Eighth Int. Conf. on IFSs, Varna, 20-21 June 2004
NIFS 10 (2004) 3, 29-32

ON SOME TEMPORAL INTUITIONISTIC FUZZY OPERATORS

Krassimir T. Atanassov
CLBME - Bulg. Academy of Sci., P.O.Box 12, Sofia-1113, Bulgaria
e-mail: krat@bas.bg

In memory of Prof. Ivan Daskalov
(26 Feb. 1933 - 12 June 2004)

1 Introduction

The temporal logic is one of the basic areas of mathematical logic developing through last century. The first intuitionistic fuzzy interpretations of the temporal logic were discussed in [1], where the temporal logic operators "always" $\mathcal{A}$ and "once" $\mathcal{O}$ are studied.

The two other temporal operators "sometimes" $\mathcal{S}$ and "at the currently" $\mathcal{C}$ are defined in [4].

Here we shall introduce some new operators are will discuss their intuitionistic fuzzy interpretations.

2 Basic concepts

Let T be a fixed set of real numbers which we shall call "time-scale" and it is strictly oriented by the relation "$<$".

Let p be a proposition and V be a truth-value function, which maps the ordered pair:

$$V(p,t) = \langle \mu(p,t), \nu(p,t) \rangle$$

to the proposition p and to the time-moment $t \in T$.

Following, e.g., [5], we note that proposition p with intuitionistic fuzzy values $\langle a, b \rangle$ is called an *"Intuitionistic Fuzzy Tautology" (IFT),* if and only if $a \geq b$.

The basic results in the intuitionistic fuzzy set theory are collected in [7].

3 Main results

Let $x \in E$ be a fixed proposition and $A \subset E$, where here and below E is a set of propositions. Firstly, we shall introduce one new (for the IFS theory) operator as follows:

$$\tau(A(T), x) = \{t \mid \mu_A(x,t) > \nu_A(x,t) \ \& \ t \in T\}.$$

Obviously, for all $x \in E$:

$$\emptyset \subset \tau(A(T), x) \subset T.$$

For x we can assert that it is *"intuitionistic fuzzy valid" (IFV)* in time-moment t, if and only if

$$\mu_A(x,t) \geq \nu_A(x,t). \qquad (*)$$

Numbers $\mu_A(x,t)$ and $\nu_A(x,t)$ can be respectively interpreted as a *"degree of validity"* and a *"degree of non-validity"*.

Let us assume that in E for each element x there exists an element $\neg x$ and let for it be valid:

$$\tau(A(T), \neg x) = \{t | \nu_A(x,t) > \mu_A(x,t) \ \& \ t \in T\}.$$

Therefore, the predicate

$$\varphi(x) = \text{"}x \text{ has always been true"}$$

will be IFV, if (*) holds for all $t \in T$.

By similarity, we can define the following predicates, too:

$$\psi(x) = \text{"}x \text{ has sometime been true, but not always"},$$

$$\chi(x) = \text{"once } x \text{ was true"},$$

$$\omega(x) = \text{"}x \text{ has never been true"}.$$

It can be easily seen that

$$\varphi(x) = 1, \text{ if and only if } \tau(A(T), x) = T,$$

$$\psi(x) = 1, \text{ if and only if } \emptyset \neq \tau(A(T), x) \neq T,$$

$$\text{and } (\exists t_1, t_2 \in \tau(A(T), x))(\exists t_3 \in T - \tau(A(T), x))(t_1 < t_3 < t_2),$$

$$\chi(x) = 1, \text{ if and only if } \emptyset \neq \tau(A(T), x) \neq T,$$

$$\text{and } (\forall t_1, t_2 \in \tau(A(T), x))(\neg \exists t_3 \in T - \tau(A(T), x))(t_1 < t_3 < t_2),$$

$$\omega(x) = 1, \text{ if and only if } \tau(A(T), x) = \emptyset.$$

All the above predicates $\varphi, \psi, \chi, \omega$ have values in set $\{0, 1\}$. Now, we can construct their IFVs.

Let below $card(X)$ be the cardinality of set X. Therefore, for the fixed elements x we can define the couple

$$\rho(x) = \langle \frac{card(\tau(A(X), x)}{card(T)}, \frac{card(\tau(A(X), \neg x)}{card(T)} \rangle.$$

It is an intuitionistic fuzzy couple, because

$$0 \leq \frac{card(\tau(A(X), x)}{card(T)} + \frac{card(\tau(A(X), \neg x)}{card(T)} \leq 1.$$

The second inequality will become an equality, if there was no time moment when for x : $\mu_A(x, t) = \nu_A(x, t)$. The set of all time-moments for which the later equality is not valid (let us note it by Δ_x) determines the *"degree of uncertainty"* for x, and of course,

$$\frac{card(\tau(A(X), x)}{card(T)} + \frac{card(\tau(A(X), \neg x)}{card(T)} + \frac{\Delta_x}{card(T)} = 1.$$

Now, we can define the following two new predicates

$$\xi(x) = 1, \text{ if and only if } \rho(x) \text{ is an IFT},$$

$$\sigma(x) = 1, \text{ if and only if } \rho(\neg x) \text{ is an IFT}.$$

These predicates can be interpreted as follows:

$$\xi(x) = \text{"}x \textit{ is often true}\text{"},$$

$$\sigma(x) = \text{"}x \textit{ is rarely true}\text{"}.$$

These two predicates can be generalized. For example, we can use the two real numbers $\alpha, \beta \in [0, 1]$ and we can define that $\langle a, b\rangle$ is (α, β)-IFT if and only if $a \geq \alpha$ and $b \leq \beta$. Then

$$\xi^*(x) = \text{"}x \textit{ is } (\alpha, \beta)\textit{-often true}\text{"},$$

$$\sigma(x) = \text{"}x \textit{ is } (\alpha, \beta)\textit{-rarely true}\text{"}.$$

Now, for them there will hold

$$\xi^*(x) \text{ is } (\alpha, \beta)\text{-often if and only if } \mu(\rho(x)) \geq \alpha \ \& \ \nu(\rho(x)) \leq \beta,$$

$$\sigma^*(x) \text{ is } (\alpha, \beta)\text{-rarely if and only if } \mu(\rho(\neg x)) \geq \alpha \ \& \ \nu(\rho(\neg x)) \leq \beta.$$

4 Conclusion

We shall discuss some applications of the concepts above described. In the recent ten years the IFSs have been applied in different areas: intuitionistic fuzzy Prolog [6] intuitionistic fuzzy expert systems [3, 5, 4] intuitionistic fuzzy tools for decision making [5], and others. In [4] it was shown that the functioning and the results of the work of each expert system from a production type can be described by a Generalized Net (GN; see [2, 8]). On the basis of the GNs possibilities as tools for modelling, the concept of an expert system was extended not only in sense of [4]. One of the directions to extend expert systems was related to providing it with the possibility to work with predicates $\varphi, \psi, \chi, \omega$. Now we can enlarge the list of these predicates adding the later two. On the other hand, the above constructions show one of the possible interpretations of these temporal predicates.

References

[1] Atanassov K., Remark on a temporal intuitionistic fuzzy logic. Second Scientific Session of the "Mathematical Foundation Artificial Intelligence" Seminar, Sofia, March 30, 1990, Preprint IM-MFAIS-1-90, Sofia, 1990, 1-5.

[2] Atanassov, K. Generalized Nets. World Scientific, Singapore, New Jersey, London, 1991.

[3] Atanassov K., Remark on intuitionistic fuzzy expert systems, BUSEFAL, Vol. 59, 1994, 71-76.

[4] Atanassov, K. Generalized Nets in Artificial Intelligence. Vol. 1: Generalized nets and Expert Systems. "Prof. M. Drinov" Academic Publishing House, Sofia, 1998.

[5] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[6] Atanassov K., Georgiev Ch., Intuitionistic fuzzy Prolog, Fuzzy sets and Systems Vol. 53 (1993), No. 1, 121-128.

[7] Nikolova M., N. Nikolov, C. Cornelis, G. Deschrijver, Survey of the research on intuitionistic fuzzy sets. Advanced Studies in Contemporary Mathematics, Vol. 4, 2002, No. 2, 127-157.

[8] Radeva, V., M. Krawczak, E. Choy, Review and bibliography on generalized nets theory and applications. Advanced Studies in Contemporary Mathematics, Vol. 4, No. 2, 173-199.