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An experimental analysis of some measures of information and knowledge for intuitionistic fuzzy sets

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Abstract: The evaluation of information and knowledge conveyed by an Atanassov's intuitionistic fuzzy set (A-IFS, for short) is discussed. We pay particular attention to the relationship between positive and negative knowledge (expressed by the entropy which may be seen as a dual measure to information), and also take into account the reliability of information expressed by the hesitation margin.
Keywords: Intuitionistic fuzzy sets, amount of information, amount of knowledge, entropy, hesitation margin.
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1 Introduction

For a piece of data represented by a fuzzy set, information conveyed consists of a membership function. Subsequently, its related knowledge is derived from information placed in the context considered, i.e. is context dependent. It may be, for example, a dual measure to entropy (as considered in Quinlan [9]). The transformation of information into knowledge is critical from a practical point of view (cf. Stewart [10]). It is a crucial task for problem solving and any analysis of a specific situation and/or problem. A notable example may here be the omnipresent problem of decision making.

In this paper we consider information conveyed by a piece of data represented by Atanassov's intuitionistic fuzzy set (A-IFS) and its related knowledge that is clearly context dependent.

Information represented by an A-IFS, may be considered just as a generalization of information conveyed by a fuzzy set, and consists of the two terms present in the definition of an A-IFS, i.e., the membership and non-membership functions ("responsible" for the positive and negative information, respectively). But, for practical purposes it seems expedient, even necessary, to also take into account a so-called hesitation margin (cf. Szmidt and Kacprzyk [14], [15], [18], [16], [19]), [20], Bustince et al. [5], [6], Szmidt and Kukier [23], [24], [25], etc.)

We show in this paper that the entropy alone, although calculated by taking into account the hesitation margin as well (cf. Szmidt and Kacprzyk [16], [19]) may be not a satisfactory dual measure of knowledge to be useful from the point of view of decision making or any specific problem solving activity that is performed via the A-IFSs. The reason is that an entropy measure answers the question about the fuzziness but does not consider reasons for the fuzziness. So, the two situations, on the one hand, the one with the maximal entropy for a membership function equal to a non-membership function (with both of them equal to 0.5), and – on the other hand – when we know absolutely nothing, are equal from the point of view of the entropy measure (in terms of the A-IFSs). However, from the point of view of decision making the two situations are quite different. This is the motivation of this paper as we propose here a new measure of knowledge for the A-IFSs. The proposed measure is not going to replace the entropy measures but may complement them by capturing additional features which are relevant when making decisions.

The new measure of knowledge is tested on several well known benchmarks commonly used in the broadly perceived analysis of data from the University of California, Irvine repository (UCI), and on one real data set.

2 Brief introduction to A-IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh [26]) given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set $A^{'}$, is an A-IFS (Atanassov [1], [3]) A is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

An additional concept for each A-IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanasov [3])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

a *hesitation margin* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [14], [15], [18], entropy (Szmidt and Kacprzyk [16], [19]), similarity (Szmidt and Kacprzyk [20]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks.

Hesitation margins turn out to be relevant for applications - in image processing (cf. Bustince et al. [5], [6]) and classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [23], [24], [25]), group decision making, negotiations, voting and other situations (cf. Szmidt and Kacprzyk papers).

In our further considerations we will use the notion of distances. In Szmidt and Kacprzyk [15], [18], Szmidt and Baldwin [11], [12], it is shown why in the calculation of distances between A-IFSs one should use all three terms describing A-IFSs.

The most often used distances between A-IFSs A, B in $X = \{x_{1,}, \ldots, x_n\}$ are:
– the normalized Hamming distance (Szmidt and Baldwin [11], [12], Szmidt and Kacprzyk [15], [18]):

$$l_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (6)$$

– and the normalized Euclidean distance (Szmidt and Baldwin [11], [12], Szmidt and Kacprzyk [15], [18]):

$$\begin{aligned} q_{IFS}(A,B) &= \\ &= (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}} \end{aligned} \quad (7)$$

For distances (6), and (7) we have $0 \leq l_{IFS}(A,B) \leq 1$, and $0 \leq q_{IFS}(A,B) \leq 1$. Clearly these distances satisfy the conditions of the metric.

Also the notation of a complement set A^C will be used

$$A^C = \{\langle x, \nu_A(x), \mu_A(x), \pi_A(x)\rangle | x \in X\} \quad (8)$$

2.1 Entropy

It is necessary to stress that the entropy we examine and then use here is a non-probabilistic-type entropy measure for the A-IFSs in the sense of De Luca and Termini [7] axioms which are intuitive and have been widely employed in the fuzzy literature. The axioms were properly reformulated for A-IFSs (see Szmidt and Kacprzyk [16]).

In our further considerations concerning the entropy, in addition to the distances, the concept of a cardinality of an A-IFS will also be useful.

Definition 1 (Szmidt and Kacprzyk [16], [17]) Let A be an A-IFS in X. First, we define the following two cardinalities of an A-IFS:

- the least ("sure") cardinality of A is equal to the so-called sigma-count (cf. Zadeh [26], [27], and is called here the $\min \sum Count$:

$$\min Card(A) \quad = \quad \min \sum Count(A) = \sum_{i=1}^{n} \mu_A(x_i) \tag{9}$$

- the biggest cardinality of A, which is possible due to π_A, is called the $\max \sum Count$, and is equal to

$$\max Card(A) \quad = \quad \max \sum Count(A) = \sum_{i=1}^{n} (\mu_A(x_i) + \pi_A(x_i)) \tag{10}$$

and, clearly, for A^c (where A^c is a complement of A) we have

$$\min Card(A^c) \quad = \quad \min \sum Count(A^c) = \sum_{i=1}^{n} \nu_A(x_i) \tag{11}$$

$$\max Card(A^c) \quad = \quad \max \sum Count(A^c) = \sum_{i=1}^{n} (\nu_A(x_i) + \pi_A(x_i)) \tag{12}$$

Then the cardinality of an A-IFS is defined as a number from the interval:

$$CardA \quad \in \quad [\min \sum Count(A), \max \sum Count(A)] \tag{13}$$

Remark: in the above formulas (9)–(13), for $i = 1$, we will use later, for simplicity, the following symbols: $\min Count(A)$ instead $\min \sum Count(A)$, $\max Count(A)$ instead $\max \sum Count(A)$, $\min Count(A^c)$ instead $\min \sum Count(A^c)$, $\max Count\ (A^c)$ instead $\max \sum Count(A^c)$.

The measure of entropy answers the question: how fuzzy is a fuzzy set? In other words, entropy $E(x)$ measures the whole missing information which may be necessary to say if an element x fully belongs or fully does not belong to our set (see Szmidt and Kacprzyk [16]).

Definition 2 A ratio-based measure of fuzziness i.e., entropy of an intuitionistic fuzzy element is given in the following way:

$$E(x) = \frac{a}{b} \tag{14}$$

where a is a *distance*(x, x_{near}) from x to the nearer point x_{near} among $M(1, 0, 0)$ and $N(0, 1, 0)$, and b is the *distance*(x, x_{far}) from x to the farer point x_{far} among $M(1, 0, 0)$ and $N(0, 1, 0)$.

Formula (14) describes the degree of fuzziness for a single element belonging to an A-IFS. For n elements belonging to an A-IFS we have

$$E = \frac{1}{n} \sum_{i=1}^{n} E(x_i). \tag{15}$$

Fortunately enough, while applying the Hamming distances in (14), the entropy of A-IFSs is the ratio of the biggest cardinalities ($\max \sum Counts$) involving only x and x^c. The following theorem was proven in (Szmidt and Kacprzyk [16]).

Theorem 1 A generalized entropy measure of an A-IFS with n elements is

$$E = \frac{1}{n} \sum_{i=1}^{n} \frac{\max Count(x_i \cap x_i^c)}{\max Count(x_i \cup x_i^c)}. \tag{16}$$

3 Measure of information and knowledge for the A-IFSs

The information concerning a particular element x belonging to (the support of) an A-IFS is equal to $\mu(x) + \nu(x)$, or, in other words: $1 - \pi(x)$. But it is one aspect of information only. For each fixed π there are different possibilities of combination between μ and ν. The combination between them influences strongly the amount of knowledge. The knowledge (for a fixed π) is different for the distant values between μ and ν, and for the close values between μ and ν. For example, if $\pi = 0.1$, the knowledge for the situation while $\mu = 0.85$ and $\nu = 0.05$ is bigger than for the case: $\mu = 0.45$ and $\nu = 0.45$. Entropy proposed by Szmidt and Kacprzyk ([16], [17]) is a good measure answering the question how fuzzy is an A-IFS (when considering the entropy one is not interested in the reasons of fuzziness). But when making decision, one is also interested in making differences between the following situations:

- we have no information at all, and
- we have a large number of arguments in favor but an equally large number of arguments in favor of the opposite statement.

In other words, we would like to have a measure making a difference between (0.5, 0.5, 0), and (0, 0, 1). To distinguish between these two types of situations, we should take into account, beside the entropy measure, also the hesitation margin π.

A good measure of the amount of the knowledge (useful from the point of view of decision making) related to a separate element $x \in X$ seems to be:

$$K(x) = 1 - 0.5(E(x) + \pi(x)) \tag{17}$$

where $E(x)$ is an entropy measure given by (14) (Szmidt and Kacprzyk [16]), $\pi(x)$ is the hesitation margin.

Measure $K(x)$ (17) makes it possible to meaningfully represent what, in our context, is meant by the amount of knowledge, and is simple both conceptually and numerically which is a big asset while solving complex real world problems.

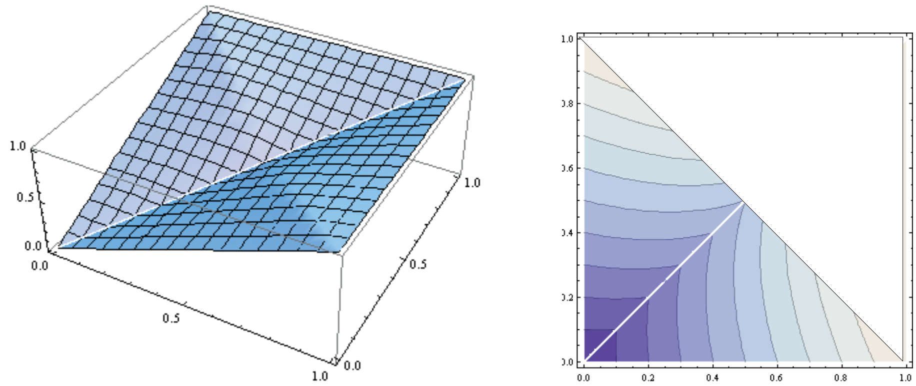
The properties of (17) are a consequence of the properties of entropy measure $E(x)$, and the fact that the lack of information $\pi(x)$ was added, and normalized, namely:

1. $0 \le K(x) \le 1$;
2. $K(x) = K(x^C)$;
3. For a fixed value of π, $K(x)$ behaves dually to the entropy measure (i.e., as $1 - E(x)$);
4. For a fixed $E(x)$, $K(x)$ increases while π decreases.

In Figure 1 we can see the shape of $K(x)$, and its contour plot.

For n elements, the total amount of knowledge K is:

$$K = \frac{1}{n} \sum_{i=1}^{n} (1 - 0.5(E(x_i) + \pi(x_i))) \tag{18}$$



a) b)

Figure 1: a) - measure $I(x)$; b) - its contourplot

To experimentally verify if the proposed measure of knowledge K (17) for the A-IFSs gives expected results, we use several well known benchmarks from the University of California, Irvine repository (UCI) (previously, in Szmidt, Kacprzyk, Bujnowski [22] we have examined Quinlan's example [9], the so-called "Saturday Morning").

3.0.1 Results for some benchmarks

We test measure "K" on several well known benchmarks commonly used in the broadly perceived analysis of data from the University of California, Irvine repository (UCI), and one real data set (IVH 3-4). We use the data sets consisting of 2 classes with different numbers of attributes and instances, namely:
– "Pima" (UCI, 768 instances, 8 attributes, 2 classes),
– "Ionosphere2" (UCI, 351 instances, 33 attributes, 2 classes),
– "Sonar" (UCI, 208 instances, 60 attributes, 2 classes),
– real data "IVH 3-4" ("IVH" means an intraventricular hemorrhage – a bleeding into the brain's ventricular system, where the cerebrospinal fluid is produced and circulates through towards the subarachnoid space. It can result from physical trauma or from hemorrhaging in stroke; 26 attributes, 2 classes).

To obtain the results we construct an A-IFS counterpart of each data set (the method is described in Szmidt and Kacprzyk [21], and in [13]).

Next, for each attribute of the data set considered, the measure of the amount of knowledge K (17), and entropy E (14)–(15) is calculated. To evaluate both measures we construct two intuitionistic fuzzy trees: the first tree constructed using the entropy, the second tree by using measure K (both measures may give different order of the attributes, and as a result, different trees). We have verified the accuracy for both trees and the results are summarized in Table 1.

The values of classification accuracy presented in Table 1 are obtained with the 10–fold cross-validation (in 10 experiment so that they are average results obtained from 100 trees). The size of the training and testing subsets was 50/50 % in each experiment.

In Table 1 we have the following information. In the first column there is the name of the

Table 1: Classification accuracy of classification for some benchmarks using entropy (14) or measure K (17) tree based techniques

Data	Pru-ned	Measure	Correctly recognized [%] class "1"	class "0"	together
Pima (5,2), training	No	Entropy	73.95 ± 6.84	83.39 ± 3.73	80.10 ± 0.57
Pima (5,2), testing	No	Entropy	67.27 ± 12.22	80.24 ± 7.01	75.72 ± 4.37
Pima (5,2), training	No	Measure K	67.35 ± 7.88	86.28 ± 3.98	79.67 ± 0.57
Pima (5,2), testing	No	Measure K	59.78 ± 11.21	82.10 ± 7.45	74.31 ± 4.34
Pima (5,2), training	Yes	Entropy	59.78 ± 4.54	87.04 ± 2.20	77.53 ± 0.50
Pima (5,2), testing	Yes	Entropy	57.58 ± 9.94	85.46 ± 5.78	75.73 ± 4.28
Pima (5,2), training	Yes	Measure K	56.95 ± 5.39	88.32 ± 2.73	77.38 ± 0.62
Pima (5,2), testing	Yes	Measure K	53.40 ± 11.31	86.30 ± 5.97	74.82 ± 4.42
Ionosphere2 (4,3), tr	No	Entropy	97.07 ± 1.69	88.44 ± 3.14	93.97 ± 0.75
Ionosphere2 (4,3), te	No	Entropy	95.10 ± 4.59	81.84 ± 11.47	90.36 ± 4.50
Ionosphere2 (4,3), tr	No	Measure K	96.20 ± 1.63	90.23 ± 3.01	94.06 ± 0.68
Ionosphere2 (4,3), te	No	Measure K	94.23 ± 5.55	83.15 ± 10.89	90.25 ± 5.18
Ionosphere2 (4,3), tr	Yes	Entropy	97.00 ± 1.80	83.38 ± 2.52	92.11 ± 1.01
Ionosphere2 (4,3), te	Yes	Entropy	94.75 ± 5.38	80.19 ± 10.93	89.54 ± 5.06
Ionosphere2 (4,3), tr	Yes	Measure K	96.31 ± 1.92	84.17 ± 2.63	91.95 ± 1.17
Ionosphere2 (4,3), te	Yes	Measure K	94.35 ± 5.56	80.53 ± 10.73	89.40 ± 5.13
Sonar (5,3), training	No	Entropy	96.41 ± 2.32	96.08 ± 2.23	96.23 ± 0.73
Sonar (5,3), testing	No	Entropy	79.24 ± 12.83	79.45 ± 11.91	79.39 ± 7.55
Sonar (5,3), training	No	Measure K	96.78 ± 2.40	96.63 ± 2.31	96.70 ± 0.91
Sonar (5,3), testing	No	Measure K	79.80 ± 14.66	81.94 ± 11.93	80.88 ± 7.76
Sonar (5,3), training	Yes	Entropy	91.41 ± 3.08	94.64 ± 2.54	93.14 ± 1.30
Sonar (5,3), testing	Yes	Entropy	75.94 ± 14.06	79.67 ± 12.09	77.94 ± 7.84
Sonar (5,3), training	Yes	Measure K	91.84 ± 2.80	94.64 ± 2.40	93.34 ± 1.38
Sonar (5,3), testing	Yes	Measure K	76.52 ± 16.26	80.27 ± 12.31	78.50 ± 8.41
IVH 3-4 (5,3), training	No	Entropy	75.86 ± 9.98	98.03 ± 0.98	96.38 ± 0.68
IVH 3-4 (5,3), testing	No	Entropy	21.33 ± 30.71	95.09 ± 4.39	89.52 ± 4.52
IVH 3-4 (5,3), training	No	Measure K	80.21 ± 12.06	97.75 ± 1.00	96.44 ± 0.57
IVH 3-4 (5,3), testing	No	Measure K	22.67 ± 30.75	94.34 ± 4.54	88.96 ± 4.65
IVH 3-4 (5,3), training	Yes	Entropy	17.79 ± 11.26	99.16 ± 2.46	93.09 ± 1.83
IVH 3-4 (5,3), testing	Yes	Entropy	5.33 ± 14.77	98.18 ± 3.82	91.24 ± 3.58
IVH 3-4 (5,3), training	Yes	Measure K	18.07 ± 11.80	99.36 ± 2.10	93.29 ± 1.59
IVH 3-4 (5,3), testing	Yes	Measure K	5.50 ± 15.18	98.29 ± 3.37	91.34 ± 3.13

tested data, the two numbers in the brackets mean the number of triangle representations used for granulation of the attributes space, and the depth of the tree, respectively. We can also find out if the results concern the training or testing data. In the second column we can see if the tree has been pruned or not. In the third column we have information which measures was applied while constructing a tree. In the last three columns there are results of classification, for each class separately, and the general accuracy (for both classes together).

Measure K gives better results (test set) for the Sonar data, for the unpruned tree of depth 3, 80.9 % of instances is correctly recognized while using measure K, while for the same kind of tree (depth 3, unpruned) built via the entropy, 79.4 % of instances is correctly classified. Class "1" of the Sonar data is equally well seen by both measures but Class "0" is better recognized while using measure K (K: 81.9 %; entropy: 79.5 %).

General accuracy for Pima data (pruned tree of depth 2, training subset) is slightly better for entropy (75.7 %) than for measure K (74.8 %) but class "0" is better recognized via applying measure K (86,3 %) than while using entropy 85.5 %).

A similar situation occurs for Ionosphere2, unpruned tree of depth 3: the general accuracy is slightly better while applying the entropy (90.4 %) than measure K (90.3 %) but class "0" is better seen while applying measure K (83.2 %) than while applying the entropy (81.8 %).

For the IVH 3-4 data, the pruned tree of depth 3, the general accuracy is slightly better for measure K (91.3 %) than for the entropy (91.2 %). The same situation is visible for recognizing both classes separately: for class "1", using the entropy we get a worse result, i.e. 5.3 % of correctly seen cases, while for measure K the 5.5 % of accuracy is obtained. For class "0", using the entropy we obtain a worse result again – 98.2 % of the correctly recognized instances than for measure K for which it is 98.3 %.

4 Conclusions

We have discussed and tested on some benchmarks from the UCI a measure of knowledge for the A-IFSs. The measure keeps the advantages of the entropy measure (reflecting the relationship of the positive and negative knowledge) but additionally also emphasizes the influence of the amount of the lacking information (expressed by the hesitation margin). The measure has been intended to be useful from the point of view of decision making.

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