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ON SOME PROPERTIES OF INTUITIONISTIC FUZZY OPERATOR $X_{a,b,c,d,e,f}$

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Over Intuitionistic Fuzzy Sets (IFSs, see [2]) different operations, relations and operators are defined. The operators are from modal, topologocal level and other types. Here, we shall discuss some properties of the modal operators.

Let $\alpha, \beta \in [0, 1]$. We will define (see [2]) six operators over a given IFS A by:

$$
\begin{array}{ll}
F_{\alpha,\beta}(A) & = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \\
 & \text{where } \alpha + \beta \leq 1 \\
G_{\alpha,\beta}(A) & = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\}, \\
H_{\alpha,\beta}(A) & = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \\
H^*_{\alpha,\beta}(A) & = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle | x \in E\}, \\
J_{\alpha,\beta}(A) & = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\}, \\
J^*_{\alpha,\beta}(A) & = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle | x \in E\},
\end{array}
$$

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [2]), the most important of which are:

$$
\begin{array}{lcl}
A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\
A \supset B & \text{iff} & B \subset A; \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}. \\
A @ B & = & \{\langle x, (\frac{\mu_A(x)+\mu_B(x))}{2}, \frac{(\nu_A(x)+\nu_B(x))}{2}\rangle | x \in E\}.
\end{array}
$$

The following assertions are proved in [2]:

Theorem 1.7.3: For every IFS A and for every α , β , γ , $\delta \in$ [0,1] such that $\alpha + \beta \leq 1$ and $\gamma + \delta \leq 1$:

$$F_{\alpha,\beta}(F_{\gamma,\delta}(A)) = F_{\alpha+\gamma-\alpha.\gamma-\alpha.\delta,\beta+\delta-\beta.\gamma-\beta.\delta}(A).$$

Theorem 1.8.1 (d): For every IFS A and for every four real numbers $\alpha, \beta, \gamma\delta \in [0, 1]$, then

$$G_{\alpha,\beta}(G_{\gamma,\delta}(A)) = G_{\alpha.\gamma,\beta.\delta}(A) = G_{\gamma,\delta}(G_{\alpha,\beta}(A)).$$

Unfortunately, similar equalities cannot be proved for the rest operators. For some of them more particular results are valid. For example, we can easity prove the validity of
Theorem 1: For every IFS A and for every $\alpha, \beta \in [0, 1]$:

$$J_{\alpha,1}(H_{1,\beta}(A)) = F_{\alpha(1-\beta),\beta}(A),$$

$$H_{1,\alpha}(J_{\beta,1}(A)) = F_{\beta,\alpha(1-\beta)}(A),$$

$$J_{1,1}(H_{\alpha,\beta}(A)) = F_{1-\beta,\beta}(A),$$

$$H_{1,1}(J_{\alpha,\beta}(A)) = F_{\alpha,1-\alpha}(A).$$

But we cannot construct similar equalities for expressions $F_{\alpha,\beta}(G_{\gamma,\delta}(A))$, $G_{\alpha,\beta}(F_{\gamma,\delta}(A))$, $H_{\alpha,\beta}(H_{\gamma,\delta}(A))$, $J_{\alpha,\beta}(J_{\gamma,\delta}(A))$, $J_{\alpha,\beta}(H_{\gamma,\delta}(A))$, $H_{\alpha,\beta}(J_{\gamma,\delta}(A))$, etc.

Following [2] we shall mention that in 1991, during a lecture given by the author, a question was launched of whether an operator can be constructed to include as partial cases all operators of modal type. On the next lecture the author gave a positive answer, defining (and publishing in [1]) the following operator which is universal for all above operators. Let:

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)),$$
$$d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$

where $a, b, c, d, e, f \in [0, 1]$ and:

$$a + e - e.f \leq 1, \tag{1}$$

$$b + d - b.c \leq 1. \tag{2}$$

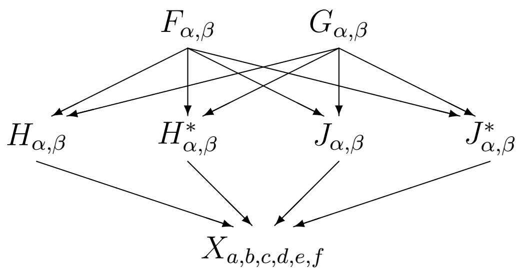
In [2] is proved the following
Theorem 1.11.1: For every IFS A and for every $a, b, c, d, e, f \in [0, 1]$ satisfying (1) and (2), $X_{a,b,c,d,e,f}(A)$ is an IFS.

All the above operators can be represented by the operator X at suitably chosen values of its parameters. These representations are the following:

$$\begin{aligned}
F_{a,b}(A) &= X_{1,a,1,1,b,1}(A), \\
G_{a,b}(A) &= X_{a,0,r,b,0,r}(A), \\
H_{a,b}(A) &= X_{a,0,r,1,b,1}(A), \\
H^*_{a,b}(A) &= X_{a,0,r,1,b,a}(A), \\
J_{a,b}(A) &= X_{1,a,1,b,0,r}(A), \\
J^*_{a,b}(A) &= X_{1,a,b,b,0,r}(A),
\end{aligned}$$

where r is an arbitrary real number $\in [0, 1]$.

In [2] all above operators of a modal type are systematized, constructing the following scheme:



In the same book they are proved the following assertions
Theorem 1.11.2: Let for $a, b, c, d, e, f, g, h, i, j, k, l \in [0, 1]$ it holds that

$$\begin{array}{lclllll} u & = & a.g & +b & -b.g & -b.c.k + b.c.k.l & \geq 0, \\ v & = & a.h & +b & -b.c.k & -b.h & > 0, \\ w & = & a.h.i & +b.c.j & -b.c.k & -b.h.i & \geq 0, \\ x & = & d.j & +e & -e.f.h & +e.f.h.i - e.j & \geq 0, \\ y & = & d.k & +e & -e.f.h & -e.k & > 0, \\ z & = & d.k.l & +e.f.g & -e.f.h & -e.k.l & \geq 0. \end{array}$$

Then:

$$X_{a,b,c,d,e,f}(X_{g,h,i,j,k,l}(A)) = X_{u,v,w/v,x,y,z/y}.$$

Theorem 1.11.3: For every two IFSs A and B and for every $a, b, c, d, e, f \in [0, 1]$ satisfying (1) and (2):

(a) $\overline{X_{a,b,c,d,e,f}(\overline{A})} = X_{d,e,f,a,b,c}(A)$,

(b) $X_{a,b,c,d,e,f}(A \cap B) \subset X_{a,b,c,d,e,f}(A) \cap X_{a,b,c,d,e,f}(B)$,

(c) $X_{a,b,c,d,e,f}(A \cup B) \supset X_{a,b,c,d,e,f}(A) \cup X_{a,b,c,d,e,f}(B)$,

(d) if $c = f = 1, a \geq b$ and $e \geq d$, then

$$X_{a,b,c,d,e,f}(A + B) \subset X_{a,b,c,d,e,f}(A) + X_{a,b,c,d,e,f}(B),$$

(e) if $c = f = 1, a \geq b$ and $e \geq d$, then

$$X_{a,b,c,d,e,f}(A.B) \supset X_{a,b,c,d,e,f}(A).X_{a,b,c,d,e,f}(B),$$

(f) $X_{a,b,c,d,e,f}(A@B) = X_{a,b,c,d,e,f}(A)@X_{a,b,c,d,e,f}(B)$.

Now, we shall formulate and prove some new propereties of operator $X_{a,b,c,d,e,f}$. It is valid
Theorem 2: For $a, b, c, d, e, f, g, h, i, j, k, l \in [0, 1]$ such that (1), (2) and

$$g + k - k.l \leq 1, \tag{3}$$

$$h + j - h.i \leq 1 \tag{4}$$

are valid, and for a given IFS A:

$$X_{a,b,c,d,e,f}(A) \cap X_{g,h,i,j,k,l}(A) \supset X_{\min(a,g),\min(b,h),\max(c,i),\max(d,j),\max(e,k),\min(f,l)}(A),$$

$$X_{a,b,c,d,e,f}(A) \cup X_{g,h,i,j,k,l}(A) \subset X_{\max(a,g),\max(b,h),\min(c,i),\min(d,j),\min(e,k),\max(f,l)}(A),$$

$$X_{a,b,c,d,e,f}(A)@X_{g,h,i,j,k,l}(A) = X_{\frac{a+g}{2}+\frac{b+h}{2}+\frac{bc+hi}{b+h}+\frac{d+j}{2}+\frac{e+k}{2}+\frac{ef+kl}{e+k}}(A).$$

Proof. Let $a, b, c, d, e, f, g, h, i, j, k, l \in [0, 1]$ satisfy the above mentioned conditions and let A be given. Then

$$X_{a,b,c,d,e,f}(A)@X_{g,h,i,j,k,l}(A)$$

$$= \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)), d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$
$$@\{\langle x, g.\mu_A(x) + h.(1 - \mu_A(x) - i.\nu_A(x)), j.\nu_A(x) + k.(1 - l.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$
$$= \{\langle x, \frac{1}{2}(a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)) + g.\mu_A(x) + h.(1 - \mu_A(x) - i.\nu_A(x))),$$
$$\frac{1}{2}(d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x)) + j.\nu_A(x) + k.(1 - l.\mu_A(x) - \nu_A(x)))\rangle | x \in E\}$$
$$= \{\langle x, \frac{a+g}{2}\mu_A(x) + \frac{b.(1 - \mu_A(x) - c.\nu_A(x)) + h.(1 - \mu_A(x) - i.\nu_A(x))}{2},$$
$$\frac{d+j}{2}\nu_A(x) + \frac{e.(1 - f.\mu_A(x) - \nu_A(x)) + k.(1 - l.\mu_A(x) - \nu_A(x))}{2}\rangle | x \in E\}$$
$$= \{\langle x, \frac{a+g}{2}\mu_A(x) + \frac{1}{2}.((b+h) - (b+h).\mu_A(x) - (bc+hi).\nu_A(x)),$$
$$\frac{d+j}{2}\nu_A(x) + \frac{1}{2}.((e+k) - (ef+kl).\mu_A(x) - (e+k).\nu_A(x))\rangle | x \in E\}$$
$$= \{\langle x, \frac{a+g}{2}\mu_A(x) + \frac{b+h}{2}.(1 - \mu_A(x) - \frac{bc+hi}{b+h}.\nu_A(x)),$$
$$\frac{d+j}{2}\nu_A(x) + \frac{e+k}{2}.(1 - \frac{ef+kl}{e+k}.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$
$$= X_{\frac{a+g}{2}+\frac{b+h}{2}+\frac{bc+hi}{b+h}+\frac{d+j}{2}+\frac{e+k}{2}+\frac{ef+kl}{e+k}}(A).$$

The validity of the other expressions is checked analogously.

Now, using X-representations if operators $F_{\alpha,\beta}, G_{\alpha,\beta}, H_{\alpha,\beta}, J_{\alpha,\beta}, H^*_{\alpha,\beta}$ and $J^*_{\alpha,\beta}$ we obtain through Theorem 1.11.2:

$$F_{a,b}(F_{c,d}(A)) = X_{1,a+c-ac-ad,1,1,b+d-bc-bd,1},$$

$$F_{a,b}(G_{c,d}(A)) = X_{a+c-ac,a,d,b+d-bd,b,c},$$

$$F_{a,b}(H_{c,d}(A)) = X_{a+c-ac,a-ad,1,1,b+d-bd,1},$$

$$F_{a,b}(H^*_{c,d}(A)) = X_{a+c-ac-ad+acd,a-ad,1,1,b+d-bd,c},$$

$$F_{a,b}(J_{c,d}(A)) = X_{1,a+c-ac,1,b+d-bd,b-bc,1},$$

$$F_{a,b}(J^*_{c,d}(A)) = X_{1,a+c-ac,d,b+d-bd,b-bc,1},$$

$$G_{a,b}(F_{c,d}(A)) = X_{a,ac,1,b,bd,1},$$

$$G_{a,b}(G_{c,d}(A)) = X_{ac,0,0,bd,0,0},$$

$$G_{a,b}(H_{c,d}(A)) = X_{ac,0,0,b,bd,1},$$

$$G_{a,b}(H^*_{c,d}(A)) = X_{ac,0,0,b,bd,c},$$

$$G_{a,b}(J_{c,d}(A)) = X_{a,ac,1,bd,0,0},$$

$$G_{a,b}(J^*_{c,d}(A)) = X_{a,ac,d,bd,0,0},$$

$$H_{a,b}(F_{c,d}(A)) = X_{a,ac,1,1,b+d-bc-bd,1},$$

$$H_{a,b}(G_{c,d}(A)) = X_{ac,0,0,b+d-bd,b,c},$$

$$H_{a,b}(H_{c,d}(A)) = X_{ac,0,0,1,b+d-bd,\frac{d+bc-bd}{b+d-bd}},$$

$$H_{a,b}(H^*_{c,d}(A)) = X_{ac,0,0,1,b+d-bd,c},$$

$$H_{a,b}(J_{c,d}(A)) = X_{a,ac,1,b+d-bd,b-bc,1},$$

$$H_{a,b}(J^*_{c,d}(A)) = X_{a,ac,d,b+d-bc-bd+bcd,b-bc,1},$$

$$H^*_{a,b}(F_{c,d}(A)) = X_{a,ac,1,1,b+d-bd,\frac{d+ab-abc-bd}{b+d-bd}},$$

$$H^*_{a,b}(G_{c,d}(A)) = X_{ac,0,0,b+d-bd,b,ac},$$

$$H^*_{a,b}(H_{c,d}(A)) = X_{ac,0,0,1,b+d-bd,\frac{d-ab+abc}{b+d-bd}},$$

$$H^*_{a,b}(H^*_{c,d}(A)) = X_{ac,0,0,1,b+d-bd,\frac{cd+abc-bcd}{b+d-bd}},$$

$$H^*_{a,b}(J_{c,d}(A)) = X_{a,ac,1,b+d-bd,b-abc,\frac{ab-abc}{b-abc}},$$

$$H^*_{a,b}(J^*_{c,d}(A)) = X_{a,ac,acd,b+d-bd-abc+abcd,b-abc,\frac{ab-abc}{b-abc}},$$

$$J_{a,b}(F_{c,d}(A)) = X_{1,a+c-ac-ad,1,b,bd,1},$$

$$J_{a,b}(G_{c,d}(A)) = X_{a+c-ac,a,d,bd,0,1},$$

$$J_{a,b}(H_{c,d}(A)) = X_{a+c-ac,a-ad,1,b,bd,1},$$

$$J_{a,b}(H^*_{c,d}(A)) = X_{a+c-ac,a-ad,1,b,bd,c},$$

$$J_{a,b}(J_{c,d}(A)) = X_{1,a+c-ac,\frac{c-ac+ad}{a+c-ac},bd,0,1},$$

$$J_{a,b}(J^*_{c,d}(A)) = X_{1,a+c-ac,d,bd,0,1},$$

$$J^*_{a,b}(F_{c,d}(A)) = X_{1,a+c-ac-abd,\frac{c+ab-ac-abd}{a+c-ac-abd},b,bd,1},$$

$$J^*_{a,b}(G_{c,d}(A)) = X_{a+c-ac,a,bd,bd,0,1},$$

$$J^*_{a,b}(H_{c,d}(A)) = X_{a+c-ac,a-abd,\frac{ab-abd}{a-abd},b,bd,1},$$

$$J^*_{a,b}(H^*_{c,d}(A)) = X_{a+c-ac-abd+abcd,a-abd,\frac{ab-abd}{a-abd},b,bd,c},$$

$$J^*_{a,b}(J_{c,d}(A)) = X_{1,a+c-ac,c-ac+abd,bd,0,1},$$

$$J^*_{a,b}(J^*_{c,d}(A)) = X_{1,a+c-ac,\frac{cd-acd+abd}{a+c-ac},bd,0,1}.$$

References

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