Notes on Intuitionistic Fuzzy Sets
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Optimal weighting method for interval-valued intuitionistic fuzzy opinions

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Abstract: In this work, we propose a method to achieve consensus in a group decision making situation, where the opinions are described by interval-valued intuitionistic fuzzy sets. Optimality is achieved by minimizing weighed incoherencies. An illustrative example is proposed.
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1 Introduction

Since unanimity is rarely achieved in group decision making, a certain level of consensus might be acceptable. The achieved consensus must take into consideration human uncertainty, to do so, we model the expressed opinions by interval-valued intuitionistic fuzzy numbers. In the rest of this manuscript the needed background for fuzzy logic is presented in Section 2, while Section 3 encompasses the used algorithm with an illustrative example.

2 Preliminaries

In classical sets, each element either belongs to a certain set or not at all, while in fuzzy set theory a certain degree of membership is tolerated [13]. Let X be a set and F be a fuzzy set in X,where F is defined as follows:

$$F = \{\langle x, \mu_F(x)\rangle \mid x \in X\},$$

where $\mu_F(x)$ is the degree of membership of x in F in the unity interval:

$$\mu_F \, : \, X \longrightarrow [0, 1].$$

Atanassov [1, 2] extended the notion of fuzzy sets to intuitionistic fuzzy sets (IFS). An intuitionistic fuzzy set A is defined as follows:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\},$$

where $\mu_A(x)$ and $\nu_A(x)$ are respectively the membership function and the non-membership function, with the following conditions:

$$\mu_A \, : \, X \longrightarrow [0, 1], \nu_A \, : \, X \longrightarrow [0, 1]$$

$$\mu_A(x) + \nu_A(x) \leq 1 \quad \forall x \in X.$$

The hesitancy function can be computed by the following formula:

$$\pi_A(x) = 1 - [\mu_A(x) + \nu_A(x)] \quad \forall x \in X.$$

The fuzzy sets were presented in order to permit human uncertainty, while it is counterintuitive to demand an exact membership function and non-membership function. In that sense Atanassov and Gargov [4] extended the IFS to interval-valued intuitionistic fuzzy sets (IVIFS) fulfilling the following:

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\},$$

where $M_A(x) \subset [0, 1]$ and $N_A(x) \subset [0, 1]$ are respectively the membership interval and the non-membership interval, and for these two intervals it holds that [4]:

$$\sup M_A(x) + \sup N_A(x) \leq 1.$$

For convenience, we note an interval-valued fuzzy number as $\beta = ([a, b], [c, d])$ where $a = \inf M_\beta$, $b = \sup M_\beta$, $c = \inf N_\beta$ and $d = \sup N_\beta$ are interval numbers.

Let $\beta_i = ([a_{\beta_i}, b_{\beta_i}], [c_{\beta_i}, d_{\beta_i}])$ be a collection of interval-valued intuitionistic fuzzy numbers, the main aggregation operators are the interval-valued intuitionistic fuzzy weighting averaging *IIFWA*, and the interval-valued intuitionistic fuzzy weighting geometric *IIFWG* [11], hence the aggregated value according to IIFWA is:

$$IIFWA_w\,(\beta_1, \beta_2, \ldots, \beta_n) = ([a, b], [c, d]),$$

where

$$a = 1 - \prod_{i=1}^{n}(1 - a_{\beta_i}), \;\; b = 1 - \prod_{i=1}^{n}(1 - b_{\beta_i}), \;\; c = 1 - \prod_{i=1}^{n} c_{\beta_i}, \;\; d = 1 - \prod_{i=1}^{n} d_{\beta_i}$$

and w_i are the weights of the respective β_i.

The main question is how to attribute the correct weight to each decision.

3 Proposed method

Several method exists in the literature to attribute the correct weights [5, 7, 8, 12, 14]. Here we propose to follow the procedure proposed in [7] to the IVIFS. The desired consensus is achieved by minimizing the following function:

$$\min_{M\times\mathbb{R}^4} \sum_{i=1}^{n} w_i^m * \Big(c - S(\beta_i, \beta)\Big),$$

where $M = \left\{ {W=(w_1,w_2,\ldots,w_n),\ w_i\geq 0 \atop \sum_{i=1}^{n} w_i=1} \right\}$, m is a positive integer ($m > 1$), $S(\beta_i, \beta)$ is the similarity between the i-th decision and the consensus, c is a real number ($c > 1$).

Several methods have been proposed to compute similarity from a distance [6, 9, 10], here we adopt the Hamming distance for IVIFS [3], and derive the similarity as by Santini and Jain [9] to ease computation $S = 1 - D$. Hence, the distance between two IVIFS β_1 and β_2 is:

$$D\Big(\beta_1, \beta_2\Big) = \frac{1}{2}\Big(|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|\Big).$$

3.1 Algorithm

Step 1: Each expert $E_i : 1 \leq i \leq n$ assesses each alternative using an IVIFS.

Step 2: Set the initial aggregation weights such that $0 \leq w_i^{(0)} \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. The iterations are labeled $l = 0, 2, \ldots$.

Step 3: Compute the aggregated consensus at Step l:

$$\beta^l = IIFWA(\beta_i).$$

Step 4: Let $W^l = \Big(w_1^{(l)}, w_2^{(l)}, \ldots, w_n^{(l)}\Big)$. Compute W^{l+1} as follows :

$$W^{l+1} = \frac{\Big(1/(c - S(\beta^l, \beta_i))\Big)^{1/(m-1)}}{\sum_{j=1}^{n} \Big(1/(c - S((\beta^l, \beta_i))\Big)^{1/(m-1)}}.$$

Step 5: If $\Big\|W^{l+1} - W^l\Big\| > \varepsilon$, set $l = l + 1$ and go to Step 3. Else Stop.

3.2 Illustrative example

Let three experts assess an alternative as follows: $\beta_1 = \Big([0.22, 0.31]; [0.23, 0.54]\Big)$, $\beta_2 = \Big([0.04, 0.21]; [0.35, 0.46]\Big)$ and $\beta_3 = \Big([0.25, 0.27]; [0.23, 0.4]\Big)$.

We choose $m = 2$, $c = 1.5$ and $W^0 = (1, 0, 0)$. Table 1 resumes the evolution of weights in each iteration.

Iteration	Expert 1	Expert 2	Expert 3
0	1	0	0
1	0.368809216192937	0.297426787252369	0.333763996554694
2	0.337704855120950	0.321717143517176	0.340578001361874
3	0.336249576125929	0.323795310037380	0.339955113836691
4	0.336159924292944	0.323955884376264	0.339884191330792
5	0.336153654216238	0.323967955371357	0.339878390412405
6	0.336153196429720	0.323968855794958	0.339877947775322
7	0.336153162568463	0.323968922812703	0.339877914618833

Table 1. Results of each iteration

4 Conclusion

In this work, we adapted Lees algorithm to achieve group consensus in the interval-valued intuitionistic fuzzy context. We restricted ourselves to the interval-valued intuitionistic fuzzy weighting averaging operator to merge opinions, used the hamming metric to compute their distances and derived similarities as a distance dual. In future research, we will investigate different combinations of aggregation operators, similarities and distances that may be more appropriate in such situations.

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