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LINEAR PROGRAMMING WITH INTUITIONISTIC FUZZY OBJECTIVE

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The notion of intuitionistic fuzzy set has originally been introduced by K. Atanassov [1] (1986) as a generalization of the notion of fuzzy set. Basically, we can identify an intuitionistic fuzzy set with a pair of mappings $<A^+,A^->$ defined on an universe E as membership and nonmembership functions.

In [2] and [3] we have investigated the convexity of intuitionistic fuzzy sets and some properties of intuitionistic fuzzy numbers.

Definition 1. An intuitionistic fuzzy set is an object of the form: $A = \{A^+,A^-\}$ where $A^+: E\to[0,1]$, $A^-: E\to[0,1]$ and: $0 \le A^+(x) + A^-(x) \le 1$. The domain of consistency of IFS A is defined by:

$$D(A) = \{x\in E \mid A^+(x) \ge A^-(x)\}$$

Definition 2. An IFS in R having the form $N = \{N^+,N^-\}$ where N^+ is a normal quasiconcave function and N^- is a normal quasiconvexe function is called an intuitionistic fuzzy number, if D(N) is finite. From definitions 1 and 2 we have:

$$N^+(x) = N^-(x) \le \frac{1}{2},\ x\in D(N).$$

Definition 3. Let $N = \{N^+,N^-\}$ be an IFN. The upper α-level and lower β-level of N are defined by:

$$N^+_\alpha = \{x\in R \mid N^+(x) \ge \alpha\},\ N^-_\beta =\{x\in R \mid N^-(x) \le \beta\}.$$

The sets D(N), N^+_α, N^-_β are convex sets in R, that is they are intervals in R. (see [3]). By the extension principle we have:

$$(N \oplus Q)(x) = \{\sup_{y+z=x} \min(N^+(y),Q^+(z)),\ \inf_{y+z=x} \max(N^-(y),Q^-(z))\}$$

$$(p \,.\, N)(x) = \{N^+(\frac{x}{p}),\ N^-(\frac{x}{p})\},\ p \neq 0$$

$$(\tilde{\max}(N,Q))(x) = \{\ \sup_{x=\max(y,z)} \min\ (N^+(y),Q^+(z)),\ \inf_{x=\max(y,z)} \max\ (N^-(y),Q^-(z))\}$$

The $N \oplus Q$, p.N, $\tilde{\max}(N,Q)$ are INF too. (see [2])

In the following we shall consider the collection of INF with a single point of maximum (resp. Minimum) and N^+ an upper semicontinuous function, N^- a lower

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semicontinuous function. We note this collection by $\mathcal{N}(R)$. If $N \in \mathcal{N}(R)$ then $D(N)$, N_α^+, N_β^- are compact intervals in R, that is:

$$D(N) = [a,b],\ N_\alpha^+ = [(N_\alpha^+)^i,(N_\alpha^+)^s],\ N_\beta^- = [(N_\beta^-)^i,(N_\beta^-)^s]$$

We assume a conventional system of nonfuzzy constraints of the form: $AX \le b$, $X \ge 0$, A_{mxn}, X_{nx1}, b_{mx1} and a fuzzy objective function:

$$f(x) = x_1 . C_1 \oplus x_2 . C_2 \oplus \ldots \oplus x_n . C_n,\ \text{where } x = {}^tX, \text{ and } c_i \in \mathcal{N}(R).$$

For every $x \in R$ the value f(x) is an IFN of the type $\mathcal{N}(R)$ [see [2],[4]]. Hence, our linear program has an intuitionistic fuzzy objective. In the sequel we find $\underset{x \in U}{\tilde{\max}} f(x)$, where U is the set of feasible solutions of constraints. We propose to approximate $\underset{x \in U}{\tilde{\max}} f(x)$ by a piecewise intuitionistic fuzzy number obtained from α-level and β-level.

$$(\underset{x \in U}{\tilde{\max}} f(x))_\alpha^+ = [\underset{x \in U}{\sup}(f(x))_\alpha^{+i}, \underset{x \in U}{\sup}(f(x))_\alpha^{+s}]$$

$$(\underset{x \in U}{\tilde{\max}} f(x))_\beta^- = [\underset{x \in U}{\sup}(f(x))_\beta^{-i}, \underset{x \in U}{\sup}(f(x))_\beta^{-s}]$$

On the other hand we have [see [4]]:

$$\underset{x \in U}{\sup}(f(x))_\alpha^{+i} = \underset{x \in U}{\sup}(x_1 C_{1\alpha}^{+i} + x_2 C_{2\alpha}^{+i} + \ldots + C_{n\alpha}^{+i})$$

In this way for every $\alpha \ge \min[A^+(x) = A^-(x)]$ we obtain two classical linear programs and for every $\beta \le \max[A^+(x) = A^-(x)]$ two classical l.p. too.

For $\alpha_1 < \alpha_2 < \ldots < \alpha_m$ we have $[a_{\alpha_1}^+, b_{\alpha_1}^+] \supseteq [a_{\alpha_2}^+, b_{\alpha_2}^+] \supseteq \ldots \supseteq [a_{\alpha_m}^+, b_{\alpha_m}^+]$ and for $\beta_1 > \beta_2 > \ldots > \beta_m$ $[a_{\beta_1}^-, b_{\beta_1}^-] \supseteq [a_{\beta_2}^-, b_{\beta_2}^-] \supseteq \ldots \supseteq [a_{\beta_m}^-, b_{\beta_m}^-]$.

Uniting the points $(a_{\alpha_1}^+, \alpha_1), \ldots, (a_{\alpha_m}^+, \alpha_m)$ by straight line segments we obtain the graphic of the increasing side of the membership function of approximation. Similarly we obtain all sides of the membership and nonmembership function of approximation.

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