y and hence an IFCS in Y. Since every IFCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Further, since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (g_\circ f)^{-1}(A)$ is intuitionistic fuzzy clopen in X. Hence $g_\circ f$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping. □

Theorem 3.25: Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings. Then the following statements hold.

(i) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Then their composition $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping.

(ii) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy continuous mapping [respectively intuitionistic fuzzy α continuous mapping, intuitionistic fuzzy pre continuous mapping, intuitionistic fuzzy α generalized continuous mapping and intuitionistic fuzzy generalized continuous mapping]. Then their composition $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy continuous mapping.

(iii) Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ an intuitionistic fuzzy weakly generalized continuous mapping. Then their composition $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy continuous mapping.

Proof: (i) Let A be an IFWGCS in Z. By hypothesis, $g^{-1}(A)$ is intuitionistic fuzzy clopen in Y and hence an IFCS in Y. Since f is an intuitionistic fuzzy continuous mapping, $f^{-1}(g^{-1}(A)) = (g_\circ f)^{-1}(A)$ is an IFCS in X. Hence $g_\circ f$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping.

(ii) Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] in Y. Since every IFCS [respectively IFαCS, IFPCS, IFαGCS and IFGCS] is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y. Then $f^{-1}(g^{-1}(A)) = (g_\circ f)^{-1}(A)$ is intuitionistic fuzzy clopen in X, by hypothesis. Thus $(g_\circ f)^{-1}(A)$ is an IFCS in X. Hence $g_\circ f$ is an intuitionistic fuzzy continuous mapping.

(iii) Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFWGCS in Y. Since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (g_\circ f)^{-1}(A)$ is intuitionistic fuzzy clopen in X. Thus $(g_\circ f)^{-1}(A)$ is an IFCS in X. Hence $g_\circ f$ is an intuitionistic fuzzy continuous mapping. □

Theorem 3.26: If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an intuitionistic fuzzy weakly generalized irresolute mapping, then $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping.

Proof: Let A be an IFWGCS in Z. By hypothesis, $g^{-1}(A)$ is an IFWGCS in Y. Since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (g_\circ f)^{-1}(A)$ is intuitionistic fuzzy clopen in X. Hence $g_\circ f$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping. □

Theorem 3.27: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ be any mapping. Then $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping if and only if g is an intuitionistic fuzzy weakly generalized irresolute mapping.

Proof: Let $g : (Y, \sigma) \to (Z, \delta)$ be an intuitionistic fuzzy weakly generalized irresolute mapping. Then the proof follows from the theorem 3.26.

Conversely, let $g_\circ f : (X, \tau) \to (Z, \delta)$ be an intuitionistic fuzzy perfectly weakly generalized continuous mapping. Let A be an IFWGCS in Z. Since $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $(g_\circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy clopen in X. Since f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping, $g^{-1}(A)$ is an IFWGCS in Y. Thus the inverse image of each IFWGCS in Z is an IFWGCS in Y. Hence g is an intuitionistic fuzzy weakly generalized irresolute mapping. □

4 Intuitionistic fuzzy perfectly weakly generalized open mappings

In this section, we introduce intuitionistic fuzzy perfectly weakly generalized open mappings and study some of their properties.

Definition 4.1: A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an *intuitionistic fuzzy perfectly weakly generalized open mapping* if the image of every IFWGOS in X is intuitionistic fuzzy clopen in Y.

Theorem 4.2: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent.

(a) f is an intuitionistic fuzzy perfectly weakly generalized open mapping,

(b) f(B) is intuitionistic fuzzy clopen in Y for every IFWGCS B in X.

Proof: (a) ⇒ (b): Let B be an IFWGCS in X. Then B^c is an IFWGOS in X. Since f is an intuitionistic fuzzy perfectly weakly generalized open mapping, $f(B^c) = (f(B))^c$ is intuitionistic fuzzy clopen in Y. This implies f(B) is intuitionistic fuzzy clopen in Y.

(b) ⇒ (a): Let B be an IFWGOS in X. Then B^c is an IFWGCS in X. By hypothesis, $f(B^c) = (f(B))^c$ is intuitionistic fuzzy clopen in Y, which implies that f(B) is intuitionistic fuzzy clopen in Y. Therefore f is an intuitionistic fuzzy perfectly weakly generalized open mapping. □

Theorem 4.3: Let $f : (X, \tau) \to (Y,\sigma)$ be a bijective mapping from an IFTS (X, τ) into an IFTS (Y,σ), then the following statements are equivalent.

(a) Inverse of f is an intuitionistic fuzzy perfectly weakly generalized continuous mapping.

(b) f is an intuitionistic fuzzy perfectly weakly generalized open mapping.

Proof: (a) ⇒ (b): Let A be an IFWGOS of X. By assumption, $(f^{-1})^{-1}(A) = f(A)$ is intuitionistic fuzzy clopen in Y. Hence f is an intuitionistic fuzzy perfectly weakly generalized open mapping.

(b) ⇒ (a): Let B be an IFWGOS in X. Then f(B) is intuitionistic fuzzy clopen in Y. That is $(f^{-1})^{-1}(B) = f(B)$ is intuitionistic fuzzy clopen in Y. Therefore f^{-1} is an intuitionistic fuzzy perfectly weakly generalized continuous mapping. □

Theorem 4.4: The composition of two intuitionistic fuzzy perfectly weakly generalized open mapping is again an intuitionistic fuzzy perfectly weakly generalized open mapping.

Proof: Suppose $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ are any two intuitionistic fuzzy perfectly weakly generalized open mapping. Let A be an IFWGOS in X. Since f is an intuitionistic fuzzy perfectly weakly generalized open mapping, f(A) is intuitionistic fuzzy clopen in Y. Hence it is an IFOS in Y. But every IFOS is an IFWGOS, which implies f(A) is an IFWGOS in Y. Since g is an intuitionistic fuzzy perfectly weakly generalized open mapping, $g(f(A)) = (g_\circ f)(A)$ is intuitionistic fuzzy clopen in Z. Thus the image of each IFWGOS in X is intuitionistic fuzzy clopen in Z. Therefore $g_\circ f: (X, \tau) \to (Z, \delta)$ is intuitionistic fuzzy perfectly weakly generalized open mapping. □

Theorem 4.5: If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy weakly generalized * open mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized open mapping, then their composition $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized open mapping.

Proof: Let A be an IFWGOS in X. Since f is an intuitionistic fuzzy weakly generalized * open mapping, f(A) is an IFWGOS in Y. Further, since g is an intuitionistic fuzzy perfectly weakly generalized open mapping, $g(f(A)) = (g_\circ f)(A)$ is intuitionistic fuzzy clopen in Z. Hence $g_\circ f : (X, \tau) \to (Z,\delta)$ is an intuitionistic fuzzy perfectly weakly generalized open mapping. □

Theorem 4.6: Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y,\sigma) \to (Z, \delta)$ be two mappings such that $g_\circ f : (X, \tau) \to (Z, \delta)$ is an intuitionistic fuzzy perfectly weakly generalized open mapping. Then the following statements hold.

(a) If f is an intuitionistic fuzzy weakly generalized irresolute mapping and surjective, then g is an intuitionistic fuzzy perfectly weakly generalized open mapping.

(b) If g is an intuitionistic fuzzy totally continuous mapping and injective, then f is an intuitionistic fuzzy perfectly weakly generalized open mapping.

Proof: (a) Let A be an IFWGOS in Y. Then $f^{-1}(A)$ is an IFWGOS in X, because f is an intuitionistic fuzzy weakly generalized irresolute mapping. Since $(g_\circ f)$ is an intuitionistic fuzzy perfectly weakly generalized open mapping, $(g_\circ f)(f^{-1}(A)) = g(A)$ is intuitionistic fuzzy clopen in Z. This shows that g is an intuitionistic fuzzy perfectly weakly generalized open mapping.

(b) Since g is injective, we have, $f(A) = g^{-1}(g_\circ f)(A)$ is true for every subset A of X. Let B be an IFWGOS in X. Therefore $(g_\circ f)(B)$ is intuitionistic fuzzy clopen in Z and hence an IFOS in Z. Since g is intuitionistic fuzzy totally continuous, $g^{-1}(g_\circ f)(A) = f(A)$ is intuitionistic fuzzy clopen in Y. This shows that f is an intuitionistic fuzzy perfectly weakly generalized open mapping. □

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