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Hexagonal Game Method model of forest fire spread with intuitionistic fuzzy estimations

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Abstract: The mathematical model for predicting the spread of a fire front in homogeneous forest from [3] is extended with intuitionistic fuzzy estimations for the area of the fire. It is based on the application of the Game Method for Modelling with hexagonal lattice.
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AMS Classification: 11C20, 03E72.

1 Introduction

This paper is a continuation of a series of papers in which the development of the fire front in homogeneous forest is modelled by the Game Method for Modelling (GMM). The existing models are based on square or hexagonal lattice. Here, the model from [3] is extended with intuitionistic fuzzy estimations for the area of the fire. The intuitionistic fuzziness is discussed, e.g., in [2]. The GMM notations are described, e.g., in [1]). Short remarks on GMM are given in the Appendix.

2 Game Method model of forest fire with intuitionistic fuzzy estimations for the area of the fire

By analogy with [3], we describe a finite grid, having the form of a hexagonal lattice, with size 11×11, in which we check the development of forest fire processes.

We assume that in the field there is a river (its territory being marked by symbol R), stone rock formations (their territory being marked by S) and on the rest part of the field there is a homogeneous forest. The digits correspond to the wood mass per one unit square. These digits are specific for different types of forests, but here the forest is homogeneous and initially, the digits are only "9". After the beginning of the fire, the digits will decrease by specific rules, described below.

If some rule determines that symbol Y must be changed with symbol Z, let us denote this fact by $Y \to Z$.

Let everywhere $r \in [0, 1]$ be a random number that is generated for the current procedure.

In the present research, we discuss a scenario without a wind. We will consider that the fire will develop in concentric circles (see Fig. 1).

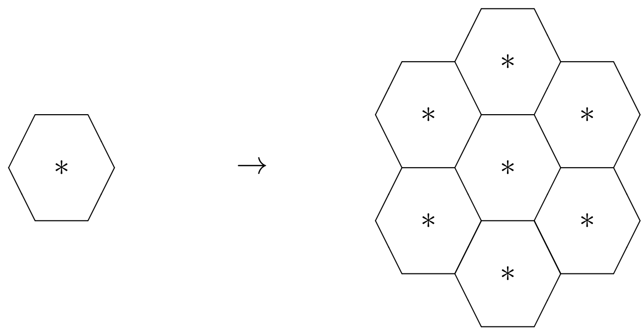


Figure 1.

The rules for the GMM are the following.

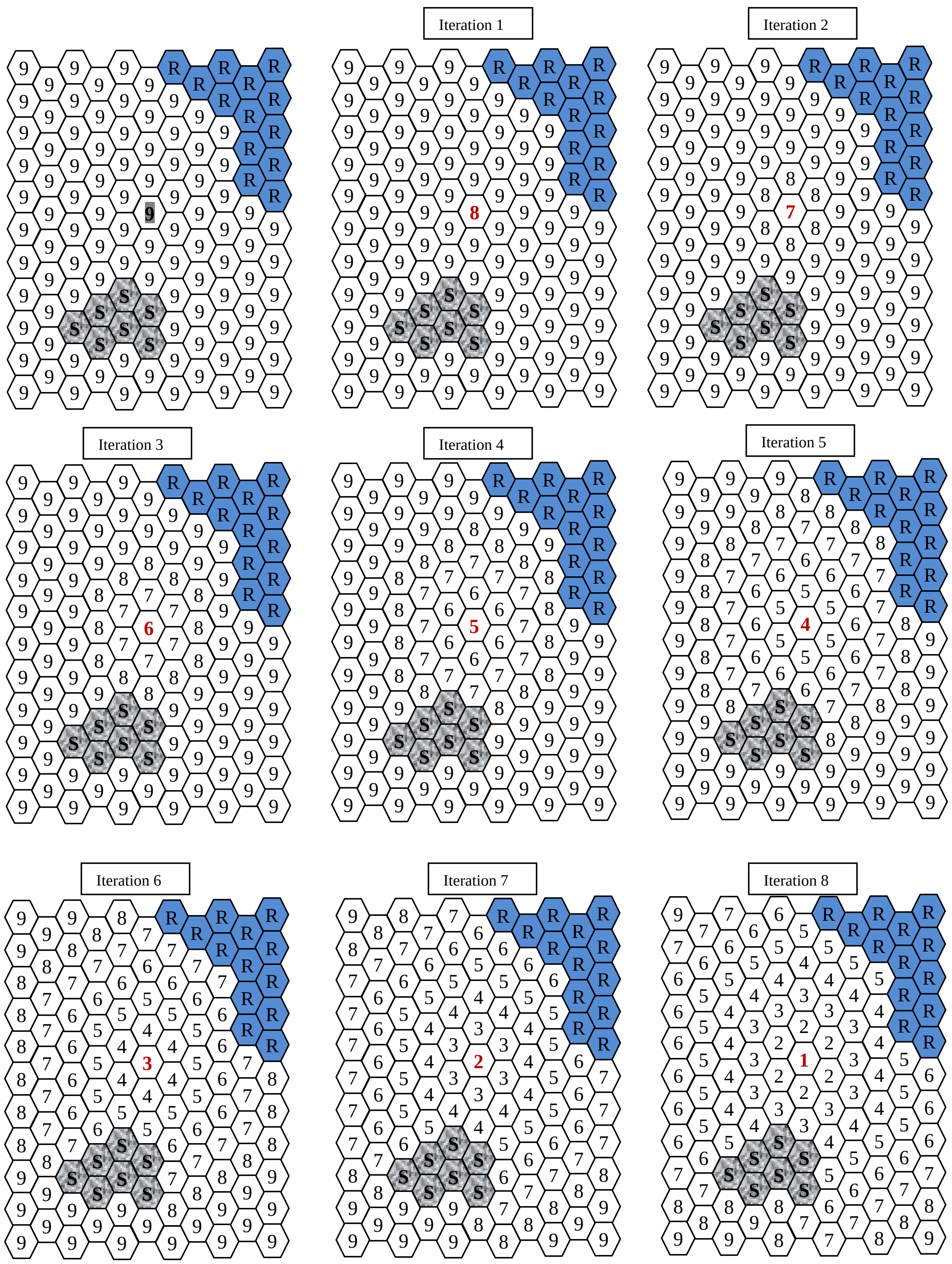
1. $R \to R$;
2. $S \to S$;
3. $0 \to 0$;
4. In the initial time-step, the fire starts from a fixed cell containing digit 9 that represents the density of the trees in that cell of the forest. On the second time-step, for the same cell $9 \to 8$. On the third time-step, for the same cell $8 \to 7$. In the same moment, all neighbouring cells of the cell with the fire change their digits with the previous digit.

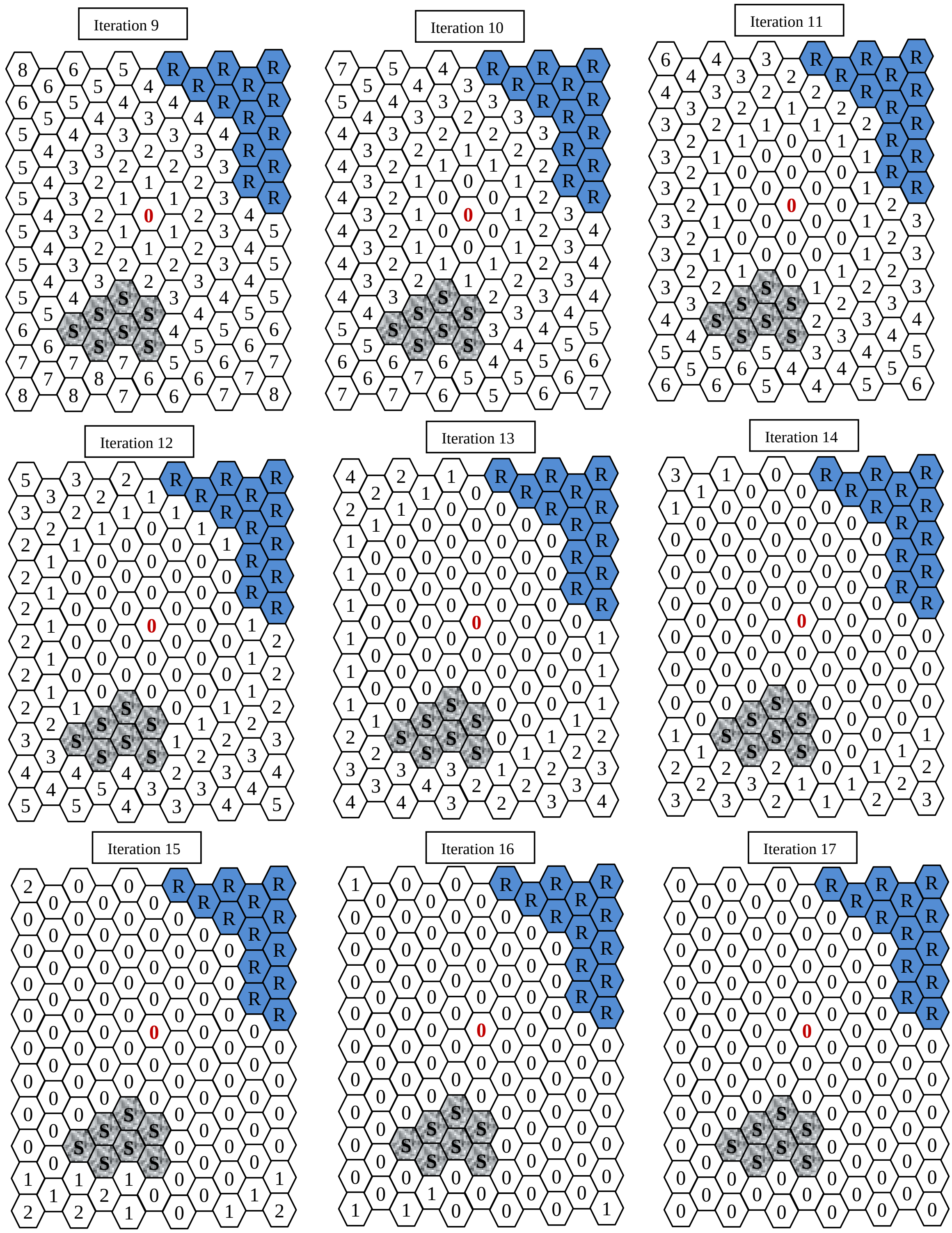
5. On the next time-steps, for the cells with fire

$$n \to \begin{cases} 0, & \text{if } n = 1 \\ \\ n - 1, & \text{if } n > 1 \end{cases}$$

6. The process continues until all cells in the region contain only digit 0. In the opposite case, go to Step 5.

The development of a forest fire is shown in the following two pages (the initial and the next 17 iterations).


Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8



Now, we determine the forms of the intuitionistic fuzzy estimations for the area of the fire by the following formulas:

$$\mu(i) = \frac{\text{number of cells with symbol “0”}}{\text{number of all cells}},$$

$$\nu(i) = \frac{\text{number of cells in which there is not fire}}{\text{number of all cells}},$$

$$\pi(i) = \frac{\text{number of cells in which there is fire}}{\text{number of all cells}}.$$

Here, i is the current number of the iteration and the number of all cells is 116. The values of functions $\mu(i), \nu(i)$ and $\pi(i)$ are given in Table 1.

Table 1.

Number of iteration	$\mu(i)$	$\nu(i)$	$\pi(i)$
0	0	1	0
1	0	0.991379	0.008621
2	0	0.939655	0.060345
3	0	0.836207	0.163793
4	0	0.698276	0.301724
5	0	0.534483	0.465517
6	0	0.379310	0.620690
7	0	0.284483	0.715517
8	0	0.206897	0.793103
9	0.008621	0.172414	0.818966
10	0.060345	0.172414	0.767241
11	0.163793	0.172414	0.663793
12	0.301724	0.172414	0.525862
13	0.465517	0.172414	0.362069
14	0.62069	0.172414	0.206897
15	0.715517	0.172414	0.112069
16	0.793103	0.172414	0.34483
17	0.827586	0.172414	0

3 Conclusion

In the next research, we will model the fire-process in an unhomogeneous forest and under conditions of wind. The intuitionistic fuzzy estimations for the area of the fire will be given.

Acknowledgements

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References

[1] Atanassov, K. *Game Method for Modelling*, "Prof. M. Drinov" Academic Publishing House, Sofia, 2011.

[2] Atanassov, K., *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.

[3] Sotirova, E., E. Velizarova, S. Fidanova, K. Atanassov, Modeling forest fire spread through a Game Method for Modeling based on hexagonal cells, In: *Large Scale Scientific Computing, Lecture Notes in Computer Science*, Springer (accepted).

Appendix: Short remarks on the Game Method for Modelling

One of the modifications of John Horton Conway's Game of Life is called "Game Method for Modelling" (GMM) [1].

Let us consider a set of symbols S and an n-dimensional simplex comprising n-dimensional cubes (when n = 2, a two-dimensional grid of squares).

We assume that material points (referred in brief as objects) can be found in some of the centres of the n-dimensional cells. The GMM-grid can be either finite or infnite. In the first case, for i-th dimension of the grid there is natural number g_i that corresponds to the number of the sequential cells of the grid in the present dimension. Therefore, when the n-dimensional GMM-grid is finite, there is a vector $\langle g_1, g_2, ..., g_n \rangle$ of the lenghts of its sides. Here, we use finite grids. We also consider a set of rules $\mathcal{A}$ as follows:

1. rules for the motion of the objects along the vertices of the simplex;

2. rules for the interactions among the objects, e.g., when they are collected in one cell.

Let the rules from the i-th type be denoted as i-rules, where $i = 1, 2$.

When $S = \{*\}$, we obtain the standard CGL.

We can call an *initial configuration* every set of (ordered) $(n+2)$-tuples with an initial component being the number of the object; the second, third, etc., until the $(n+1)^{-\mathrm{st}}$ – its coordinates; and the $(n+2)^{-\mathrm{nd}}$ – its corresponding symbol from S.

We can call a *final configuration* the ordered set of $(n+2)$-tuples having the above form and being a result of the modifications that occured during a certain number of applications of the rules from $\mathcal{A}$ over a (fixed) initial configuration.

The single application of a rule from $\mathcal{A}$ over a given configuration K is called an *elementary step* in the transformation of the model and is denoted by $A_1(K)$.

When we have some initial configuration, we obtain new configurations in a stepwise manner. We must determine some constructive criteria for stopping the process. For example, such a condition may be the following.

1. The rules of the GMM are applied over the initial configuration and its derivates for exactly n iterations.
2. A predefined configuration is obtained on the GMM-grid. For example, if we model a process of interaction between some objects, the process should stop when the grid contains only one of these objects.
3. A previous configuration is obtained on the GMM-grid, i.e., the process oscillates. This is criterion is applicable for deterministic processes.

Let us consider a rule P which juxtaposes to a set of configurations M a single configuration $P(M)$ being the mean of the given ones. We will call this rule a *concentrate rule.* The concentration can be made either over the values of the symbols from S for the objects, or over their coordinates (not over both of them imultaneously). In [1], different formulas for P are given. Here, we suppose that as a result of applying the rule P over the set of configurations M, we will obtain a new configuration $P(M)$, for which the $(i_k; j_k)$-th place is occupied by a digit calculated as:

$$d_{i,j} = \left[\frac{1}{s}\sum_{k=1}^{s} d_{i,j}^k\right],$$

where for any real number $x = a + \alpha$, where a is a natural number and $\alpha \in [0, 1)$: $[x] = a$.

If K is an initial configuration, let $A_1(K)$ be the configuration, obtained in a result of application of the rules from A over K (for one step). Let $A_{n+1}(K) = A_1(A_n(K))$ for every natural number $n \geq 1$.

Let $\mathcal{B}$ be a criterion derived from physical or mathematical considerations. For two given configurations K_1 and K_2, it answers to the question whether they are close enough to each other or not. For two configurations K_1 and K_2 lying in a planar rectangle with lengths p and q, we can use the following criterion:

$$\mathcal{B}(K_1, K_2) = \frac{1}{p.q}\sum_{i=1}^{p}\sum_{j=1}^{q}|d_{i,j}^1 - d_{i,j}^2| < C,$$

where C is a predefined constant. For the set of configurations M and the set of rules $\mathcal{A}$, we define the set of configurations

$$\mathcal{A}(M) = \{L|(\exists K \in M)(L = A(K))\}.$$

The rules $\mathcal{A}$ are called statistically correct, if for a large enough (from a statistical point of view) natural number N:

$$(\forall n > N)(\forall M = \{K_1, K_2, ..., K_n\})(\forall m \geq 1)$$
$$(\mathcal{B}(A_m(P(M)), P(\{L_i|L_i = A_m(K_i), 1 \leq i \leq n\})) < C).$$