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Solution of system of first order linear differential equations in intuitionistic fuzzy environment

Neha Singhal[1] and S. P. Sharma[2]

Department of Mathematics
Indian Institute of Technology Roorkee, Roorkee, India
[1] nehasinghal.iitr@gmail.com
[2] sspprfma@iitr.ac.in

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Abstract: The purpose of this manuscript is to present a method for solving system of linear ordinary differential equations in intuitionistic fuzzy environment. In the present study, authors discussed method for getting Intuitionistic fuzzy solution of the system of differential equations with intuitionistic fuzzy coefficients and initial conditions. Present study finds industrial application to evaluate the availability of the system.

Keywords: Availability, Differential equation, Intuitionistic fuzzy number, Intuitionistic fuzzy solution, Markov model.
AMS Classification: 03E72, 90B25, 62N05, 60K20.

1 Introduction

In real world, most of problems we deal with contains uncertain and imprecise information. To deal with these types of uncertainties Zadeh [1] introduced the concept of fuzzy set theory. A lot of work has been done in the field of fuzzy set theory. Fuzzy set theory uses membership function to handle the uncertainty. But in real world, there are many situations concerned with the degree of hesitation. Then in 1983 Atanassov [2, 3] introduced the notion of intuitionistic fuzzy set (IFS) theory as the generalization of fuzzy set theory. Many authors [4, 5] worked in theoretical as well as in practical applications of intuitionistic fuzzy set theory.

Further from the theoretical as well as application point of view, importance of differential equation is well-known. Fuzzy differential equations have also been studied [6, 7, 8].

In this paper, authors aim to study the solution of system of intuitionistic linear differential equations. A system of linear ordinary differential equations can be written as

$$\frac{dY(t)}{dt} = AY(t) + g(t), \tag{1}$$

where $(Y(t))^T = (y_1(t), y_2(t), ..., y_n(t))$, $(g(t))^T = (g_1(t), g_2(t), ..., g_n(t))$ and $A = [a_{ij}]$ is an $n \times n$ matrix of constants. All the $g_i(t)$'s are assumed continuous on some interval I. The independent variable $t \in I$, where $I = [0, T], T > 0$ or $[0, \infty)$ and $(Y(0))^T = (\gamma_1, \gamma_2, ..., \gamma_n)$. In intuitionistic fuzzy environment, authors have considered elements of matrix A and initial conditions as intuitionistic fuzzy numbers. For simplicity, triangular fuzzy numbers have been taken into account. Ettoussi et al. [9] discussed about existence and uniqueness of a solution of the intuitionistic fuzzy differential equation by approximation method. Nirmala and Pandian [10] discussed the numerical approach for solving Intuitionistic fuzzy differential equations under generalized differentiability concept.

In Section 3 of the present paper, intuitionistic fuzzy solution of the system of differential equations with intuitionistic fuzzy coefficients and initial conditions has been found by using (α, β)-cut method. In Section 4, an industrial application of this system of intuitionistic fuzzy linear differential equations has been presented. Conclusion of the article is discussed in Section 5. For brevity, some basic definitions have been discussed first.

2 Preliminaries

In IFS theory, the element x in the universe X is associated with its membership (called acceptance) as well as non-membership (called rejection) value such that their sum always belongs to the unit interval $[0, 1]$.

Mathematically, let X be a universe of discourse. Then the IFS $\tilde{A}$ in X is stated as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\}$ where the functions $\mu_{\tilde{A}} : X \to [0, 1]$ and $\nu_{\tilde{A}} : X \to [0, 1]$ are subjected to the condition $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1,\ \forall x \in X$. The values $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ symbolize the degree of acceptance and rejection of element x in set $\tilde{A}$, respectively.

2.1 (α, β)–Cut

An $(\alpha, \beta)-$cut of IFS $\tilde{A}$ denoted as $\tilde{A}[\alpha, \beta]$ or here as $(\tilde{A}^\alpha, \tilde{A}_\beta)$ is defined by $\tilde{A}[\alpha, \beta] = \tilde{A}^\alpha \cap \tilde{A}_\beta$, where $\tilde{A}^\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$ and $\tilde{A}_\beta = \{x \in X \mid \nu_{\tilde{A}}(x) \leq \beta\}$ for $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ such that $\alpha + \beta \leq 1$.

In this paper, we separately define $\tilde{A}^0$ as the closure of the union of all $\tilde{A}^\alpha$'s for $\alpha \in (0, 1]$. Similarly, $\tilde{A}_1$ as the closure of the union of all $\tilde{A}_\beta$'s for $\beta \in [0, 1)$.

2.2 Convex intuitionistic fuzzy set

An IFS $\tilde{A}$ in some continuous subset X of R is convex iff its membership function $\mu_{\tilde{A}}(x)$ is fuzzy convex while non-membership function $\nu_{\tilde{A}}(x)$ is fuzzy concave, i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda x_2)) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$

and

$$\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda x_2)) \leq max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2))$$

$\forall x_1, x_2 \in X, \;\; 0 \leq \lambda \leq 1.$

2.3 Normal intuitionistic fuzzy set

An IFS $\tilde{A}$ in X is normal if there exists at least one point $x_0 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1$.

2.4 Intuitionistic fuzzy number (IFN)

An intuitionistic fuzzy subset $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in R\}$ of the real line R is called Intuitionistic Fuzzy Number (IFN) if

1. $\tilde{A}$ is normal and convex IFS,
2. $\mu_{\tilde{A}}(x)$ is upper semi-continuous and $\nu_{\tilde{A}}(x)$ is lower semi-continuous,
3. $A = \{x \in R \mid \nu_{\tilde{A}}(x) < 1\}$ is bounded.

2.5 Triangular intuitionistic fuzzy number (TIFN)

A TIFN $\tilde{A}$ with parameters $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ is a subset of IFS in R, denoted as $\tilde{A} = \left\langle (a_1, a_2, a_3); (a_1', a_2, a_3')\right\rangle$ with membership and non-membership functions defined respectively by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, & a_2 \leq x \leq a_3' \\ 1, & \text{otherwise} \end{cases}.$$

3 Proposed method

The suggested approach is to find the solution of system of intuitionistic fuzzy differential equations:

$$\frac{d\tilde{Y}(t)}{dt} = \tilde{A}\tilde{Y}(t) + g(t), \text{ with initial conditions } \tilde{Y}(0)^T = (\tilde{\gamma}_1, \tilde{\gamma}_2, ..., \tilde{\gamma}_n) \quad (2)$$

where

(i) $\tilde{\gamma}_i$'s are triangular intuitionistic fuzzy numbers.

(ii) $\tilde{A} = [\tilde{a}_{ij}]$ an $n \times n$ matrix of triangular intuitionistic fuzzy numbers.

(iii) $(g(t))^T = (g_1(t), g_2(t), ..., g_n(t))$, with all the $g_i(t)$'s for $i = 1, 2..., n$ as continuous functions on the interval I.

Let $(\tilde{Y}(t))^T = (\tilde{y}_1(t), \tilde{y}_2(t), ..., \tilde{y}_n(t))$ be the intuitionistic fuzzy function where all the $\tilde{y}_i(t)$'s are intuitionistic fuzzy subsets of real numbers for $t \in I$. Let $\tilde{y}_i(t)[\alpha, \beta] = \tilde{y}_i(t)^\alpha \cap \tilde{y}_i(t)_\beta$ where $\tilde{y}_i(t)^\alpha$ and $\tilde{y}_i(t)_\beta$ are closed and bounded intervals for all t and i. Let

$$\tilde{y}_i(t)^\alpha = [(\tilde{y}_i(t))^\alpha_{(L)}, (\tilde{y}_i(t))^\alpha_{(R)}],$$

$$\tilde{y}_i(t)_\beta = [(\tilde{y}_i(t))_{\beta(L)}, (\tilde{y}_i(t))_{\beta(R)}],$$

where $(\tilde{y}_i(t))^\alpha_{(L)}$ and $(\tilde{y}_i(t))^\alpha_{(R)}$ are functions of α and t, $(\tilde{y}_i(t))_{\beta(L)}$ and $(\tilde{y}_i(t))_{\beta(R)}$ are functions of β and t.

Assume that all $(\tilde{y}_i)^\alpha_{(L)}, (\tilde{y}_i)^\alpha_{(R)}, (\tilde{y}_i)_{\beta(L)}$ and $(\tilde{y}_i)_{\beta(R)}$ are continuously differentiable functions on t for all α and β, $1 \leq i \leq n$. Now substitute the (α, β)-cuts of $\tilde{Y}(t)$ into Eq. (2). Then using the concepts of interval arithmetic, system of intuitionistic differential equations (2) reduces to the following differential equations (3)-(6):

$$(\tilde{y}_i'(t))^\alpha_{(L)} = \sum_{j=1}^{n} b_{ij} x_j + g_i(t), \tag{3}$$

where $b_{ij} x_j = \min((\tilde{a}_{ij})^\alpha_{(L)}(\tilde{y}_j)^\alpha_{(L)}, (\tilde{a}_{ij})^\alpha_{(L)}(\tilde{y}_j)^\alpha_{(R)},$

$$(\tilde{a}_{ij})^\alpha_{(R)}(\tilde{y}_j)^\alpha_{(L)}, (\tilde{a}_{ij})^\alpha_{(R)}(\tilde{y}_j)^\alpha_{(R)})$$

$$(\tilde{y}_i'(t))^\alpha_{(R)} = \sum_{j=1}^{n} c_{ij} x_j + g_i(t), \tag{4}$$

where $c_{ij} x_j = \max((\tilde{a}_{ij})^\alpha_{(L)}(\tilde{y}_j)^\alpha_{(L)}, (\tilde{a}_{ij})^\alpha_{(L)}(\tilde{y}_j)^\alpha_{(R)},$

$$(\tilde{a}_{ij})^\alpha_{(R)}(\tilde{y}_j)^\alpha_{(L)}, (\tilde{a}_{ij})^\alpha_{(R)}(\tilde{y}_j)^\alpha_{(R)})$$

$$(\tilde{y}_i'(t))_{\beta(L)} = \sum_{j=1}^{n} b'_{ij} x_j + g_i(t), \tag{5}$$

where $b'_{ij} x_j = \min((\tilde{a}_{ij})_{\beta(L)}(\tilde{y}_j)_{\beta(L)}, (\tilde{a}_{ij})_{\beta(L)}(\tilde{y}_j)_{\beta(R)},$

$$(\tilde{a}_{ij})_{\beta(R)}(\tilde{y}_j)_{\beta(L)}, (\tilde{a}_{ij})_{\beta(R)}(\tilde{y}_j)_{\beta(R)})$$

$$(\tilde{y}_i'(t))_{\beta(R)} = \sum_{j=1}^{n} c'_{ij} x_j + g_i(t), \tag{6}$$

where $c'_{ij} x_j = \max((\tilde{a}_{ij})_{\beta(L)}(\tilde{y}_j)_{\beta(L)}, (\tilde{a}_{ij})_{\beta(L)}(\tilde{y}_j)_{\beta(R)},$

$$(\tilde{a}_{ij})_{\beta(R)}(\tilde{y}_j)_{\beta(L)}, (\tilde{a}_{ij})_{\beta(R)}(\tilde{y}_j)_{\beta(R)})$$

with the initial conditions $(\tilde{y}_i(0))^{\alpha}_{(L)} = (\tilde{\gamma})^{\alpha}_{(L)}, (\tilde{y}_i(0))^{\alpha}_{(R)} = (\tilde{\gamma})^{\alpha}_{(R)}, (\tilde{y}_i(0))_{\beta(L)} = (\tilde{\gamma})_{\beta(L)}$ and $(\tilde{y}_i(0))_{\beta(R)} = (\tilde{\gamma})_{\beta(R)}$ for $1 \leq i \leq n$.

Then these converted ordinary differential equations (3)-(6) for each $\alpha, \beta \in [0, 1]$ can be solved by standard methods. Clearly $\tilde{Y}(t)$ is an intuitionistic fuzzy solution for all t if the obtained values of $(\tilde{y}_i(t))^{\alpha}_{(L)}$, $(\tilde{y}_i(t))^{\alpha}_{(R)}$, $(\tilde{y}_i(t))_{\beta(L)}$ and $(\tilde{y}_i(t))_{\beta(R)}$ define the (α, β)-cuts

$$([(\tilde{y}_i(t))^{\alpha}_{(L)}, (\tilde{y}_i(t))^{\alpha}_{(R)}], [(\tilde{y}_i(t))_{\beta(L)}, (\tilde{y}_i(t))_{\beta(R)}])$$

of triangular intuitionistic fuzzy numbers. Thus, we can say that $\tilde{Y}(t)$ is an intuitionistic fuzzy solution of Eq.(2) if following conditions are met:

1. $\frac{\partial(\tilde{y}_i)^{\alpha}_{(L)}}{\partial\alpha} > 0$ and $\frac{\partial(\tilde{y}_i)^{\alpha}_{(R)}}{\partial\alpha} < 0$,
   i.e., $(\tilde{y}_i)^{\alpha}_{(L)}$ is increasing and $(\tilde{y}_i)^{\alpha}_{(R)}$ is decreasing as α increases.

2. $\frac{\partial(\tilde{y}_i)_{\beta(L)}}{\partial\beta} < 0$ and $\frac{\partial(\tilde{y}_i)_{\beta(R)}}{\partial\beta} > 0$,
   i.e., $(\tilde{y}_i)_{\beta(L)}$ is decreasing and $(\tilde{y}_i)_{\beta(R)}$ is increasing as β increases.

3. $(\tilde{y}_i)^{\alpha}_{(L)} = (\tilde{y}_i)^{\alpha}_{(R)}$ for $\alpha = 1$ and $(\tilde{y}_i)_{\beta(L)} = (\tilde{y}_i)_{\beta(R)}$ for $\beta = 0$,

for all $\alpha, \beta \in [0, 1]$, $t \in I$ and $1 \leq i \leq n$.

4 Application

Plastic manufacturing plant is a complex repairable system composed of numerous complex components. To reduce the complexity for the behavior analysis of the system, Piston manufacturing plant is composed of two subsystems S_1 and S_2 as described in [11].

As an illustration of the suggested method, we have considered the subsystem S_2 of Piston manufacturing plant described in [11]. System S_2 is composed of many subsystems out of which major subsystems whose failure shows complete breakdown of the system, are of importance. These are described below.

- Subsystem A that denotes the finish pin hole boring machine.
- Subsystem B that denotes the finish crown and cavity which is used to give finishing of the upper part of the piston i.e. crown.
- Subsystem C that denotes the valve milling machine to create the valve recession of the piston.
- Subsystem D which is a chamfering machine used to round off the corners of the piston for its smooth running.
- Subsystem E that denotes the circlip grooving machine for making the circlip grooves on the piston.
- Subsystem F which is the cleaning machine that helps to clean the piston.

In addition to these subsystems in S_2, there are subsystems described below which are considered to have no failure.

- System G which is deburring machine that is used to neaten and smooth the rough edges of the piston.
- System H which is the surface treatment equipment whose operation is to coat the piston.
- System I is final inspection of the manufactured product.

4.1 Notations

In this subsection, notations that are used for examining the availability of the system are given below.

○	Represents working state of the system.
▭	Represents failed state of the system.
a, b, c, d, e, f	Failed states of the subsystem.
A, B, C, D, E, F	Working states of the subsystem.
$\bar{A}$	Reduced state of the subsystem A.
$P_1(t)$	Probability of working of the system in full capacity at time 't'.
$P_2(t)$	Probability of working of the system in reduced state at time 't'.
$P_3(t)$ to $P_{13}(t)$	Probability of failed state of the system at time 't'.
$\lambda_i, i = 1, 2..., 7$	Failure rates of $B, C, D, E, F, \bar{A}, A$, respectively.
$\mu_i, i = 1, 2..., 7$	Repair rates of $B, C, D, E, F, \bar{A}, A$, respectively.
Av	Availability of the system.

Transition diagram of this system S_2 is presented in Figure 1.

4.2 Mathematical formulation

Using the concepts of probability and markov modeling, following intuitionistic fuzzy differential equations corresponding to the transition diagram (Figure 1) are formulated as:

$$\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_1(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_2(t) \oplus \tilde{\mu}_7 \tilde{P}_{13}(t) \tag{7}$$

$$\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_2(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t) \tag{8}$$

$$\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+2}(t) = \tilde{\lambda}_i \tilde{P}_1(t), \qquad i = 1, 2, ..., 5 \tag{9}$$

$$\frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+7}(t) = \tilde{\lambda}_i \tilde{P}_2(t) \qquad i = 1, 2, ..., 6 \tag{10}$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6} \tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5} \tilde{\lambda}_j \oplus \tilde{\lambda}_7 \oplus \tilde{\mu}_6$, and initial conditions as
$\tilde{P}_1(0) = \langle (0.95, 0.96, 0.97); (0.945, 0.96, 0.975) \rangle$

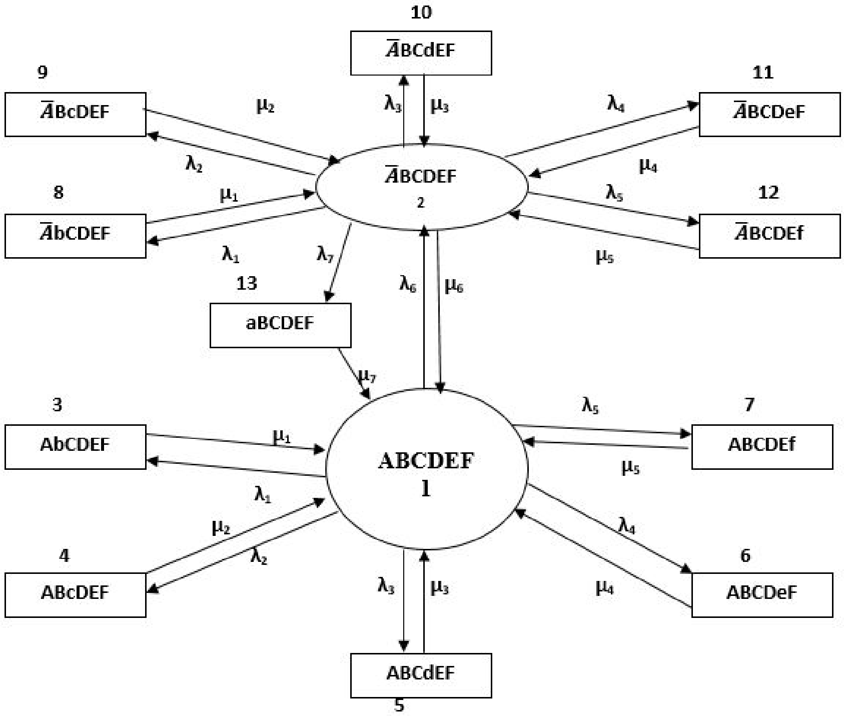

Figure 1. Transition diagram of the system

$\tilde{P}_2(0) = \langle(0.003, 0.004, 0.005); (0.0025, 0.004, 0.0055)\rangle$ and
$\tilde{P}_j(0) = \langle(0, 0, 0, 0); (0, 0, 0, 0)\rangle$ for $j = 3$ to 13.

Herein we take the failure and repair rates of the system S_2 as taken by [11], i.e.,

$$\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7) = (0.0014, 0.0003, 0.0001, 0.0003, 0.0001, 0.0208, 0.004)$$

$$\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7) = (0.33, 0.5, 0.67, 0.35, 3.03, 0.222, 0.125)$$

and we have taken the spread of $\pm 10\%$ and $\pm 12\%$ for degree of acceptance and rejection respectively in failure and repair rates treated as triangular intuitionistic fuzzy numbers. Now system of intuitionistic fuzzy differential equations (7)-(10) has been converted into the system of ordinary differential equations by the proposed method. The reduced system of differential equations is solved here by Runge-Kutta fourth order method.

Availability function $\tilde{A}v(t)$ of the system S_2 in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ can be obtained by

$$\tilde{A}v(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t).$$

4.3 Results

Solution of differential equations (7)-(10) and availability function are shown in Tables 1 and 2, respectively. It can be observed that the obtained solutions in Table 1, define the (α, β)-cuts of the triangular intuitionistic fuzzy numbers. It is observed from the Table 2 that $\tilde{A}v^{\alpha}_{(L)}$ is increasing and $\tilde{A}v^{\alpha}_{(R)}$ is decreasing as α increases and the two coincide at $\alpha = 1$ while $\tilde{A}v_{\beta(L)}$ and $\tilde{A}v_{\beta(R)}$ coincide at $\beta = 0$. $\tilde{A}v_{\beta(L)}$ is decreasing and $\tilde{A}v_{\beta(R)}$ is increasing as β increases.

Table 1. Solution of Intuitionistic fuzzy differential equations at $t = 360$h

j	$\tilde{P}_j^\alpha$ for $\alpha = 0$		$\tilde{P}_j^\alpha$ for $\alpha = 0.2$		$\tilde{P}_j^\alpha$ for $\alpha = 0.4$		$\tilde{P}_j^\alpha$ for $\alpha = 0.6$		$\tilde{P}_j^\alpha$ for $\alpha = 0.8$		$\tilde{P}_j^\alpha$ for $\alpha = 1$	
	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$	$\tilde{P}_{j(L)}^\alpha$	$\tilde{P}_{j(R)}^\alpha$
1	0.8635411	0.8744877	0.8644743	0.8732563	0.8654497	0.8720506	0.8664646	0.8708720	0.8675165	0.8697223	0.8686031	0.8686031
2	0.0751349	0.0840745	0.0761616	0.0832926	0.0771535	0.0824897	0.0781128	0.0816646	0.0790418	0.0808160	0.0799422	0.0799422
3	0.0034599	0.0038786	0.0035080	0.0038419	0.0035544	0.0038043	0.0035993	0.0037656	0.0036428	0.0037259	0.0036850	0.0036850
4	0.0004893	0.0005485	0.0004961	0.0005434	0.0005027	0.0005380	0.0005090	0.0005325	0.0005152	0.0005269	0.0005212	0.0005212
5	0.0001217	0.0001365	0.0001234	0.0001352	0.0001250	0.0001338	0.0001267	0.0001325	0.0001282	0.0001311	0.0001296	0.0001296
6	0.0069905	0.0078363	0.0070876	0.0077623	0.0071813	0.0076863	0.0072721	0.0076082	0.0073599	0.0075278	0.0074451	0.0074451
7	0.0000269	0.0000301	0.0000273	0.0000299	0.0000277	0.0000296	0.0000280	0.0000293	0.0000283	0.0000290	0.0000287	0.0000287
8	0.0003010	0.0003729	0.0003091	0.0003664	0.0003169	0.0003599	0.0003245	0.0003531	0.0003319	0.0003462	0.0003391	0.0003391
9	0.0000426	0.0000527	0.0000437	0.0000518	0.0000448	0.0000509	0.0000459	0.0000499	0.0000469	0.0000490	0.0000480	0.0000480
10	0.0000106	0.0000131	0.0000109	0.0000129	0.0000111	0.0000127	0.0000114	0.0000124	0.0000117	0.0000122	0.0000119	0.0000119
11	0.0006082	0.0007534	0.0006244	0.0007404	0.0006402	0.0007271	0.0006556	0.0007134	0.0006706	0.0006995	0.0006852	0.0006852
12	0.0000023	0.0000029	0.0000024	0.0000028	0.0000024	0.0000027	0.0000025	0.0000027	0.0000026	0.0000027	0.0000026	0.0000026
13	0.0022707	0.0028127	0.0023312	0.0027641	0.0023901	0.0027143	0.0024475	0.0026635	0.0025035	0.0026115	0.0025582	0.0025582

j	$\tilde{P}_{j\beta}$ for $\beta = 1$		$\tilde{P}_{j\beta}$ for $\beta = 0.8$		$\tilde{P}_{j\beta}$ for $\beta = 0.6$		$\tilde{P}_{j\beta}$ for $\beta = 0.4$		$\tilde{P}_{j\beta}$ for $\beta = 0.2$		$\tilde{P}_{j\beta}$ for $\beta = 0$	
	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$	$\tilde{P}_{j\beta(L)}$	$\tilde{P}_{j\beta(R)}$
1	0.8588424	0.8795961	0.8606648	0.8773233	0.8625587	0.8750844	0 0.8645170	0.8728825	0.8665336	0.8707208	0.8686031	0.8686031
2	0.0744718	0.0844088	0.0756720	0.0835757	0.0768136	0.0827151	0.0779026	0.0818245	0.0789440	0.0809013	0.0799422	0.0799422
3	0.0034292	0.0038939	0.0034853	0.0038549	0.0035387	0.0038147	0.0035896	0.0037730	0.0036383	0.0037298	0.0036850	0.0036850
4	0.0004850	0.0005507	0.0004929	0.0005452	0.0005005	0.0005395	0.0005077	0.0005336	0.0005146	0.0005275	0.0005212	0.0005212
5	0.0001206	0.0001370	0.0001226	0.0001356	0.0001245	0.0001342	0.0001262	0.0001327	0.0001280	0.0001312	0.0001296	0.0001296
6	0.0069283	0.0078672	0.0070417	0.0077885	0.0071496	0.0077072	0.0072524	0.0076230	0.0073508	0.0075358	0.0074451	0.0074451
7	0.0000267	0.0000303	0.0000271	0.0000299	0.0000275	0.0000296	0.0000279	0.0000293	0.0000283	0.0000290	0.0000287	0.0000287
8	0.0002974	0.0003737	0.0003064	0.0003672	0.0003169	0.0003606	0.0003235	0.0003537	0.0003315	0.0003465	0.0003391	0.0003391
9	0.0000420	0.0000528	0.0000433	0.0000519	0.0000448	0.0000509	0.0000457	0.0000500	0.0000469	0.0000490	0.0000480	0.0000480
10	0.0000105	0.0000131	0.0000107	0.0000129	0.0000111	0.0000127	0.0000114	0.0000124	0.0000117	0.0000122	0.0000119	0.0000119
11	0.0006008	0.0007550	0.0006191	0.0007420	0.0006367	0.0007285	0.0006535	0.0007146	0.0006706	0.0007002	0.0006852	0.0006852
12	0.0000023	0.0000029	0.0000024	0.0000029	0.0000024	0.0000028	0.0000025	0.0000028	0.0000026	0.0000027	0.0000026	0.0000026
13	0.0022429	0.0028185	0.0023114	0.0027700	0.0023770	0.0027197	0.0024398	0.0026678	0.0025002	0.0026140	0.0025582	0.0025582

Table 2. System availability at $t = 360$h

$\alpha, \beta \downarrow$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}_{\beta(L)}$	$\tilde{Av}_{\beta(R)}$
0	0.9386760	0.9585622	0.9485453	0.9485453
0.1	0.9396550	0.9575550	0.9470103	0.9500826
0.2	0.9406359	0.9565489	0.9454776	0.9516221
0.3	0.94161867	0.9555440	0.9439473	0.9531636
0.4	0.9426032	0.9545403	0.9424196	0.9547070
0.5	0.9435895	0.9535378	0.9408945	0.9562524
0.6	0.9445775	0.9525366	0.9393723	0.9577995
0.7	0.9455671	0.9515368	0.9378530	0.9593484
0.8	0.9465583	0.9505382	0.9363368	0.9608990
0.9	0.9475511	0.9495411	0.9348238	0.9624512
1.0	0.9485453	0.9485453	0.9333142	0.9640049

This shows that availability of the system is in the form of TIFN and availability of the system S_2 lies in the interval $[0.9386760\ 0.9585622]$.

Also availability function at $t = 360$h can be approximated in the form of a triangular intuitionistic fuzzy number whose membership and non-membership functions are respective.

$$\mu_{\tilde{Av}}(x) = \begin{cases} \frac{x-0.9386760}{0.0098693}, & 0.9386760 \leq x \leq 0.9485453 \\ \frac{0.9585622-x}{0.100169}, & 0.9485453 \leq x \leq 0.9585622 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{\tilde{Av}}(x) = \begin{cases} \frac{0.9485453-x}{0.0152311}, & 0.9333142 \leq x \leq 0.9485453 \\ \frac{x-0.9485453}{0.0154596}, & 0.9485453 \leq x \leq 0.9640049 \\ 1, & \text{otherwise} \end{cases}.$$

5 Conclusion

In this article, authors have discussed a method for solving intuitionistic fuzzy differential equations having intuitionistic fuzzy coefficients and initial conditions. The concepts of (α, β)-cuts of triangular intuitionistic fuzzy number have been used to check whether the obtained solution defines a intuitionistic fuzzy number. The system of intuitionistic fuzzy differential equations pertaining to a system deals with the uncertainty and provides more realistic results to the system analyst.

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