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Intuitionistic fuzzy rw-closed sets and intuitionistic fuzzy rw-continuity

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Abstract: The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy rw-closed sets in itutionistic fuzzy topological space. The class of all intuitionistic fuzzy rw-closed sets lies between the class of all intuitionistic fuzzy w-closed sets and class of all intuitionistic fuzzy rg-closed sets. We also introduce the concepts of intuitionistic fuzzy rw- open sets, intuitionistic fuzzy rw-continuity and intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed mappings in intuitionistic fuzzy topological spaces.

Keywords: Intuitionistic fuzzy rw-closed sets, Intuitionistic fuzzy rw-open sets, Intuitionistic fuzzy rw-connectedness, Intuitionistic fuzzy rw-compactness, intuitionistic fuzzy rw-continuous mappings, Intuitionistic fuzzy rw-open mappings and intuitionistic fuzzy rw-closed mappings.

AMS Classification: 03E72

1 Introduction

After the introduction of fuzzy sets by Zadeh [28] in 1965 and fuzzy topology by Chang [5] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for IFS. In 1997 Coker [6] introduced the concept of IF topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [24], fuzzy separation axioms [3], fuzzy continuity [9], fuzzy g-closed sets [16], fuzzy g-continuity [17], fuzzy rg-closed sets [18] have been generalized for IF topological spaces. Recently authors of this paper introduced the concept of IF w-closed sets[20] in IF topology.

In the present paper we extend the concepts of rw-closed sets due to Benchalli and Walli [4] in IF topological spaces . The class of intuitionistic fuzzy rw-closed sets is properly placed between the class of IF w-closed sets and IF rg-closed sets. We also introduced the concepts of IF rw-open sets, IF rw-connectedness, IF rw-compactness and IF rw-continuity, and obtain some of their characterization and properties.

2 Preliminaries

Let X be a nonempty fixed set. An IFS A[1] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x)+ \gamma_A(x) \leq 1$ for each $x \in X$. The IFSs $\tilde{0} = \{< x, 0, 1 > : x \in X\}$ and $\tilde{1} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole IFS on X. An IFS $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an IFS $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for each $x \in X$. The complement of an IFS $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the IFS $A^c = \{ <x,\gamma_A(x), \mu_A(x)>: x \in X\}$. The intersection (resp. union) of any arbitrary family of IFSs $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x)> : x \in X, (i \in \wedge)\}$ of X be the IFS $\cap A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x)> : x \in X\}$ (resp. $\cup A_i = \{<x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x)>: x \in X\}$). Two IFSs $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said be q-coincident (A_qB for short) if and only if there exists an element $x \in X$, such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of IFSs on a non empty set X is called an intuitionistic fuzzy topology [6] on X if the intuitionistic fuzzy sets $\tilde{0}, \tilde{1} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It is denoted by cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by int(A) [6].

Lemma 2.1 [6]**:** Let A and B be any two IFS of an IF topological space $(X, \Im)$. Then:

(a) $(A_qB) \Leftrightarrow A \subseteq B^c$.
(b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl(A) = A
(c) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int(A) = A.
(d) $cl(A^c) = (int(A))^c$.
(e) $int(A^c) = (cl(A))^c$.
(f) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$.
(g) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$.
(h) $cl(A \cup B) = cl(A) \cup cl(B)$.
(i) $int(A \cap B) = int(A) \cap int(B)$

Definition 2.1 [7]**:** Let X is a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta \leq 1$ then,

(a) $c(\alpha,\beta) = <x, c_\alpha, c_{1-\beta}>$ is called an IF point in X, where α denotes the degree of membership of $c(\alpha, \beta)$, and β denotes the degree of nonmembership of $c(\alpha,\beta)$.
(b) $c(\beta) = <x, 0, 1-c_{1-\beta}>$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2 [8]: A family $\{G_i : i \in \wedge\}$ of IFSs in X is called an intuitionistic fuzzy open cover of X if $\cup\{G_i : i \in \wedge\} = \tilde{1}$ and a finite subfamily of an intuitionistic fuzzy open cover $\{G_i: i \in \wedge\}$of X which also an intuitionistic fuzzy open cover of X is called a finite sub cover of $\{G_i: i \in \wedge\}$.

Definition 2.3 [8]: An intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy compact if every intuitionistic fuzzy open cover of X has a finite sub cover.

Definition 2.4 [24]: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open and intuitionistic fuzzy closed .

Definition 2.5 [9]: An IFS A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O \subseteq A \subseteq cl(O)$.
(b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.

(c) An intuitionistic fuzzy regular open of X if int(cl(A)) = A .
(d) An intuitionistic fuzzy regular closed of X if cl(int(A))= A.
(e) An intuitionistic fuzzy pre open if A $\subseteq$ int(cl(A)) .
(f) An intuitionistic fuzzy pre closed if cl(int(A)) $\subseteq$ A

Definition 2.6 [22]: An intuitionistic fuzzy topological space (X, $\Im$) is called intuitionistic fuzzy regular semi open if there is a regular open set U such that U $\subseteq$ A $\subseteq$ cl(U).

Theorem 2.1 [22]: Let(X,$\Im$) be an intuitionistic fuzzy topological spaces and A be an IFS in X then, the following conditions are equivalent:
(a) A is intuitionistic fuzzy regular semi open
(b) A is both intuitionistic fuzzy semi open and intuitionistic fuzzy semi closed.
(c) A^c is intuitionistic fuzzy regular semi open

Definition 2.7 [9] If A is an IFS in intuitionistic fuzzy topological space(X,$\Im$) then
(a) scl(A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi closed}
(b) pcl(A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy pre closed}

Definition 2.8: An IFS A of an intuitionistic fuzzy topological space (X,$\Im$) is called:
(a) Intuitionistic fuzzy g-closed if cl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[16]
(b) Intuitionistic fuzzy g-open if its complement A^c is intuitionistic fuzzy g-closed.[16]
(c) Intuitionistic fuzzy rg-closed if cl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[18]
(d) Intuitionistic fuzzy rg-open if its complement A^c is intuitionistic fuzzy rg-closed.[18]
(e) Intuitionistic fuzzy w-closed if cl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[20]
(f) Intuitionistic fuzzy w -open if its complement A^c is intuitionistic fuzzy w-closed.[20]
(g) Intuitionistic fuzzy gpr-closed if pcl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[21]
(h) Intuitionistic fuzzy gpr -open if its complement A^c is intuitionistic fuzzy gpr-closed. [21]

Remark 2.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true.[16]

Remark 2.2: Every intuitionistic fuzzy g- closed set is intuitionistic fuzzy rg-closed but its converse may not be true.[18]

Remark 2.3: Every intuitionistic fuzzy w-closed (resp. Intuitionistic fuzzy w-open) set is intuitionistic fuzzy g-closed (intuitionistic fuzzy g-open) but its converse may not be true.[20]

Remark 2.4: Every intuitionistic fuzzy g-closed (resp. Intuitionistic fuzzy g-open) set is intuitionistic fuzzy gpr-closed (intuitionistic fuzzy gpr-open) but its converse may not be true[21]

Definition 2.9 [6]: Let X and Y are two nonempty sets and f: X $\rightarrow$ Y is a function:
(a) If B = {<y, $\mu_B(y)$, $\gamma_B(y)$> : y $\in$ Y}is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by
$f^{-1}(B)$ = <x, $f^{-1}(\mu_B)$ (x), $f^{-1}(\gamma_B)$ (x)>: x $\in$ X}.
(b) If A = {<x, $\lambda_A(x)$, $\nu_A(x)$> : x $\in$ X}is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by
f (A) = {<y, f (λ_A) (y), f(ν_A) (y)>: y $\in$ Y}
Where $f(\nu_A) = 1 - f(1- \nu_A)$.

Definition 2.10 [9]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X \to Y$ be a function. Then f is said to be

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.
(b) Intuitionistic fuzzy closed if the image of each intuitionisic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.
(c) Intuitionistic fuzzy open if the image of each intuitionisic fuzzy open set in X is an intuitionistic fuzzy open set in Y.

Definition 2.11 [27]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X \to Y$ be a function. Then f is said to be intuitionistic fuzzy almost continuous if inverse image of every intuitionistic fuzzy regular closed set of Y is intuitionistic fuzzy closed in X.

Definition 2.12 [23]: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: $X \to Y$ be a function. Then f is said to be intuitionistic fuzzy almost irresolute if inverse image of every intuitionistic fuzzy regular semi open set of Y is intuitionistic fuzzy semi open in X.

Definition 2.13: Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let $f : X \to Y$ be a function. Then f is said to be

(a) Intutionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.[17]
(b) Intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X.[20]
(c) Intuitionistic fuzzy w-open if image of every open set of X is intuitionistic fuzzy w-open in Y.[20]
(d) Intuitionistic fuzzy w-closed if image of every closed set of X is intuitionistic fuzzy w-closed in Y.[20]
(e) Intutionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg –closed in X.[19]
(f) Intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X.[21]

Remark 2.5: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [17].

Remark 2.6: Every intuitionistic fuzzy w- continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [20].

Remark 2.7: Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rg-continuous, but the converse may not be true [16].

Remark 2.8: Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gpr-continuous, but the converse may not be true [21].

3 Intuitionistic fuzzy rw-closed set

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called an intuitionistic fuzzy rw-closed if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open in X.
First we prove that the class of intuitionistic fuzzy rw-closed sets properly lies between the class of intuitionistic fuzzy w-closed sets and the class of intuitionistic fuzzy rg-closed sets.

Theorem 3.1: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy rw-closed.

Proof: The proof follows from the Definition 3.1 and the fact that every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy semi open.

Remark 3.1: The converse of Theorem 3.1 need not be true as from the following example.
Example 3.1: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$,U $\tilde{\mathbf{1}}$, } be an intuitionistic fuzzy topology on X, where U = {<a, 0.7, 0.2>, <b, 0.6, 0.3>}.Then the IFS A = {<a, 0.7, 0.2>, <b, 0.8, 0.1>} is intuitionistic fuzzy rw-closed but it is not intuitionistic fuzzy w-closed.

Theorem 3.2: Every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy rg-closed.
Proof: The proof follows from the Definition 3.1 and the fact that every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open.

Remark 3.2: The converse of Theorem 3.2 need not be true as from the following example.
Example 3.2: Let X = {a, b, c, d} and intuitionistic fuzzy sets O, U, V, W defined as follows

O = {<a, 0.9, 0.1>, <b, 0, 1>, <c, 0, 1>, <d, 0, 1>}
U = {<a, 0, 1,>, <b, 0.8, 0.1>, <c, 0, 1>, <d, 0, 1>}
V = {<a, 0.9, 0.1>, <b, 0.8, 0.1,>, <c, 0, 1>, <d, 0, 1>}
W = {<a, 0.9, 0.1>, <b, 0.8, 0.1>, <c, 0.7, 0.2>, <d, 0, 1>}

$\Im$ = {$\tilde{\mathbf{0}}$, O, U, V, W $\tilde{\mathbf{1}}$,} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0, 1>, <b, 0, 1>, <c, 0.7, 0.2>, <d, 0, 1>} is intuitionistic fuzzy rg-closed but it is not intuitionistic fuzzy rw-closed.

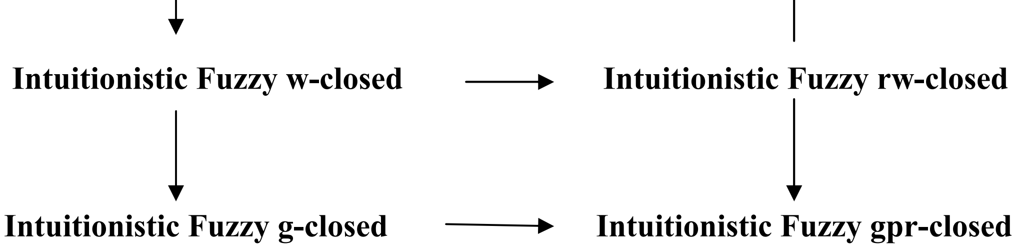
Theorem 3.3: Every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy gpr-closed.
Proof: Let A is an intuitionistic fuzzy rw closed set in intuitionistic fuzzy topological space (X,$\Im$) Let A $\subseteq$ O where O is intuitionistic fuzzy regular open in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open and A is intuitionistic fuzzy rw-closed set, we have cl(A) $\subseteq$ O. Since every intuitionistic fuzzy closed set is intuitionistic fuzzy pre closed ,pcl(A) $\subseteq$ cl(A) . Hence pcl(A)$\subseteq$ O which implies that A is intuitionistic fuzzy gpr-closed.

Remark 3.3: The converse of Theorem 3.3 need not be true as from the following example.
Example 3.3: Let X = {a, b, c, d, e} and intuitionistic fuzzy sets O, U, V defined as follows

O = {<a, 0.9, 0.1>, <b, 0.8, 0.1>, <c, 0, 1>, <d, 0, 1>, <e, 0, 1>}
U = {<a, 0, 1>, <b, 0, 1>, <c, 0.8, 0.1>, <d, 0.7, 0.2>, <e, 0, 1>}
V = {<a, 0.9, 0.1>, <b, 0.8, 0.1>, <c, 0.8, 0.1>, <d, 0.7, 0.2>, <e, 0, 1>}

Let $\Im$ = {$\tilde{\mathbf{0}}$, O, U, V, $\tilde{\mathbf{1}}$ } be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={<a, 0.9, 0.1>, <b, 0, 1>, <c, 0, 1>, <d, 0, 1>, <e, 0, 1>} is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy rw-closed.

Remark 3.4: From Remarks 2.1, 2.2, 2.3, 2. , 3.1, 3.2, 3.3 and Theorems 3.1, 3.2, 3.3 we reach at the following diagram of implications:



Theorem 3.1: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy rw-closed if and only if $\neg(AqF) \Rightarrow \neg(cl(A)qF)$ for every intuitionistic fuzzy regular semi open set F of X.

Proof: Necessity: Let F be an intuitionistic fuzzy regular semi open set of X and $\neg(AqF)$. Then by Lemma 2.1(a) and Theorem 2.1, $A \subseteq F^c$ and F^c intuitionistic fuzzy regular semi open in X. Therefore $cl(A) \subseteq F^c$ by Definiton 3.1 because A is intuitionistic fuzzy rw-closed. Hence by Lemma 2.1(a), $\neg(cl(A)qF)$.

Sufficiency: Let O be an intuitionistic fuzzy regular semi open set of X such that $A \subseteq O$ i.e. $A \subseteq (O)^c)^c$. Then by Lemma 2.1(a), $\neg(A_qO^c)$ and O^c is an intuitionistic fuzzy regular semi open set in X. Hence, by hypothesis $\neg(cl(A)_qO^c)$. Therefore, by Lemma 2.1(a), $cl(A) \subseteq((O)^c)^c$ i.e. $cl(A) \subseteq O$. Hence, A is intuitionistic fuzzy rw-closed in X.

Theorem 3.2: Let A be an intuitionistic fuzzy rw-closed set in an IF topological space $(X,\Im)$ and $c(\alpha,\beta)$ be an IF point of X, such that $c(\alpha,\beta)_q\, cl(int(A))$ then $cl(int(c(\alpha,\beta))qA$.

Proof: If $\neg cl(int(c(\alpha,\beta))_qA$ then by Lemma 2.1(a), $cl(int(c(\alpha,\beta) \subseteq A^c$ which implies that $A \subseteq (cl(c(\alpha,\beta)))^c$ and so $cl(A) \subseteq (cl(int(c(\alpha,\beta)))^c \subseteq (c(\alpha,\beta))^c$, because A is intuitionistic fuzzy rw-closed in X. Hence by Lemma 2.1(a), $\neg(c(\alpha,\beta)_q\, (cl(int(A)))$, a contradiction.

Theorem 3.3: Let A and B are two intuitionistic fuzzy rw-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$, then $A\cup B$ is intuitionistic fuzzy rw-closed.

Proof: Let O be an intuitionistic fuzzy regular semi open set in X, such that $A \cup B \subseteq O$. Then, $A \subseteq O$ and $B \subseteq O$. So, $cl(A) \subseteq O$ and $cl(B) \subseteq O$. Therefore, $cl(A) \cup cl(B) = cl(A\cup B) \subseteq O$. Hence $A\cup B$ is intuitionistic fuzzy rw-closed.

Remark 3.5: The intersection of two intuitionistic fuzzy rw-closed sets in an intuitionistic fuzzy topological space $(X,\Im)$ may not be intuitionistic fuzzy rw-closed. For,

Example 3.4: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = {<a, 0.9, 0.1>, <b, 0, 1>, <c, 0, 1>, <d, 0, 1>}

U = {<a, 0, 1,>, <b, 0.8, 0.1>, <c, 0, 1>, <d, 0, 1>}

V = {<a, 0.9, 0.1>, <b, 0.8, 0.1,>, <c, 0, 1>, <d, 0, 1>}

W = {<a, 0.9,0.1,>, <b, 0.8, 0.1>, <c, 0.7, 0.2>, <d, 0, 1>}

$\Im = \{\tilde{0}$, O, U, V, W $\tilde{1}$,} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a, 0.9, 0.1>, <b, 0.8, 0.1>, <c, 0, 1>, <d, 0, 1>} and B = {<a, 0.9, 0.1>, <b, 0, 1>, <c, 0.7, 0.2>, <d, 0.9, 0.1>} are intuitionistic fuzzy rw-closed in $(X,\Im)$ but $A \cap B$ is not intuitionistic fuzzy rw-closed.

Theorem 3.4: Let A be an intuitionistic fuzzy rw-closed set in an intuitionistic fuzzy topological space $(X,\Im)$ and $A \subseteq B \subseteq cl(A)$. Then B is intuitionistic fuzzy rw-closed in X.

Proof: Let O be an intuitionistic fuzzy regular semi open set in X such that $B \subseteq O$. Then, $A \subseteq O$ and since A is intuitionistic fuzzy rw-closed, $cl(A) \subseteq O$. Now $B \subseteq cl(A) \Rightarrow cl(B) \subseteq cl(A) \subseteq O$. Consequently B is intuitionistic fuzzy rw-closed.

Theorem 3.5: If A is intuitionistic fuzzy regular open and intuitionistic fuzzy rw-closed set, then A is intuitionistic fuzzy regular closed and hence intuitionistic fuzzy clopen.

Proof: Suppose A is intuitionistic fuzzy regular open and intuitionistic fuzzy rw-closed set. As every intuitionistic fuzzy regular open set is intuitionistic fuzzy regular semi open and $A \subseteq A$, we have $cl(A) \subseteq A$. Also $A \subseteq cl(A)$. Therefore $cl(A) = A$. That means A is intuitionistic fuzzy closed. Since A is intuitionistic regular open, them A is intuitionistic fuzzy open. Now $cl(int(A)) = cl(A)) = A$. Therefore, A is intuitionistic fuzzy regular closed and intuitionistic fuzzy closed.

Theorem 3.6: If A is an intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed in intuitionistic fuzzy topological space $(X,\Im)$, then A is intuitionistic fuzzy rw-closed in X.

Proof: Let A is an intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed in X. We prove that A is an intuitionistic fuzzy rw-closed in X. Let U be any intuitionistic fuzzy regular semi open set in X such that A $\subseteq$ U. Since A is intuitionistic fuzzy regular open and intuitionistic fuzzy rg-closed, we have cl(A) $\subseteq$ A. Then cl(A) $\subseteq$ A $\subseteq$ U. Hence A is intuitionistic fuzzy rw-closed in X.

Theorem 3.7: If A is an intuitionistic regular semi open and intuitionistic fuzzy rw-closed in intuitionistic fuzzy topological space (X,$\Im$), then A is intuitionistic fuzzy closed in X.
Proof: Suppose A is both intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw-closed set in X. Now A $\subseteq$ A. Then cl(A) $\subseteq$ A. Hence A is intuitionistic fuzzy closed in X.

Corollary 3.1: If A is an intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw-closed in intuitionistic fuzzy topological space (X,$\Im$). Suppose that F is intuitionistic fuzzy closed in X then A $\cap$ F is intuitionistic fuzzy rw-closed in X.
Proof: Suppose A is both intuitionistic fuzzy regular semi open and intuitionistic fuzzy rw-closed set in X and F is intuitionistic fuzzy closed in X. By Theorem 3.7, A is intuitionistic fuzzy closed in X . So A $\cap$ F is intuitionistic fuzzy closed in X . Hence A $\cap$ F is intuitionistic fuzzy rw-closed in X .

Theorem 3.8: If A is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in intuitionistic fuzzy topological space (X,$\Im$) . Then A is intuitionistic fuzzy rw-closed set in X.
Proof: Let A is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in X. Let A $\subseteq$ U, where U is intuitionistic fuzzy regular semi open in X. Now A $\subseteq$ A. By hypothesis cl(A) $\subseteq$ A. That is cl(A) $\subseteq$ U. Thus, A is intuitionistic fuzzy rw-closed in X.

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called intuitionistic fuzzy rw-open if and only if its complement A^c is intuitionistic fuzzy rw-closed.

Remark 3.6: Every intuitionistic fuzzy w-open set is intuitionistic fuzzy rw-open but its converse may not be true.
Example 3.5: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}}$,} be an intuitionistic fuzzy topology on X, where U = {<a, 0.7, 0.2>, <b, 0.6, 0.3>}. Then the intuitionistic fuzzy set A = {<a, 0.2, 0.7>, <b, 0.1, 0.8>} is intuitionistic fuzzy rw-open in (X, $\Im$) but it is not intuitionistic fuzzy w-open in (X, $\Im$).

Theorem 3.9: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy rw-open if F$\subseteq$ int(A) whenever F is intuitionistic fuzzy regular semi open and F $\subseteq$ A.
Proof: Follows from Definition 3.1 and Lemma 2.1

Theorem 3.10: Let A be an intuitionistic fuzzy rw-open set of an intuitionistic fuzzy topological space (X,$\Im$) and int(A) $\subseteq$ B $\subseteq$ A. Then B is intuitionistic fuzzy rw-open.
Proof: Suppose A is an intuitionistic fuzzy rw-open in X and int(A) $\subseteq$ B $\subseteq$A. $\Rightarrow A^c \subseteq B^c \subseteq (int(A))^c \Rightarrow A^c \subseteq B^c \subseteq cl(A^c)$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy rw-closed it follows from Theorem 3.4 that B^c is intuitionistic fuzzy rw-closed. Hence, B is intuitionistic fuzzy rw-open.

Theorem 3.11: Let A be an intuitionistic fuzzy w-closed set in an intuitionistic fuzzy topological space (X,$\Im$) and f: (X,$\Im$) $\rightarrow$ (Y,$\Im^*$) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping then f (A) is an intuitionistic rw-closed set in Y.
Proof: Let A be an intuitionistic fuzzy w-closed set in X and f: (X,$\Im$) $\rightarrow$ (Y,$\Im^*$) is an intuitionistic fuzzy almost irresolute and intuitionistic fuzzy closed mapping. Let f(A) $\subseteq$ G where G is intuitionistic fuzzy regular semi open in Y then A $\subseteq$ f^{-1}(G) and f^{-1}(G) is

intutionistic fuzzy semi open in X because f is intuitionistic fuzzy almost irresolute. Now A be an intuitionistic fuzzy w-closed set in X, cl(A) $\subseteq f^{-1}(G)$. Thus, f(cl(A)) $\subseteq$ G and f(cl(A)) is an intuitionistic fuzzy closed set in Y (since cl(A) is intuitionistic fuzzy closed in X and f is intuitionistic fuzzy closed mapping). It follows that cl(f (A) $\subseteq$ cl(f (cl(A))) = f (cl(A)) $\subseteq$ G. Hence cl(f (A)) $\subseteq$ G whenever f (A) $\subseteq$ G and G is intuitionistic fuzzy regular semi open in Y. Hence f (A) is intuitionistic fuzzy rw-closed set in Y.

Theorem 3.12: Let(X,$\Im$) be an intuitionistic fuzzy topological space and IFRSO(X) (resp. IFC(X)) be the family of all intuitionistic fuzzy regular semi open (resp. intuitionistic fuzzy closed) sets of X. Then IFRSO(X) $\subseteq$ IFC(X) if and only if every intuitionistic fuzzy set of X is intuitionistic fuzzy rw-closed.

Proof: Necessity: Suppose that IFRSO(X) $\subseteq$ IFC(X) and let A be any intuitionistic fuzzy set of X such that A$\subseteq$ U $\in$ IFRSO(X) i.e. U is intuitionistic fuzzy regular semi open. Then, cl(A) $\subseteq$ cl(U) = U because U $\in$ IFRSO(X) $\subseteq$ IFC(X). Hence cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy regular semi open. Hence A is rw-closed set.

Sufficiency: Suppose that every intuitionistic fuzzy set of X is intuitionistic fuzzy rw-closed. Let U $\in$ IFRSO(X), then since U $\subseteq$ U and U is intuitionistic fuzzy rw-closed, cl(U) $\subseteq$ U then U $\in$ IFC(X).Thus IFRSO(X) $\subseteq$ IFC(X).

Definition 3.3: An intuitionistic fuzzy topological space (X $\Im$) is called intuitionistic fuzzy rw-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed .

Theorem 3.13: Every intuitionistic fuzzy rw-connected space is intuitionistic fuzzy connected.

Proof: Let (X, $\Im$) be an intuitionistic fuzzy rw-connected space and suppose that (X, $\Im$) is not intuitionistic fuzzy connected. Then there exists a proper IFS A (A$\neq$ $\tilde{\mathbf{0}}$, A$\neq$ **1**) such that A is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic rw-open (resp. intuitionistic fuzzy rw-closed), X is not intuitionistic fuzzy rw-connected, a contradiction.

Remark 3.7: Converse of Theorem 3.13 may not be true for

Example 3.6: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$, U, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where U ={<a, 0.5, 0.5>, < b, 0.4, 0.6 >}.Then, intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy connected but not intuitionistic fuzzy rw-connected because there exists a proper intuitionistic fuzzy set A={<a,0.5,0.5>,<b, 0.5,0.5>} which is both intuitionistic fuzzy rw-closed and intuitionistic rw-open in X.

Theorem 3.14: An intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy rw-connected if and only if there exists no non zero intuitionistic fuzzy rw-open sets A and B in X such that A=B^c.

Proof: Necessity: Suppose that A and B are intuitionistic fuzzy rw-open sets such that A$\neq$ $\tilde{\mathbf{0}}$$\neq$ B and A = B^c. Since A=B^c, B is an intuitionistic fuzzy rw-open set which implies that B^c = A is intuitionistic fuzzy rw-closed set and B$\neq$ $\tilde{\mathbf{0}}$ this implies that $B^c$$\neq$ $\tilde{\mathbf{1}}$ i.e. A$\neq$ $\tilde{\mathbf{1}}$. Hence, there exists a proper intuitionistic fuzzy set A(A$\neq$ $\tilde{\mathbf{0}}$, A$\neq$ $\tilde{\mathbf{1}}$) such that A is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed. But this is contradiction to the fact that X is intuitionistic fuzzy rw-connected.

Sufficiency: Let (X,$\Im$) is an intuitionistic fuzzy topological space and A is both intuitionistic fuzzy rw-open set and intuitionistic fuzzy rw-closed set in X such that $\tilde{\mathbf{0}}$ $\neq$ A $\neq$ $\tilde{1}$. Now take B = A^c. In this case B is an intuitionistic fuzzy rw-open set and A $\neq$ $\tilde{\mathbf{1}}$.This implies that B = A^c $\neq$ $\tilde{0}$, which is a contradiction. Hence there is no proper IFS of X which is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed. Therefore, intuitionistic fuzzy topological (X,$\Im$) is intuitionistic fuzzy rw-connected

Definition 3.3: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set X. Then rw-interior and rw-closure of A are defined as follows.

$$rwcl(A) = \cap \{K: K \text{ is an intuitionistic fuzzy rw-closed set in X and } A \subseteq K\}$$
$$rwint(A) = \cup \{G: G \text{ is an intuitionistic fuzzy rw-open set in X and } G \subseteq A\}$$

Remark 3.8: It is clear that $A \subseteq rwcl(A) \subseteq cl(A)$ for any intuitionistic fuzzy set A.

Theorem 3.15: An intuitionistic fuzzy topological space $(X, \Im)$ is intuitionistic fuzzy rw-connected if and only if there exists no non zero intuitionistic fuzzy rw-open sets A and B in X such that $B = A^c$, $B = (rwcl(A))^c$, $A = (rwcl(B))^c$.

Proof: Necessity: Assume that there exists intuitionistic fuzzy sets A and B such that $A \neq \tilde{0} \neq B$ in X such that $B = A^c$, $B = (rwcl(A))^c$, $A = (rwcl(B))^c$. Since $(rwcl(A))^c$ and $(rwcl(B))^c$ are intuitionistic fuzzy rw-open sets in X, which is a contradiction.

Sufficiency: Let A is both an intuitionistic fuzzy rw-open set and intuitionistic fuzzy rw-closed set such that $\tilde{0} \neq A \neq \tilde{1}$. Taking $B= A^c$, we obtain a contradiction.

Definition 3.4: An intuitionistic fuzzy topological space $(X,\Im)$ is said to be intuitionistic fuzzy rw-$T_{1/2}$ if every intuitionistic fuzzy rw-closed set in X is intuitionistic fuzzy closed in X.

Theorem 3.16: Let $(X,\Im)$ be an intuitionistic fuzzy rw-$T_{1/2}$ space, then the following conditions are equivalent:

(a) X is intuitionistic fuzzy rw-connected.
(b) X is intuitionistic fuzzy connected.

Proof: (a) $\Rightarrow$ (b) follows from Theorem 3.13

(b) $\Rightarrow$ (a): Assume that X is intuitionistic fuzzy rw-$T_{1/2}$ and intuitionistic fuzzy rw-connected space. If possible, let X be not intuitionistic fuzzy rw-connected, then there exists a proper intuitionistic fuzzy set A such that A is both intuitionistic fuzzy rw-open and rw-closed. Since X is intuitionistic fuzzy rw-$T_{1/2}$, A is intuitionistic fuzzy open and intuitionistic fuzzy closed which implies that X is not intuitionistic fuzzy connected, a contradiction.

Definition 3.4: A collection $\{A_i : i \in \Lambda\}$ of intuitionistic fuzzy rw-open sets in intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy rw-open cover of intuitionistic fuzzy set B of X if $B \subseteq \cup\{ A_i : i \in \Lambda\}$

Definition 3.5: An intuitionistic fuzzy topological space $(X,\Im)$ is said to be intuitionistic fuzzy rw-compact if every intuitionistic fuzzy rw-open cover of X has a finite sub cover.

Definition 3.6 : An intuitionistic fuzzy set B of intuitionistic fuzzy topological space $(X, \Im)$ is said to be intuitionistic fuzzy rw-compact relative to X, if for every collection $\{A_i : i \in \Lambda\}$ of intuitionistic fuzzy rw-open subset of X such that $B \subseteq \cup\{A_i : i \in \Lambda\}$ there exists finite subset Λ_o of Λ such that $B \subseteq \cup\{A_i : i \in \Lambda_o\}$.

Definition 3.7: A crisp subset B of intuitionistic fuzzy topological space $(X, \Im)$ is said to be intuitionistic fuzzy rw-compact if B is intuitionistic fuzzy rw-compact as intuitionistic fuzzy subspace of X .

Theorem 3.16: A intuitionistic fuzzy rw-closed crisp subset of intuitionistic fuzzy rw-compact space is intuitionistic fuzzy rw-compact relative to X.

Proof: Let A be an intuitionistic fuzzy rw-closed crisp subset of intuitionistic fuzzy rw-compact space $(X,\Im)$. Then A^c is intuitionistic fuzzy rw-open in X. Let M be a cover of A by intuitionistic fuzzy rw-open sets in X. Then the family $\{M, A^c \}$ is intuitionistic fuzzy rw-open cover of X. Since X is intuitionistic fuzzy rw-compact, it has a finite sub cover say $\{G_1, G_2, ..., Gn\}$. If this sub cover contains A^c, we discard it. Otherwise, leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy rw-open sub cover of A. Therefore A is intuitionistic fuzzy rw-compact relative to X.

4 Intuitionistic fuzzy rw-continuity

Definition 4.1: A mapping $f : (X,\Im). \rightarrow (Y, \sigma)$ is intuitionistic fuzzy rw-continuous if inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy rw-closed set in X.

Theorem 4.1: A mapping $f : (X,\Im). \rightarrow (Y,\sigma)$ is intuitionistic fuzzy rw-continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy rw-open in X.

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

Remark 4.1 Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy rw-continuous, but converse may not be true. For,

Example 4.1 Let X = {a, b}, Y ={x, y} and intuitionistic fuzzy sets U and V are defined as follows:

U = {<a, 0.7, 0.2>, <b, 0.6, 0.3>}
V = {<x, 0.7, 0.2>, <y, 0.8, 0.1>}

Let $\Im = \{\tilde{0}, \mathbf{U}, \tilde{1}\}$ and $\sigma = \{\tilde{\mathbf{0}}, \mathbf{V}, \tilde{\mathbf{1}}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\Im). \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy rw-continuous but not intuitionistic fuzzy continuous.

Remark 4.2 Every intuitionistic fuzzy rw-continuous mapping is intuitionistic fuzzy rg-continuous, but converse may not be true. For,

Example 4.2: Let X = {a, b, c, d } Y ={p, q r, s} and intuitionistic fuzzy sets O,U,V,W,T are defined as follows:

O = {<a, 0.9,0.1> ,<b, 0,1>,<c, 0,1>,<d, 0,1>}
U = {<a, 0,1,> ,<b, 0.8,0.1>,<c, 0,1>,<d, 0,1>}
V = {<a, 0.9,0.1> ,<b, 0.8,0.1,>,<c, 0,1>,<d, 0,1>}
W = {<a, 0.9,0.1,> ,<b, 0.8,0.1>,<c, 0.7,0.2>,<d, 0,1>}
T = {<p,0,1>,<q,0,1> <r,0.7,0.2>,<s,0,1>}

Let $\Im = \{\tilde{\mathbf{0}}, O, U, V, W, \tilde{\mathbf{1}}\}$ and $\sigma = \{\tilde{\mathbf{0}}, T, \mathbf{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X,\Im) \rightarrow (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$ is intuitionistic fuzzy rg-continuous but not intuitionistic fuzzy rw-continuous.

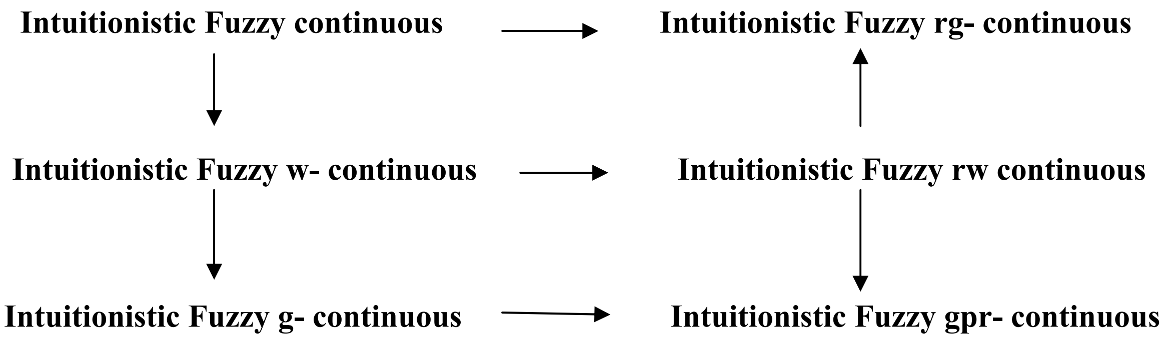
Remark 4.3 Every intuitionistic fuzzy rw-continuous mapping is intuitionistic fuzzy gpr-continuous, but converse may not be true. For,

Example 4.3: Let X = {a, b, c, d, e } Y ={p, q r, s, t} and intuitionistic fuzzy sets O,U,V,W are defined as follows:

O = {<a, 0.9,0.1> , <b, 0.8,1>, <c, 0,1>,<d, 0,1>,<e,0,1>}
U = {<a, 0,1,> ,<b, 0,1>,<c, 0.8,1>, <d, 0.7,0.2>,<e,0,1>}
V = {<a, 0.9,0.1> ,<b, 0.8,0.1,>, <c, 0.8,0.1>,<d, 0.7,0.2>,<e,0,1>}
W = {<a, 0.9,0.1,> ,<b, 0,1>, <c, 0.8,0.1>, <d, 0.7,0.2>,<e,0,1>}

Let $\Im = \{\tilde{\mathbf{0}}, O, U, V, \tilde{\mathbf{1}}\}$ and $\sigma = \{\tilde{\mathbf{0}}, W, \tilde{\mathbf{1}}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f: (X,\Im). \rightarrow (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$, $f(e) = t$ is intuitionistic fuzzy gpr-continuous but not intuitionistic fuzzy rw-continuous.

Remark 4.3: From the Remarks 2.5, 2.6, 2.7, 2.8, 4.1, 4.2, 4.3 we reach the following diagram of implications:



Theorem 4.2: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw- continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy open set V of Y such that $f(c(\alpha,\beta)) \subseteq V$ there exists a intuitionistic fuzzy rw-open set U of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) \subseteq V$.
Proof: Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta)) \subseteq V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy rw-open set of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 4.3: Let $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy open set V of Y such that $f(c(\alpha,\beta))qV$, there exists a intuitionistic fuzzy rw-open set U of X such that $c(\alpha,\beta)qU$ and $f(U) \subseteq V$.
Proof: Let $c(\alpha,\beta)$ be intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta))q\, V$. Put $U = f^{-1}(V)$. Then by hypothesis U is intuitionistic fuzzy rw-open set of X such that $c(\alpha,\beta)q\, U$ and $f(U)= f(f^{-1}(V)) \subseteq V$.

Theorem 4.4: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous, then $f(rwcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.
Proof: Let A be an intuitionistic fuzzy set of X. Then $cl(f(A))$ is an intuitionistic fuzzy closed set of Y. Since f is intuitionistic fuzzy rw-continuous, $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy rw-closed in X. Clearly $A \subseteq f^{-1}(cl((A))$. Therefore $wcl(A) \subseteq wcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$. Hence $f(rwcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.

Theorem 4.5: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous and $g : (Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy continuous. Then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rw-continuous.
Proof: Let A is an intuitionistic fuzzy closed set in Z. then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy rw-closed in X. Hence gof is intuitionistic fuzzy rw-continuous.

Theorem 4.6: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous and $g : (Y,\sigma) \to (Z,\mu)$ is intuitionistic fuzzy g-continuous and (Y,σ) is IF $T_{1/2}$ then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rw-continuous.
Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionistic fuzzy g-closed in Y. Since Y is $T_{1/2}$, then $g^{-1}(A)$ is intuitionstic fuzzy closed in Y. Hence, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy rw-closed in X. Hence gof is intuitionistic fuzzy w-continuous.

Theorem 4.7: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rg-irresolute and $g : (Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy rw-continuous. Then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rg-continuous.

Proof: Let A is an intuitionistic fuzzy closed set in Z, then $g^{-1}(A)$ is intuitionistic fuzzy rw-closed in Y, because g is intuitionistic fuzzy rw-continuous. Since every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy rg-closed set, therefore $g^{-1}(A)$ is intuitionistic fuzzy rg-closed in Y .Then $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy rg-closed in X, because f is intuitionistic fuzzy rg-irresolute. Hence $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rg-continuous.

Theorem 4.8: An intuitionistic fuzzy rw-continuous image of a intuitionistic fuzzy rw-compact space is intuitionistic fuzzy compact.

Proof: Let. $f : (X,\Im). \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous map from a intuitionistic fuzzy rw-compact space $(X,\Im)$ onto a intuitionistic fuzzy topological space (Y, σ). Let $\{Ai : i \in \Lambda\}$ be an intuitionistic fuzzy open cover of Y then $\{f^{-1}(Ai) : i \in \Lambda\}$ is a intuitionistic fuzzy rw-open cover of X. Since X is intuitionistic fuzzy rw-compact it has finite intuitionistic fuzzy sub cover say $\{ f^{-1}(A_1) , f^{-1}(A_2) ,----f^{-1}(An) \}$. Since f is onto $\{A_1, A_2, \ldots, A_n\}$ is an intuitionistic fuzzy open cover of Y and so (Y, σ) is intuitionistic fuzzy compact.

Theorem 4.9: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-continuous surjection and X is intuitionistic fuzzy rw-connected then Y is intuitionistic fuzzy connected.

Proof: Suppose Y is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy set of X, which is both intuitionistic fuzzy rw-open and intuitionistic fuzzy rw-closed, because f is intuitionistic fuzzy rw-continuous surjection. Hence, X is not intuitionistic fuzzy rw-connected, which is a contradiction.

Definition 4.2: A mapping $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-open if the image of every intuitionistic fuzzy open set of X is intuitionistic fuzzy rw-open set in Y.

Remark 4.5: Every intuitionistic fuzzy w-open map is intuitionistic fuzzy rw-open but converse may not be true. For,

Example 4.4: Let X = {a, b}, Y = {x , y} and the intuitionistic fuzzy set U and V are defined as follows :

U = {<a, 0.7, 0.2 >, <b, 0.8, 0.1>}

V = {<x, 0.7, 0.2 >, <y, 0.6, 0.3>}

Then $\Im = \{\tilde{\mathbf{0}}, \mathbf{U}, \tilde{\mathbf{1}}\}$ and $\sigma = \{\tilde{\mathbf{0}}, \mathbf{V}, \mathbf{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then, the mapping $f : (X,\Im) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy rw-open but it is not intuitionistic fuzzy w-open.

Theorem 4.10: A mapping $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-open if and only if for every intuitionistic fuzzy set U of X $f(\text{int}(U)) \subseteq \text{rwint}(f(U))$.

Proof: Necessity: Let f be an intuitionistic fuzzy rw-open mapping and U is an intuitionistic fuzzy open set in X. Now $\text{int}(U) \subseteq U$ which implies that f (int(U) $\subseteq f(U)$. Since f is an intuitionistic fuzzy rw-open mapping, f(Int(U) is intuitionistic fuzzy rw-open set in Y such that f(Int(U) $\subseteq f(U)$ therefore f(Int(U) $\subseteq$ rwint $f(U)$.

Sufficiency: For the converse suppose that U is an intuitionistic fuzzy open set of X. Then f (U) = f (Int(U) $\subseteq$ rwint $f(U)$. But wint(f (U)) $\subseteq f$ (U). Consequently f (U) = wint(U) which implies that $f(U)$ is an intuitionistic fuzzy rw-open set of Y and hence f is an intuitionistic fuzzy rw-open.

Theorem 4.11: If $f : (X,\Im) \to (Y, \sigma)$ is an IF rw-open map then int(f^{-1}(G) $\subseteq f^{-1}$(rwint(G) for every IFS G of Y.

Proof: Let G is an intuitionistic fuzzy set of Y. Then int f^{-1}(G) is an intuitionistic fuzzy open set in X. Since *f* is intuitionistic fuzzy rw-open *f*(int f^{-1}(G)) is intuitionistic fuzzy rw-open in Y and hence *f*(Int f^{-1}(G)) $\subseteq$ rwint(*f*(f^{-1}(G)) $\subseteq$ rwint(G). Thus int f^{-1}(G) $\subseteq$ f^{-1}(rwint(G).

Theorem 4.12: A mapping $f : (X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy rw-open if and only if for each IFS S of Y and for each intuitionistic fuzzy closed set U of X containing f^{-1}(S) there is a intuitionistic fuzzy rw-closed V of Y such that S $\subseteq$ V and f^{-1}(V) $\subseteq$ U.
Proof: Necessity: Suppose that *f* is an intuitionistic fuzzy rw-open map. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy closed set of X such that f^{-1}(S) $\subseteq$ U. Then V = $(f^{-1}(U^c))^c$ is intuitionistic fuzzy rw-closed set of Y such that f^{-1}(V) $\subseteq$ U.
Sufficiency: For the converse suppose that F is an intuitionistic fuzzy open set of X. Then $f^{-1}((f(F))^c \subseteq F^c$ and F^c is intuitionistic fuzzy closed set in X. By hypothesis there is an intuitionistic fuzzy rw-closed set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ .Therefore F $\subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies *f*(F) = V^c. Since V^c is intuitionistic fuzzy rw-open set of Y. Hence *f* (F) is intuitionistic fuzzy rw-open in Y and thus *f* is intuitionistic fuzzy rw-open map.

Definition 4.3: A mapping $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy rw-closed if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy rw-closed set in Y.

Remark 4.6: Every intuitionistic fuzzy closed map is intuitionistic fuzzy rw-closed but converse may not be true. For,
Example 4.5: Let X = {a, b}, Y = {x, y}
Then the mapping $f : (X,\Im) \to (Y, \sigma)$ defined in Example 4.4 is intuitionistic fuzzy rw-closed but it is not intuitionistic fuzzy w-closed.

Theorem 4.13: A mapping $f : (X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy rw-closed if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy open set U of X containing f^{-1}(S) there is a intuitionistic fuzzy rw-open set V of Y such that S $\subseteq$ V and f^{-1}(V) $\subseteq$ U.
Proof: Necessity: Suppose that *f* is an intuitionistic fuzzy rw-closed map. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy open set of X such that f^{-1}(S) $\subseteq$ U. Then V =Y - $f^{-1}(U^c)$ is intuitionistic fuzzy rw-open set of Y such that f^{-1}(V) $\subseteq$ U.
Sufficiency: For the converse suppose that F is an intuitionistic fuzzy closed set of X. Then $(f(F))^c$ is an intuitionistic fuzzy set of Y and F^c is intuitionistic fuzzy open set in X such that $f^{-1}((f(F))^c) \subseteq F^c$. By hypothesis there is an intuitionistic fuzzy rw-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore, $F \subseteq (f^{-1}(V))^c$. Hence, $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies *f*(F) = V^c. Since V^c is intuitionistic fuzzy rw-closed set of Y. Hence *f*(F) is intuitionistic fuzzy rw-closed in Y and thus *f* is intuitionistic fuzzy w-closed map.

Theorem 4.14: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy almost irresolute and intuitionistic fuzzy rw-closed map and A is an intuitionistic fuzzy w-closed set of X, then *f* (A) intuitionistic fuzzy rw-closed.
Proof: Let *f*(A) $\subseteq$ O where O is an intuitionistic fuzzy regular semi open set of Y. Since *f* is intuitionistic fuzzy almost irresolute therefore f^{-1}(O) is an intuitionistic fuzzy semi open set of X such that A $\subseteq$ f^{-1}(O). Since A is intuitionistic fuzzy w-closed of X which implies that cl(A) $\subseteq$ (f^{-1}(O)) and hence *f*(cl(A)) $\subseteq$ O which implies that cl(*f*(cl(A))) $\subseteq$ O therefore cl(*f* ((A))) $\subseteq$ O whenever *f*(A) $\subseteq$ O where O is an intuitionistic fuzzy regular semi open set of Y. Hence *f*(A) is an intuitionistic fuzzy rw-closed set of Y.

Theorem 4.15: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy regular semi irresolute and intuitionistic fuzzy rw-closed map and A is an intuitionistic fuzzy rw-closed set of X, then *f*(A) is intuitionistic fuzzy rw-closed.

Proof: Let f(A) $\subseteq$ O where O is an intuitionistic fuzzy regular semi open set of Y. Since f is intuitionistic fuzzy regular semi irresolute therefore f^{-1}(O) is an intuitionistic fuzzy regular semi open set of X such that A $\subseteq f^{-1}$(O). Since A is intuitionistic fuzzy rw-closed of X which implies that cl(A) $\subseteq$ (f^{-1}(O)) and hence f(cl(A) $\subseteq$ O which implies that cl(f(cl(A))) $\subseteq$ O therefore cl(f ((A))) $\subseteq$ O whenever f(A) $\subseteq$ O where O is an intuitionistic fuzzy regular semi open set of Y. Hence f(A) is an intuitionistic fuzzy rw-closed set of Y.

Theorem 4.16: If $f : (X,\Im) \to (Y, \sigma)$ is intuitionistic fuzzy closed and $g : (Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy rw-closed. Then $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rw-closed.
Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space$(X,\Im)$. Then f(H) is an intuitionistic fuzzy closed set of (Y, σ) because f is an inuituionistic fuzzy closed map. Now (gof)(H) = g(f(H)) is an intuitionistic fuzzy rw-closed set in the intuitionistic fuzzy topological space Z because g is an intuitionistic fuzzy rw-closed map. Thus, $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy rw-closed.

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