

INTUITIONISTIC FUZZY STRONGLY PREIRRESOLUTE CONTINUOUS MAPPINGS

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ABSTRACT. We introduce (r, s) -intuitionistic fuzzy strongly preopen and (r, s) -intuitionistic fuzzy strongly preclosed sets in intuitionistic fuzzy topological spaces in Šostak sense. We investigate some properties of them. IF strong preirresolute, IF strongly preirresolute open and IF strongly preirresolute closed mappings between intuitionistic fuzzy topological spaces are defined. Their properties and the relationships between these mappings are investigated.

1. INTRODUCTION AND PRELIMINARIES

Šostak [13] introduced the concept of a fuzzy topological structure, as an extension of both of crisp topology and Chang fuzzy topology [6]. Kim et al., [10] introduced the concept of r -fuzzy strongly preopen and r -fuzzy strongly preclosed sets in Šostak fuzzy topology as an extension of fuzzy strongly preopen and fuzzy strongly preclosed sets given by Biljana Krsteska [5].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2,3,4]. Recently, Coker and his colleagues [7,8] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Samanta et al., [11,12] introduced the intuitionistic gradation of openness as an extension of intuitionistic fuzzy topology [8] and Šostak fuzzy topology [9,13].

In this paper, we introduced (r, s) -intuitionistic fuzzy strongly preopen and (r, s) -intuitionistic fuzzy strongly preclosed sets in an intuitionistic fuzzy topological space [12]. We investigate some properties of them. The classes of IF strongly precontinuous, IF strongly preirresolute continuous and IF strongly preirresolute open

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(closed) mapping are introduced. We show that IF continuity implies IF strong precontinuity and IF strongly preirresolute continuity implies IF strong precontinuity but the inverse of them is not true. Also, we obtain some properties of IF strongly preirresolute continuous mappings.

Throughout this paper, let X be a nonempty set $I = [0, 1]$ and $I_o = (0, 1]$. For $\alpha \in I$, $\bar{\alpha}(x) = \alpha$ for all $x \in X$. All the other notations and the other definitions are standard in the fuzzy set theory.

Definition 1.1 ([13]). A mapping $\tau : I^X \rightarrow I$ is called an *fuzzy topology* on X if it satisfies the following conditions:

- (O1) $\tau(\bar{0}) = \tau(\bar{1}) = 1$,
- (O2) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.
- (O3) $\tau(\bigvee_{i \in \Gamma} \mu_i) \geq \bigwedge_{i \in \Gamma} \tau(\mu_i)$, for any $\{\mu\}_{i \in \Gamma} \subset I^X$.

The pair (X, τ) is called a *fuzzy topological space* (for short, fts).

Definition 1.2 [12]. The pair (τ, τ^*) where, $\tau, \tau^* : I^X \rightarrow I$ is called an IGO on X if it satisfies the following conditions:

- (IGO1) $\tau(\lambda) + \tau^*(\lambda) \leq 1$, $\forall \lambda \in I^X$,
- (IGO2) $\tau(\bar{0}) = \tau(\bar{1}) = 1$, $\tau^*(\bar{0}) = \tau^*(\bar{1}) = 0$,
- (IGO3) $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$, $\lambda_i \in I^X, i = 1, 2$.
- (IGO4) $\tau(\bigvee_{i \in \Delta} \lambda_i) \geq \bigwedge_{i \in \Delta} \tau(\lambda_i)$ and $\tau^*(\bigvee_{i \in \Delta} \lambda_i) \leq \bigvee_{i \in \Delta} \tau^*(\lambda_i)$, $\lambda_i \in I^X, i \in \Delta$.

The triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological space (ifts, for short).

Theorem 1.3[1]. Let (X, τ, τ^*) be an ifts. Then for each $r \in I_o, s \in I_1, \lambda \in I^X$ we define an operator $C_{\tau, \tau^*} : I^X \times I_o \times I_1 \rightarrow I^X$ as follows

$$C_{\tau, \tau^*}(\lambda, r, s) = \bigwedge \{\mu \in I^X : \lambda \leq \mu, \tau(\bar{1} - \mu) \geq r, \tau^*(\bar{1} - \mu) \leq s\}.$$

For $\lambda, \mu \in I^X$ and $r, r_1 \in I_o, s, s_1 \in I_1$, the operator C_{τ, τ^*} satisfies the following conditions:

- (C1) $C_{\tau, \tau^*}(\bar{0}, r, s) = \bar{0}$.
- (C2) $\lambda \leq C_{\tau, \tau^*}(\lambda, r, s)$.
- (C3) $C_{\tau, \tau^*}(\lambda, r, s) \vee C_{\tau, \tau^*}(\mu, r, s) = C_{\tau, \tau^*}(\lambda \vee \mu, r, s)$.

(C4) $C_{\tau, \tau^*}(\lambda, r, s) \leq C_{\tau, \tau^*}(\lambda, r_1, s_1)$ if $r \leq r_1, s_1 \geq s$.

(C5) $C_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s) = C_{\tau, \tau^*}(\lambda, r, s)$.

Theorem 1.4[1]. Let (X, τ, τ^*) be an ifts. Then for each $r \in I_o, s \in I_1, \lambda \in I^X$ we define an operator $I_{\tau, \tau^*} : I^X \times I_o \times I_1 \rightarrow I^X$ as follows

$$I_{\tau, \tau^*}(\lambda, r, s) = \bigvee \{\mu \in I^X : \lambda \geq \mu, \tau(\mu) \geq r, \tau^*(\mu) \leq s\}.$$

For $\lambda, \mu \in I^X$ and $r, r_1 \in I_o, s, s_1 \in I_1$, the operator I_{τ, τ^*} satisfies the following conditions:

- (I1) $I_{\tau, \tau^*}(\bar{1} - \lambda, r, s) = \bar{1} - C_{\tau, \tau^*}(\lambda, r, s)$.
- (I2) $I_{\tau, \tau^*}(\bar{1}, r, s) = \bar{1}$.
- (I3) $\lambda \geq I_{\tau, \tau^*}(\lambda, r, s)$.
- (I4) $I_{\tau, \tau^*}(\lambda, r, s) \wedge I_{\tau, \tau^*}(\mu, r, s) = I_{\tau, \tau^*}(\lambda \wedge \mu, r, s)$.
- (I5) $I_{\tau, \tau^*}(\lambda, r, s) \geq I_{\tau, \tau^*}(\lambda, r_1, s_1)$ if $r \leq r_1, s \geq s_1$.
- (I6) $I_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s) = I_{\tau, \tau^*}(\lambda, r, s)$.

Definition 1.5. [12] Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping.

- (1) f is called *intuitionistic fuzzy continuous* (IF continuous, for short) iff $\eta(\mu) \leq \tau(f^{-1}(\mu))$ and $\eta^*(\mu) \geq \tau^*(f^{-1}(\mu))$ for each $\mu \in I^Y$.
- (2) f is called *intuitionistic fuzzy open* (IF open, for short) iff $\tau(\lambda) \leq \eta(f(\lambda))$ and $\tau^*(\lambda) \geq \eta^*(f(\lambda))$ for each $\lambda \in I^X$.
- (3) f is called *intuitionistic fuzzy closed* (IF closed, for short) iff $\tau(\bar{1} - f(\lambda)) \leq \eta(\bar{1} - f(\lambda))$ and $\tau^*(\bar{1} - f(\lambda)) \geq \eta^*(\bar{1} - f(\lambda))$ for each $\lambda \in I^X$.

2. (r, s) -INTUITIONISTIC FUZZY STRONGLY PREOPEN AND (r, s) -INTUITIONISTIC FUZZY STRONGLY PRECLOSED SETS

Definition 2.1. Let (X, τ, τ^*) be an ifts. For $\lambda \in I^X$ and $r \in I_o, s \in I_1$.

- (1) λ is called (r, s) -intuitionistic fuzzy preopen ((r, s) -ifpo, for short) iff

$$\lambda \leq I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s).$$

- (2) λ is called (r, s) -intuitionistic fuzzy preclosed ((r, s) -ifpc, for short) iff

$$\lambda \geq C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s).$$

(3) The (r, s) -intuitionistic fuzzy preinterior of λ , denoted by $PI_{\tau, \tau^*}(\lambda, r, s)$ is defined by

$$PI_{\tau, \tau^*}(\lambda, r, s) = \bigvee \{\nu \in I^X : \nu \leq \lambda, \nu \text{ is } (r, s) - \text{ifpo}\}.$$

(4) The (r, s) -intuitionistic fuzzy preclosure of λ , denoted by $PC_{\tau, \tau^*}(\lambda, r, s)$ is defined by

$$PC_{\tau, \tau^*}(\lambda, r, s) = \bigwedge \{\nu \in I^X : \nu \geq \lambda, \nu \text{ is } (r, s) - \text{ifpc}\}.$$

Theorem 2.2. Let (X, τ, τ^*) be an ifts. For $\lambda \in I^X$ and $r \in I_o, s \in I_1$. Then,

- (1) $\lambda \vee C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s) \leq PC_{\tau, \tau^*}(\lambda, r, s)$.
- (2) $PI_{\tau, \tau^*}(\lambda, r, s) \leq \lambda \wedge I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s)$.
- (3) $I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda, r, s), r, s) \leq I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s)$.
- (4) $I_{\tau, \tau^*}(C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s), r, s) \leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda, r, s), r, s)$.
- (5) $PC_{\tau, \tau^*}(\bar{1} - \lambda, r, s) = \bar{1} - PI_{\tau, \tau^*}(\lambda, r, s)$, $PI_{\tau, \tau^*}(\bar{1} - \lambda, r, s) = \bar{1} - PC_{\tau, \tau^*}(\lambda, r, s)$

Proof. (1) Since, $PC_{\tau, \tau^*}(\lambda, r, s)$ is (r, s) -ifpc, we have

$$C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s) \leq C_{\tau, \tau^*}(I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda, r, s), r, s), r, s) \leq PC_{\tau, \tau^*}(\lambda, r, s).$$

Thus, $\lambda \vee C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s) \leq PC_{\tau, \tau^*}(\lambda, r, s)$.

- (2) It can be shown as (1).
- (3) It follows from the relation $PC_{\tau, \tau^*}(\lambda, r, s) \leq C_{\tau, \tau^*}(\lambda, r, s)$.
- (4) From (1) we have

$$\begin{aligned} I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda, r, s), r, s) &\geq I_{\tau, \tau^*}(\lambda \vee C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s)), r, s \\ &\geq I_{\tau, \tau^*}(C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, r, s), r, s), r, s). \end{aligned}$$

(5) straightforward.

Definition 2.3. Let (X, τ, τ^*) be an ifts. For $\lambda \in I^X$ and $r \in I_o, s \in I_1$.

(1) λ is called (r, s) -intuitionistic fuzzy strongly preopen ((r, s) -ifspo, for short) iff

$$\lambda \leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda, r, s), r, s).$$

(2) λ is called (r, s) -intuitionistic fuzzy strongly preclosed ((r, s) -ifspc, for short) iff

$$C_{\tau, \tau^*}(PI_{\tau, \tau^*}(\lambda, r, s), r, s) \leq \lambda.$$

(3) The (r, s) -intuitionistic fuzzy strongly preinterior of λ , denoted by $SPI_{\tau, \tau^*}(\lambda, r, s)$ is defined by

$$SPI_{\tau, \tau^*}(\lambda, r, s) = \bigvee \{\nu \in I^X : \nu \leq \lambda, \nu \text{ is } (r, s) - \text{ifspo}\}.$$

(4) The (r, s) -intuitionistic fuzzy strongly preclosure of λ , denoted by $SPC_{\tau, \tau^*}(\lambda, r, s)$ is defined by

$$SPC_{\tau, \tau^*}(\lambda, r, s) = \bigwedge \{\nu \in I^X : \nu \geq \lambda, \nu \text{ is } (r, s) - \text{ifspc}\}.$$

Theorem 2.4. Let (X, τ, τ^*) be an ifts. For $\lambda, \mu \in I^X$ and $r \in I_0, s \in I_1$.

(1) If $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$, then λ is (r, s) -ifspo.

(2) If λ is (r, s) -ifspo, then λ is (r, s) -ifpo.

Proof. It follows easily from Theorem 2.2.

The following examples show that the converse of the above theorem is not true in general.

Example 2.5. Let $X = \{a, b, c\}$. Define the fuzzy sets $\lambda_1, \lambda_2, \mu \in I^X$ as follows

$$\lambda_1(a) = 0.3 \quad ; \quad \lambda_1(b) = 0.2 \quad ; \quad \lambda_1(c) = 0.7$$

$$\lambda_2(a) = 0.8 \quad ; \quad \lambda_2(b) = 0.8 \quad ; \quad \lambda_2(c) = 0.4$$

$$\mu(a) = 0.8 \quad ; \quad \mu(b) = 0.7 \quad ; \quad \mu(c) = 0.6.$$

Define the IGO (τ, τ^*) as follows

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0}, \bar{1}, \\ \frac{1}{3}, & \text{if } \lambda = \lambda_1, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_2, \\ \frac{2}{3}, & \text{if } \lambda = \lambda_1 \wedge \lambda_2, \\ \frac{2}{3}, & \text{if } \lambda = \lambda_1 \vee \lambda_2, \\ 0, & \text{otherwise,} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0}, \bar{1}, \\ \frac{2}{3}, & \text{if } \lambda = \lambda_1, \\ \frac{1}{2}, & \text{if } \lambda = \lambda_2, \\ \frac{1}{3}, & \text{if } \lambda = \lambda_1 \wedge \lambda_2, \\ \frac{1}{3}, & \text{if } \lambda = \lambda_1 \vee \lambda_2, \\ 1, & \text{otherwise.} \end{cases}$$

For ifts (X, τ, τ^*) , for $0 < r \leq \frac{1}{3}$ and $\frac{2}{3} \leq s < 1$, μ is (r, s) -ifspo set from

$$\mu \leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\mu, r, s), r, s) = \bar{1}.$$

For $0 < r \leq \frac{1}{3}$ and $\frac{2}{3} \leq s < 1$, $\tau(\mu) < r$, $\tau^*(\mu) > s$

Example 2.6. Let X be a nonempty set. We define IGO (τ, τ^*) on X as follows

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0}, \bar{1}, \\ \frac{2}{3}, & \text{if } \lambda = \overline{0.2}, \\ \frac{1}{3}, & \text{if } \lambda = \overline{0.4}, \\ \frac{1}{2}, & \text{if } \lambda = \overline{0.5}, \\ \frac{3}{4}, & \text{if } \lambda = \overline{0.6}, \\ 0, & \text{otherwise,} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{0}, \bar{1}, \\ \frac{1}{3}, & \text{if } \lambda = \overline{0.2}, \\ \frac{2}{3}, & \text{if } \lambda = \overline{0.4}, \\ \frac{1}{2}, & \text{if } \lambda = \overline{0.5}, \\ \frac{1}{4}, & \text{if } \lambda = \overline{0.6}, \\ 1, & \text{otherwise.} \end{cases}$$

For $0 < r \leq \frac{1}{3}$ and $\frac{2}{3} \leq s < 1$, $\overline{0.7}$ is (r, s) -ifpo set but it is not (r, s) -ifspo, from the followings

$$\overline{0.7} \leq I_{\tau, \tau^*}(C_{\tau, \tau^*}(\overline{0.7}, r, s), r, s) = \bar{1}.$$

$$\overline{0.7} > I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\overline{0.7}, r, s), r, s) = I_{\tau, \tau^*}(\overline{0.7}, r, s) = \overline{0.4}.$$

Theorem 2.7. Let (X, τ, τ^*) be an ifts. For $r \in I_0, s \in I_1$.

- (1) Any union of (r, s) -ifspo sets is (r, s) -ifspo.
- (2) Any intersection of (r, s) -ifspc sets is (r, s) -ifspc.

Proof. (1) Let $\{\lambda_\alpha : \alpha \in \Gamma\}$ be a family of (r, s) -ifspo sets. For each $\alpha \in \Gamma$, $\lambda_\alpha \leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda_\alpha, r, s), r, s)$. Hence we have

$$\begin{aligned} \bigvee_{\alpha \in \Gamma} \lambda_\alpha &\leq \bigvee_{\alpha \in \Gamma} (I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\lambda_\alpha, r, s), r, s)) \\ &\leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\bigvee_{\alpha \in \Gamma} \lambda_\alpha, r, s), r, s). \end{aligned}$$

So, $\bigvee_{\alpha \in \Gamma} \lambda_\alpha$ is (r, s) -ifspo.

- (2) It is easily proved the same manner.

Remark 2.8. The intersection of two (r, s) -ifspo sets need not be (r, s) -ifspo. The union of two (r, s) -ifspc sets need not be (r, s) -ifspc. We will show it from Example 2.9.

Example 2.9. If we consider the ifts (X, τ, τ^*) in Example 2.5. The fuzzy set ρ defined as

$$\rho(a) = 0.4 \quad ; \quad \rho(b) = 0.2 \quad ; \quad \rho(c) = 0.8$$

is $(\frac{1}{3}, \frac{2}{3})$ -ifspo set, but $\lambda_2 \wedge \rho$ is not $(\frac{1}{3}, \frac{2}{3})$ -ifspo set. Also, $(\bar{1} - \lambda_2) \vee (\bar{1} - \rho)$ is not $(\frac{1}{3}, \frac{2}{3})$ -ifspc of (X, τ, τ^*) .

Theorem 2.10. Let (X, τ, τ^*) be an ifts. For $\lambda, \mu \in I^X$ and $r \in I_0, s \in I_1$ the following statements hold:

- (1) $C_{\tau, \tau^*}(\lambda, r, s)$ is (r, s) -ifspc.
- (2) λ is (r, s) -ifspo iff $\lambda = SPI_{\tau, \tau^*}(\lambda, r, s)$.
- (3) λ is (r, s) -ifspc iff $\lambda = SPC_{\tau, \tau^*}(\lambda, r, s)$.
- (4) $I_{\tau, \tau^*}(\lambda, r, s) \leq SPI_{\tau, \tau^*}(\lambda, r, s) \leq PI_{\tau, \tau^*}(\lambda, r, s) \leq \lambda \leq PC_{\tau, \tau^*}(\lambda, r, s)$
 $\leq SPC_{\tau, \tau^*}(\lambda, r, s) \leq C_{\tau, \tau^*}(\lambda, r, s)$.
- (5) $SPI_{\tau, \tau^*}(\bar{1} - \lambda, r, s) = \bar{1} - SPC_{\tau, \tau^*}(\lambda, r, s)$ and
 $SPC_{\tau, \tau^*}(\bar{1} - \lambda, r, s) = \bar{1} - SPI_{\tau, \tau^*}(\lambda, r, s)$.
- (6) $C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s)) = SPC_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s) = C_{\tau, \tau^*}(\lambda, r, s)$.

Proof. (1),(2),(3),(4) it follows from the definitions.

(5) For all $\lambda \in I^X$, $r \in I_0, s \in I_1$, we have the following:

$$\begin{aligned} \bar{1} - SPI_{\tau, \tau^*}(\lambda, r, s) &= \bar{1} - \bigvee \{\nu : \nu \leq \lambda, \nu \text{ is } (r, s) - \text{ifspo}\} \\ &= \bigwedge \{\bar{1} - \nu : \bar{1} - \lambda \leq \bar{1} - \nu, \bar{1} - \nu \text{ is } (r, s) - \text{ifspc}\} \\ &= SPC_{\tau, \tau^*}(\bar{1} - \lambda, r, s). \end{aligned}$$

(6) From Theorem 2.10(1,3), $SPC_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, r, s), r, s) = C_{\tau, \tau^*}(\lambda, r, s)$. We only show that

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s) = C_{\tau, \tau^*}(\lambda, r, s).$$

Since $\lambda \leq SPC_{\tau, \tau^*}(\lambda, r, s)$,

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s) \geq C_{\tau, \tau^*}(\lambda, r, s).$$

Suppose that

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s) \not\leq C_{\tau, \tau^*}(\lambda, r, s).$$

There exist $x \in X$ and $r \in I_0, s \in I_1$ such that

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s)(x) > C_{\tau, \tau^*}(\lambda, r, s)(x).$$

By the definition of C_{τ, τ^*} , there exists $\rho \in I^X$ with $\lambda \leq \rho$ and $\tau(\bar{1} - \rho) \geq r$ and $\tau^*(\bar{1} - \rho) \leq s$ such that

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s)(x) > \rho(x) \geq C_{\tau, \tau^*}(\lambda, r, s)(x).$$

On the other hand, since $\rho = C_{\tau, \tau^*}(\rho, r, s)$, $\lambda \leq \rho$ implies

$$\begin{aligned} SPC_{\tau, \tau^*}(\lambda, r, s) &\leq SPC_{\tau, \tau^*}(\rho, r, s) \\ &= SPC_{\tau, \tau^*}(C_{\tau, \tau^*}(\rho, r, s), r, s) \\ &= C_{\tau, \tau^*}(\rho, r, s) = \rho. \end{aligned}$$

Thus,

$$C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s) \leq \rho.$$

It is a contradiction. Hence $C_{\tau, \tau^*}(SPC_{\tau, \tau^*}(\lambda, r, s), r, s) \leq C_{\tau, \tau^*}(\lambda, r, s)$.

3. INTUITIONISTIC FUZZY STRONGLY PREIRRESOLUTE CONTINUOUS MAPPINGS

Definition 3.1. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping.

(1) f is called intuitionistic fuzzy strongly precontinuous (IF strongly precontinuous, for short) iff $f^{-1}(\mu)$ is (r, s) -ifspo set of X for each $\mu \in I^Y$, $r \in I_0$, $s \in I_1$ with $\eta(\mu) \geq r$ and $\eta^*(\mu) \leq s$.

(2) f is called intuitionistic fuzzy strongly preirresolute (IF strongly preirresolute, for short) continuous iff $f^{-1}(\mu)$ is (r, s) -ifspo set of X for each (r, s) -ifspo $\mu \in I^Y$.

(3) f is called intuitionistic fuzzy strongly preirresolute open (IF strongly preirresolute open, for short) iff $f(\lambda)$ is (r, s) -ifspo set of Y for each (r, s) -ifspo $\lambda \in I^Y$.

(4) f is called intuitionistic fuzzy strongly preirresolute closed (IF strongly preirresolute closed, for short) iff $f(\lambda)$ is (r, s) -ifspc set of Y for each (r, s) -ifspc $\lambda \in I^Y$.

(5) f is called intuitionistic fuzzy strongly preopen (IF strongly preopen, for short) iff $f(\lambda)$ is (r, s) -ifspo set of Y for each $\lambda \in I^Y$ with $\tau(\lambda) \geq r$, $\tau^*(\lambda) \leq s$).

(6) f is called intuitionistic fuzzy strongly preclosed (IF strongly preclosed, for short) iff $f(\lambda)$ is (r, s) -ifspc set of Y for each $\lambda \in I^Y$ with $\tau(\bar{\lambda}) \geq r$, $\tau^*(\bar{\lambda}) \leq s$.

(7) f is called intuitionistic fuzzy strongly preirresolute homeomorphism iff f is bijective and both of f and f^{-1} are IF strongly preirresolute continuous.

Remark 3.2. (1) Every IF continuous (resp. IF open and IF closed) mapping is IF strongly precontinuous (resp. IF strongly preopen and IF strongly preclosed).

(2) Every IF strongly preirresolute continuous mapping is IF strongly precontinuous from Theorem 2.4(1). However, the converse of (1) and (2) may be false see Example 3.3 and Example 3.4.

(3) IF strongly preirresolute continuous and IF continuous mappings are independent notions.

Example 3.3. We consider Example 2.5. If we put

$$\eta(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise,} \end{cases} \quad \eta^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{2}{3}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise,} \end{cases}$$

then $id_X : (X, \tau, \tau^*) \rightarrow (X, \eta, \eta^*)$ is IF precontinuous but id_X is not IF strongly precontinuous. Also, if we put

$$\sigma(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ 0, & \text{otherwise.} \end{cases} \quad \sigma^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ 1, & \text{otherwise.} \end{cases}$$

We obtain $PC_{\sigma, \sigma^*} : I^X \times I_\circ \rightarrow I^X$ as follows

$$PC_{\sigma, \sigma^*}(\lambda, r, s) = \begin{cases} \frac{1}{r} & \text{if } \lambda \geq \nu, 0 < r \leq \frac{1}{3}, \frac{2}{3} \leq s < 1 \\ \lambda & \text{otherwise.} \end{cases}$$

Moreover, for each $\lambda \geq \nu$ and $0 < r \leq \frac{1}{3}, \frac{2}{3} \leq s < 1$, λ is (r, s) -ifspo in (X, σ, σ^*) and (X, η, η^*) . Thus, the identity mapping $id_X : (X, \tau, \tau^*) \rightarrow (X, \sigma, \sigma^*)$ is IF strongly precontinuous and IF strongly preirresolute continuous but id_X is not IF continuous.

Example 3.4. Let $X = \{a, b, c\}$. Define the fuzzy sets $\mu_1, \mu_2, \mu_3, \mu_4 \in I^X$ as follows

$$\begin{aligned} \mu_1(a) &= 0.5 & ; & \mu_1(b) = 0.3 & ; & \mu_1(c) = 0.6 \\ \mu_2(a) &= 0.3 & ; & \mu_2(b) = 0.4 & ; & \mu_2(c) = 0.3 \\ \mu_3(a) &= 0.5 & ; & \mu_3(b) = 0.4 & ; & \mu_3(c) = 0.6 \\ \mu_4(a) &= 0.5 & ; & \mu_4(b) = 0.5 & ; & \mu_4(c) = 0.6 \end{aligned}$$

Define IGO's (τ, τ^*) and (η, η^*) on X as follows

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \mu_1, \\ \frac{1}{2}, & \text{if } \lambda = \mu_2, \\ \frac{2}{3}, & \text{if } \lambda = \mu_1 \wedge \mu_2, \\ \frac{2}{3}, & \text{if } \lambda = \mu_1 \vee \mu_2, \\ 0, & \text{otherwise,} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{2}, & \text{if } \lambda = \mu_1, \\ \frac{1}{2}, & \text{if } \lambda = \mu_2, \\ \frac{1}{3}, & \text{if } \lambda = \mu_1 \wedge \mu_2, \\ \frac{1}{3}, & \text{if } \lambda = \mu_1 \vee \mu_2, \\ 1, & \text{otherwise,} \end{cases}$$

$$\eta(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{1}{3}, & \text{if } \lambda = \mu_3, \\ 0, & \text{otherwise,} \end{cases} \quad \eta^*(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{0}, \underline{1}, \\ \frac{2}{3}, & \text{if } \lambda = \mu_3, \\ 1, & \text{otherwise.} \end{cases}$$

Then $id_X : (X, \tau, \tau^*) \rightarrow (X, \eta, \eta^*)$ is IF strongly precontinuous but not IF strongly preirresolute continuous. Furthermore, id_X is IF continuous mapping which is not IF strongly preirresolute continuous, because

$$\mu_4 \leq I_{\eta, \eta^*}(PC_{\eta, \eta^*}(\mu_4, \frac{1}{3}), \frac{1}{3}) = \underline{1},$$

$$\mu_4 \not\leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(\mu_4, \frac{1}{3}), \frac{1}{3}) = I_{\tau, \tau^*}(\underline{1} - \mu_2, \frac{1}{3}, \frac{1}{3}) = \mu_3.$$

Theorem 3.5. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X$, $\mu \in I^Y$, $r \in I_o$, $s \in I_1$. The following statements are equivalent.

- (1) f is IF strongly preirresolute continuous.
- (2) For each (r, s) -ifspc $\mu \in I^Y$, $f^{-1}(\mu)$ is (r, s) -ifspc.
- (3) $f(SPC_{\tau, \tau^*}(\lambda, r, s)) \leq SPC_{\eta, \eta^*}(f(\lambda), r, s)$.
- (4) $SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s))$.
- (5) $f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s)) \leq SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.
- (6) $C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s) \leq f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s))$.
- (7) $f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s)) \leq I_{\tau, \tau^*}(PC_{\tau, \tau^*}(f^{-1}(\mu), r, s))$.
- (8) $f(C_{\tau, \tau^*}(PI_{\tau, \tau^*}(\lambda, r, s), r, s)) \leq SPC_{\eta, \eta^*}(f(\lambda), r, s)$.

Proof. (1) \Leftrightarrow (2) It is easily proved from Definition 2.1, and $f^{-1}(\underline{1} - \mu) = \underline{1} - f^{-1}(\mu)$.

(2) \Rightarrow (3) Suppose there exist $\lambda \in I^X$ and $r \in I_o$, $s \in I_1$ such that

$$f(SPC_{\tau, \tau^*}(\lambda, r, s)) \not\leq SPC_{\eta, \eta^*}(f(\lambda), r, s).$$

There exist $y \in Y$ and $t \in I_o$ such that

$$f(SPC_{\tau, \tau^*}(\lambda, r, s))(y) > t > SPC_{\eta, \eta^*}(f(\lambda), r, s)(y).$$

If $f^{-1}(\{y\}) = \phi$, it is a contradiction because $f(SPC_{\tau, \tau^*}(\lambda, r, s))(y) = 0$.

If $f^{-1}(\{y\}) \neq \phi$, there exists $x \in f^{-1}(\{y\})$ such that

$$f(SPC_{\tau, \tau^*}(\lambda, r, s))(y) \geq SPC_{\tau, \tau^*}(\lambda, r, s)(x) > t > SPC_{\eta, \eta^*}(f(\lambda), r, s)(f(x)). \quad (\text{A})$$

Since $SPC_{\eta,\eta^*}(f(\lambda), r, s)(f(x)) < t$, there exists (r, s) -ifspc $\mu \in I^Y$ with $f(\lambda) \leq \mu$ such that

$$SPC_{\eta,\eta^*}(f(\lambda), r, s)(f(x)) \leq \mu(f(x)) < t.$$

Moreover, $f(\lambda) \leq \mu$ implies $\lambda \leq f^{-1}(\mu)$. From (2), $f^{-1}(\mu)$ is (r, s) -ifspc. Thus, $SPC_{\tau,\tau^*}(\lambda, r, s)(x) \leq f^{-1}(\mu)(x) = \mu(f(x)) < t$. It is a contradiction for (A).

(3) \Rightarrow (4) For all $\mu \in I^Y$, $r \in I_o$, put $\lambda = f^{-1}(\mu)$. From (3), we have

$$f(SPC_{\tau,\tau^*}(f^{-1}(\mu), r, s)) \leq SPC_{\eta,\eta^*}(f(f^{-1}(\mu)), r, s) \leq SPC_{\eta,\eta^*}(\mu, r, s).$$

It implies

$$SPC_{\tau,\tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(f(SPC_{\tau,\tau^*}(f^{-1}(\mu), r, s))) \leq f^{-1}(SPC_{\eta,\eta^*}(\mu, r, s)).$$

(4) \Rightarrow (5) It is easily proved from Theorem 2.10(5).

(5) \Rightarrow (1) Let μ be (r, s) -ifspo set of Y . From Theorem 2.10(2),

$\mu = SPI_{\eta,\eta^*}(\mu, r, s)$. By (5),

$$f^{-1}(\mu) \leq SPI_{\tau,\tau^*}(f^{-1}(\mu), r, s).$$

On the other hand, by Theorem 2.10(4),

$$f^{-1}(\mu) \geq SPI_{\tau,\tau^*}(f^{-1}(\mu), r, s).$$

Thus, $f^{-1}(\mu) = SPI_{\tau,\tau^*}(f^{-1}(\mu), r, s)$, that is, $f^{-1}(\mu)$ is (r, s) -ifspo.

(1) \Rightarrow (6) Let $\mu \in I^Y$ and $r \in I_o, s \in I_1$. According to the assumption $f^{-1}(SPC_{\eta,\eta^*}(\mu, r, s))$ is (r, s) -ifspc set of X . Hence,

$$\begin{aligned} f^{-1}(SPC_{\eta,\eta^*}(\mu, r, s)) &\geq C_{\tau,\tau^*}(PI_{\tau,\tau^*}(f^{-1}(SPC_{\eta,\eta^*}(\mu, r, s)), r, s), r, s) \\ &\geq C_{\tau,\tau^*}(PI_{\tau,\tau^*}(f^{-1}(\mu), r, s)). \end{aligned}$$

(6) \Rightarrow (7) : It can be proved by using (4,5).

(7) \Rightarrow (8) : Let $\lambda \in I^X$ and $r \in I_o, s \in I_1$. Let us put $\mu = f(\lambda)$; then $\lambda \leq f^{-1}(\mu)$.

According to the assumption

$$\begin{aligned} \underline{1} - I_{\tau,\tau^*}(PC_{\tau,\tau^*}(\underline{1} - \lambda, r, s), r, s) &\leq \underline{1} - I_{\tau}(PC_{\tau,\tau^*}(f^{-1}(\underline{1} - \mu), r, s), r, s) \\ &\leq \underline{1} - f^{-1}(SPI_{\eta,\eta^*}(\underline{1} - \mu, r, s)). \end{aligned}$$

Thus,

$$\begin{aligned} C_{\tau, \tau^*}(PI_{\tau, \tau^*}(\lambda, r, s), r, s) &\leq C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s) \\ &\leq f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)). \end{aligned}$$

Hence,

$$\begin{aligned} f(C_{\tau, \tau^*}(PI_{\tau, \tau^*}(\lambda, r, s), r, s)) &\leq ff^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)) \\ &\leq SPC_{\eta, \eta^*}(\mu, r, s) \\ &= SPC_{\eta, \eta^*}(f(\lambda), r, s). \end{aligned}$$

(8) \Rightarrow (1) : Let $r \in I_o, s \in I_1$ and μ be (r, s) -ifspc set of Y . According to the assumption,

$$\begin{aligned} f(C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s)) &\leq SPC_{\eta, \eta^*}(ff^{-1}(\mu), r, s) \\ &\leq SPC_{\eta, \eta^*}(\mu, r, s) \\ &= \mu. \end{aligned}$$

Then

$$C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s) \leq f^{-1}f(C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s)) \leq f^{-1}(\mu)$$

. Thus $f^{-1}(\mu)$ is (r, s) -ifspc set of X , hence f is IF strongly preirresolute continuous.

The following theorem is similarly proved as Theorem 3.5.

Theorem 3.6. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X, \mu \in I^Y, r \in I_o, s \in I_1$. The following statements are equivalent.

- (1) f is IF strongly precontinuous.
- (2) $f(SPC_{\tau, \tau^*}(\lambda, r, s)) \leq C_{\eta, \eta^*}(f(\lambda), r, s)$.
- (3) $SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(C_{\eta, \eta^*}(\mu, r, s))$.
- (4) $f^{-1}(I_{\eta, \eta^*}(\mu, r, s)) \leq SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.
- (5) $C_{\tau, \tau^*}(PI_{\tau, \tau^*}(f^{-1}(\mu), r, s), r, s) \leq f^{-1}(C_{\eta, \eta^*}(\mu, r, s))$.
- (6) $f(C_{\tau, \tau^*}(PI_{\tau, \tau^*}(\lambda, r, s), r, s)) \leq C_{\eta, \eta^*}(f(\lambda), r, s)$.

Theorem 3.7. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . If f is IF strongly preirresolute continuous, then

$$f^{-1}(\mu) \leq SPI_{\tau, \tau^*}(f^{-1}(I_{\eta, \eta^*}(PC_{\eta, \eta^*}(\mu, r, s), r, s)), r, s),$$

for each μ is (r, s) -ifspo in Y and $r \in I_o, s \in I_1$.

Proof. Let $r \in I_o, s \in I_1$ and μ be (r, s) -ifspo set of Y . Then,

$f^{-1}(\mu) \leq f^{-1}(I_{\eta, \eta^*}(PC_{\eta, \eta^*}(\mu, r, s), r, s))$. Since $f^{-1}(\mu)$ is (r, s) -ifspo set of X , we have

$$f^{-1}(\mu) \leq SPI_{\tau, \tau^*}(f^{-1}(I_{\eta, \eta^*}(PC_{\eta, \eta^*}(\mu, r, s), r, s)), r, s).$$

Theorem 3.8. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a bijective mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . The mapping f is IF strongly precontinuous iff $I_{\eta, \eta^*}(f(\lambda), r, s) \leq f(SPI_{\tau, \tau^*}(\lambda, r, s))$, for each $\lambda \in I^X$ and $r \in I_o, s \in I_1$.

Proof. We suppose that f is IF strongly precontinuous. For any $\lambda \in I^X$ and $r \in I_o, s \in I_1$, $f^{-1}(I_{\eta, \eta^*}(f(\lambda), r, s))$ is a (r, s) -ifspo set. From Theorem 3.6(4) and the fact that f is injective we have

$$\begin{aligned} f^{-1}(I_{\eta, \eta^*}(f(\lambda), r, s)) &\leq SPI_{\eta, \eta^*}(f^{-1}f(\lambda), r, s) \\ &= SPI_{\tau, \tau^*}(\lambda, r, s). \end{aligned}$$

Again, since f is surjective, we obtain

$$I_{\eta, \eta^*}(f(\lambda), r, s) = ff^{-1}(I_{\eta, \eta^*}(f(\lambda), r, s)) \leq f(SPI_{\tau, \tau^*}(\lambda, r, s)).$$

(Conversely) Let $\mu \in I^Y$ and $r \in I_o, s \in I_1$ with $\eta(\mu) \geq r$ and $\eta^*(\mu) \leq s$. Then $I_{\eta, \eta^*}(\mu, r, s) = \mu$. According to assumption,

$$f(SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)) \geq I_{\eta, \eta^*}(ff^{-1}(\mu), r, s) = I_{\eta, \eta^*}(\mu, r, s) = \mu.$$

This implies that

$$f^{-1}f(SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)) \geq f^{-1}(\mu).$$

Since f is injective we obtain

$$SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s) = f^{-1}f(SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)) \geq f^{-1}(\mu).$$

Hence $SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s) = f^{-1}(\mu)$, so $f^{-1}(\mu)$ is a (r, s) -ifspo set. Thus f is IF strongly precontinuous.

Theorem 3.9. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X$, $\mu \in I^Y$, $r \in I_o$, $s \in I_1$. The following statements are equivalent:

- (1) f is IF strongly preirresolute open.
- (2) $f(SPI_{\tau, \tau^*}(\lambda, r, s)) \leq SPI_{\eta, \eta^*}(f(\lambda), r, s)$.
- (3) $SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s))$.
- (4) For any $\mu \in I^Y$ and any (r, s) -ifspc $\lambda \in I^X$ with $f^{-1}(\mu) \leq \lambda$, there exists a (r, s) -ifspc $\rho \in I^Y$ with $\mu \leq \rho$ such that $f^{-1}(\rho) \leq \lambda$.

Proof. (1) \Rightarrow (2) : For each $\lambda \in I^X$, since $SPI_{\tau, \tau^*}(\lambda, r, s) \leq \lambda$ from Theorem 2.10(4), we have $f(SPI_{\tau, \tau^*}(\lambda, r, s)) \leq f(\lambda)$. From (1), $f(SPI_{\tau, \tau^*}(\lambda, r, s))$ is (r, s) -ifspo. Hence, $f(SPI_{\tau, \tau^*}(\lambda, r, s)) \leq SPI_{\eta, \eta^*}(f(\lambda), r, s)$.

(2) \Rightarrow (3) : For all $\mu \in I^Y$, $r \in I_o$, $s \in I_1$, put $\lambda = f^{-1}(\mu)$ from (2). Then

$$f(SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s)) \leq SPI_{\eta, \eta^*}(f(f^{-1}(\mu)), r, s) \leq SPI_{\eta, \eta^*}(\mu, r, s).$$

It implies $SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s))$.

(3) \Rightarrow (4) : Let λ be (r, s) -ifspc set of X such that $f^{-1}(\mu) \leq \lambda$. Since $\underline{1} - \lambda \leq f^{-1}(\underline{1} - \mu)$ and $SPI_{\tau, \tau^*}(\underline{1} - \lambda, r, s) = \underline{1} - \lambda$,

$$SPI_{\tau, \tau^*}(\underline{1} - \lambda, r, s) = \underline{1} - \lambda \leq SPI_{\tau, \tau^*}(f^{-1}(\underline{1} - \mu), r, s).$$

From (3),

$$\underline{1} - \lambda \leq SPI_{\tau, \tau^*}(f^{-1}(\underline{1} - \mu), r, s) \leq f^{-1}(SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)).$$

It implies

$$\begin{aligned} \lambda &\geq \underline{1} - f^{-1}(SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)) \\ &= f^{-1}(\underline{1} - SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)) \\ &= f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)). \end{aligned}$$

Hence there exists a (r, s) -ifspc $SPC_{\eta, \eta^*}(\mu, r, s) \in I^Y$ with $\mu \leq SPC_{\eta, \eta^*}(\mu, r, s)$ such that $f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)) \leq \lambda$.

(4) \Rightarrow (1) : Let ω be (r, s) -ifspo set of X . Put $\mu = \underline{1} - f(\omega)$ and $\lambda = \underline{1} - \omega$ such that λ is (r, s) -ifspc. We obtain

$$f^{-1}(\mu) = f^{-1}(\underline{1} - f(\omega)) = \underline{1} - f^{-1}(f(\omega)) \leq \underline{1} - \omega = \lambda.$$

From (4), there exists (r, s) -ifspc set ρ with $\mu \leq \rho$ such that $f^{-1}(\rho) \leq \lambda = \underline{1} - \omega$. It implies $\omega \leq \underline{1} - f^{-1}(\rho) = f^{-1}(\underline{1} - \rho)$. Thus, $f(\omega) \leq f(f^{-1}(\underline{1} - \rho)) \leq \underline{1} - \rho$. On the other hand, since $\mu \leq \rho$,

$$f(\omega) = \underline{1} - \mu \geq \underline{1} - \rho.$$

Hence $f(\omega) = \underline{1} - \rho$, that is, $f(\omega)$ is (r, s) -ifspo.

The following three theorems are similarly proved as Theorem 3.9.

Theorem 3.10. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X$, $\mu \in I^Y$, $r \in I_o$, $s \in I_1$. The following statements are equivalent:

- (1) f is IF strongly preopen.
- (2) $f(I_{\tau, \tau^*}(\lambda, r, s)) \leq SPI_{\eta, \eta^*}(f(\lambda), r, s)$.
- (3) $I_{\tau, \tau^*}(f^{-1}(\mu), r, s) \leq f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s))$.
- (4) For any $\lambda \in I^Y$ and any $\mu \in I^X$ with $\tau(\underline{1} - \mu) \geq r$ and $\tau^*(\underline{1} - \mu) \leq s$ such that $f^{-1}(\lambda) \leq \mu$, there exists a (r, s) -ifspc $\rho \in I^Y$ with $\lambda \leq \rho$ such that $f^{-1}(\rho) \leq \mu$.

Theorem 3.11. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. The following statements are equivalent:

- (1) f is IF strongly preirresolute closed.
- (2) $SPC_{\eta, \eta^*}(f(\lambda), r, s) \leq f(SPC_{\tau, \tau^*}(\lambda, r, s))$.

Theorem 3.12. Let (X, τ, τ^*) and (Y, η, η^*) be ifts's. Let $f : X \rightarrow Y$ be a mapping. For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. The following statements are equivalent:

- (1) f is IF strongly preclosed.
- (2) $SPC_{\eta, \eta^*}(f(\lambda), r, s) \leq f(C_{\tau, \tau^*}(\lambda, r, s))$.

Theorem 3.13. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a bijective mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\mu \in I^Y$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly preirresolute closed.
- (2) $f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)) \leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.
- (3) f is IF strongly preirresolute open.
- (4) f^{-1} is IF strongly preirresolute continuous.

Proof. (1) \Rightarrow (2) : Let f be IF strongly preirresolute closed. From Theorem 3.11(2), $f(SPC_{\tau, \tau^*}(\lambda, r, s)) \geq SPC_{\eta, \eta^*}(f(\lambda), r, s)$, for each $\lambda \in I^X$ and $r \in I_o$, $s \in I_1$. For

all $\mu \in I^Y$, $r \in I_o$, $s \in I_1$, put $\lambda = f^{-1}(\mu)$ from (1). Since f is onto, $f(f^{-1}(\mu)) = \mu$. Thus,

$$f(SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)) \geq SPC_{\eta, \eta^*}(f(f^{-1}(\mu)), r, s) = SPC_{\eta, \eta^*}(\mu, r, s).$$

Since f is injective, it implies

$$SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s) = f^{-1}(f(SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s))) \geq f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)).$$

(2) \Rightarrow (1) : From (2), put $\mu = f(\lambda)$ for each $\lambda \in I^X$. Since f is injective

$$f^{-1}(SPC_{\eta, \eta^*}(f(\lambda), r, s)) \leq SPC_{\tau, \tau^*}(f^{-1}(f(\lambda)), r, s) = SPC_{\tau, \tau^*}(\lambda, r, s).$$

Since f is onto, $SPC_{\eta, \eta^*}(f(\lambda), r, s) \leq f(SPC_{\tau, \tau^*}(\lambda, r, s))$. From Theorem 3.11(2), f is IF strongly preirresolute closed

(2) \Leftrightarrow (3) : From Theorem 3.9(3) and Theorem 2.10(5), it is proved from:

$$\begin{aligned} & f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s)) \leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s) \\ \Leftrightarrow & f^{-1}(\underline{1} - SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)) \leq \underline{1} - SPI_{\tau, \tau^*}(\underline{1} - f^{-1}(\mu), r, s). \\ \Leftrightarrow & \underline{1} - f^{-1}(SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)) \leq \underline{1} - SPI_{\tau, \tau^*}(f^{-1}(\underline{1} - \mu), r, s). \\ \Leftrightarrow & f^{-1}(SPI_{\eta, \eta^*}(\underline{1} - \mu, r, s)) \geq SPI_{\tau, \tau^*}(f^{-1}(\underline{1} - \mu), r, s). \end{aligned}$$

(2) \Leftrightarrow (4) : From Theorem 2.10(3), it is trivial.

From the above theorems, we easily prove the following corollary:

Corollary 3.14. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a bijective mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\lambda \in I^X$, $\mu \in I^Y$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly preirresolute homeomorphism.
- (2) f is IF strongly preirresolute continuous and IF strongly preirresolute open.
- (3) f is IF strongly preirresolute continuous and IF strongly preirresolute closed.
- (4) $f(SPI_{\tau, \tau^*}(\lambda, r, s)) = SPI_{\eta, \eta^*}(f(\lambda), r, s)$.
- (5) $f(SPC_{\tau, \tau^*}(\lambda, r, s)) = SPC_{\eta, \eta^*}(f(\lambda), r, s)$.
- (6) $SPI_{\tau, \tau^*}(f^{-1}(\mu), r, s) = f^{-1}(SPI_{\eta, \eta^*}(\mu, r, s))$.
- (7) $SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s) = f^{-1}(SPC_{\eta, \eta^*}(\mu, r, s))$.

Theorem 3.15. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly preirresolute open.
- (2) $f(SPI_{\tau, \tau^*}(\lambda, r, s)) \leq I_{\eta, \eta^*}(PC_{\tau, \tau^*}(f(\lambda), r, s), r, s)$.

Proof. (1) \Rightarrow (2) : Let $\lambda \in I^X$ and $r \in I_o$, $s \in I_1$. Then $SPI_{\tau, \tau^*}(\lambda, r, s)$ is (r, s) -ifspo set of X . By (1), $f(SPI_{\tau, \tau^*}(\lambda, r, s))$ is (r, s) -ifspo set of Y . Hence

$$\begin{aligned} f(SPI_{\tau, \tau^*}(\lambda, r, s)) &\leq I_{\eta, \eta^*}(f(SPI_{\tau, \tau^*}(\lambda, r, s)), r, s), r, s \\ &\leq I_{\eta, \eta^*}(PC_{\eta, \eta^*}(f(\lambda), r, s), r, s). \end{aligned}$$

(2) \Rightarrow (1) : Let λ be (r, s) -ifspo set of X . From

$f(\lambda) = f(SPI_{\tau, \tau^*}(\lambda, r, s)) \leq I_{\eta, \eta^*}(PC_{\eta, \eta^*}(f(\lambda), r, s), r, s)$ it follows that $f(\lambda)$ is (r, s) -ifspo set of Y . Hence, f is IF strongly preirresolute open.

Theorem 3.16. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly preirresolute closed.
- (2) $C_{\eta, \eta^*}(PI_{\eta, \eta^*}(f(\lambda), r, s)) \leq f(SPC_{\tau, \tau^*}(\lambda, r, s))$.

Theorem 3.17. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly pre-open.
- (2) $f(I_{\tau, \tau^*}(\lambda, r, s)) \leq I_{\eta, \eta^*}(PC_{\eta, \eta^*}(f(\lambda), r, s), r, s)$.

Theorem 3.18. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . For each $\lambda \in I^X$, $r \in I_o$, $s \in I_1$. Then the following statements are equivalent:

- (1) f is IF strongly pre-closed.
- (2) $C_{\eta, \eta^*}(PI_{\tau, \tau^*}(f(\lambda), r, s), r, s) \leq f(C_{\tau, \tau^*}(\lambda, r, s))$.

Theorem 3.19. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping. Then, f is IF strongly preirresolute open iff for each $\nu \in I^Y$ and each (r, s) -ifspc set $\lambda \in I^X$, $r \in I_o$, $s \in I_1$, when $f^{-1}(\nu) \leq \lambda$, there exists a (r, s) -ifspc set $\mu \in I^Y$ such that $\nu \leq \mu$ and $f^{-1}(\mu) \leq \lambda$.

Proof. Suppose that f is IF strongly preirresolute open mapping, $\nu \in I^Y$ and λ is (r, s) -ifspc set of X , $r \in I_o, s \in I_1$ such that $f^{-1}(\nu) \leq \lambda$. Then, $f(\underline{1} - \lambda) \leq f(f^{-1}(\underline{1} - \nu)) \leq \underline{1} - \nu$. Since f is IF strongly preirresolute open, then $f(\underline{1} - \lambda)$ is (r, s) -ifspo set of Y . Hence,

$$f(\underline{1} - \lambda) \leq SPI_{\eta, \eta^*}(\underline{1} - \nu, r, s).$$

Thus

$$\underline{1} - \lambda \leq f^{-1}(f(\underline{1} - \lambda)) \leq f^{-1}(SPI_{\eta, \eta^*}(\underline{1} - \nu), r, s).$$

It follows that

$$\lambda \geq f^{-1}(\underline{1} - SPI_{\eta, \eta^*}(\underline{1} - \nu, r, s)) = f^{-1}(SPC_{\eta, \eta^*}(\nu, r, s)).$$

The result follows for $\mu = SPC_{\eta, \eta^*}(\nu, r, s)$.

Conversely, let ω be (r, s) -ifspo set of X . We claim that $f(\omega)$ is (r, s) -ifspo set of Y . From $\omega \leq f^{-1}(f(\omega))$ it follows that $\underline{1} - \omega \geq \underline{1} - f^{-1}f(\omega)$, where $\underline{1} - \omega$ is (r, s) -ifspc set of X . Hence there is ν is (r, s) -ifspc set of Y such that $\nu \geq f(\underline{1} - \omega)$ and $f^{-1}(\nu) \leq \underline{1} - \omega$. Since $\nu \geq f(\underline{1} - \omega)$, it follows that $\nu \geq SPC_{\eta, \eta^*}(f(\underline{1} - \omega), r, s)$ or $\underline{1} - \nu \leq \underline{1} - SPC_{\eta, \eta^*}(f(\underline{1} - \omega), r, s) = SPI_{\eta, \eta^*}(f(\omega), r, s)$. From $f^{-1}(\nu) \leq \underline{1} - \omega$ we obtain $f^{-1}(\underline{1} - \nu) \geq \omega$ or $\underline{1} - \nu \geq ff^{-1}(\underline{1} - \nu) \geq f(\omega)$. Since $f(\omega) \leq \underline{1} - \nu \leq SPI_{\eta, \eta^*}(f(\omega), r, s)$, we have $f(\omega) = SPI_{\eta, \eta^*}(f(\omega), r, s)$. Thus, $f(\omega)$ is (r, s) -ifspo set of Y , hence f is IF strongly preirresolute open.

Theorem 3.20. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . If f is IF strongly preirresolute open, then

- (1) For each $\mu \in I^Y$ and $r \in I_o, s \in I_1$, $f^{-1}(C_{\eta, \eta^*}(PI_{\eta, \eta^*}(\mu, r, s), r, s) \leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.
- (2) For each μ is (r, s) -ifpo set of Y and $r \in I_o, s \in I_1$, $f^{-1}(C_{\eta, \eta^*}(\mu, r, s)) \leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.

Proof. (1) Let $\mu \in I^Y$ and $r \in I_o, s \in I_1$. Then $SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)$ is (r, s) -ifspo set of X . From Theorem 3.19, it follows that there exists r -ifspc set ν of Y such that $\mu \leq \nu$ and $f^{-1}(\nu) \leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s)$. Thus

$$\begin{aligned} f^{-1}(C_{\eta, \eta^*}(PI_{\eta, \eta^*}(\mu, r, s))) &\leq f^{-1}(C_{\eta, \eta^*}(PI_{\eta, \eta^*}(\nu, r, s), r, s)) \\ &\leq f^{-1}(\nu) \\ &\leq SPC_{\tau, \tau^*}(f^{-1}(\mu), r, s). \end{aligned}$$

(2) It follows immediately from (1).

The following theorem is similarly proved as Theorem 3.20.

Theorem 3.21. Let $f : (X, \tau, \tau^*) \rightarrow (Y, \eta, \eta^*)$ be a mapping from an ifts (X, τ, τ^*) into an ifts (Y, η, η^*) . If f is IF strongly pre-open then

- (1) For each $\mu \in I^Y$ and $r \in I_o, s \in I_1$, $f^{-1}(C_{\eta, \eta^*}(PI_{\eta, \eta^*}(\mu, r, s), r, s) \leq C_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.
- (2) For each μ is (r, s) -ifpo set of Y and $r \in I_o, s \in I_1$, $f^{-1}(C_{\eta, \eta^*}(\mu, r, s)) \leq C_{\tau, \tau^*}(f^{-1}(\mu), r, s)$.

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