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# Assigning the parameters for Intuitionistic Fuzzy Sets

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**Abstract**

In this article we propose two ways of assigning the parameters for intuitionistic fuzzy sets: by asking experts, and from relative frequency distributions (histograms).

# 1 Introduction

Intutionistic fuzzy sets, with independent memberships and non-memberships, are generalization of fuzzy sets. They make it possible to represent imperfect knowledge in a more adequate way. But to use intuitionistic fuzzy sets one should be able to assign their parameters. In this article we propose two ways of assigning the parameters for intuitionistic fuzzy sets: by asking experts, and from relative frequency distributions (histograms).

# 2 Intuitionistic Fuzzy Set Theory

Let us start with basic concepts related to fuzzy sets.

**Definition 1** *A fuzzy set $A'$ in $X = \{x\}$ is given by (Zadeh [33]):*

$$A' = \{< x, \mu_A(x) > | x \in X\} \tag{1}$$

*where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set $A'$; $\mu_A \in [0,1]$.*

The intuitionistic fuzzy set (IFS) theory is based both on extensions of corresponding definitions of fuzzy sets objects and definitions of new objects and their properties (Atanassov [1, 2, 3, 4, 5]).

**Definition 2** *An intuitionistic fuzzy set A in X is given by (Atanassov [1, 5]):*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

*where*

$$\mu_A : X \to [0,1]$$
$$\nu_A : X \to [0,1]$$

*with the condition*

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad \forall x \in X$$

*The numbers* $\mu_A(x)$, $\nu_A(x) \in [0,1]$ *denote the degree of membership and non-membership of* $x$ *to* $A$*, respectively.*

Obviously, each fuzzy set $A'$ corresponds to the following intuitionistic fuzzy set:

$$A = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\} \tag{3}$$

For each intuitionistic fuzzy set in X, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

the intuitionistic fuzzy index (or a hesitation margin) of $x$ in $A$ and, it expresses a lack of knowledge of whether $x$ belongs to $A$ or not (Atanassov [2, 3, 4, 5]).

It is obvious, that

$$0 \leq \pi_A(x) \leq 1 \quad for \quad each \quad x \in X$$

For each fuzzy set $A'$ in $X$, evidently,

$$\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0, \quad for \quad each \quad x \in X$$

In our further considerations we will use the notion of the complement elements, which definition is a simple consequence of a complement set $A^C$

$$A^C = \{< x, \nu_A, \mu_A > | x \in X\} \tag{5}$$

The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge what leads to describing many real problems in a more adequate way.

Intuitionistic fuzzy sets based models may be adequate mainly in the situations when we face human testimonies, opinions, etc. involving answers of three types: yes, no, abstaining i.e. which can not be classified (because of different reasons, eg. "I do not know", "I am not sure", "I do not want to answer", "I am not satisfied with any of the options" etc.).

**Example 1** Let us assume that we have a set $X$ of $n$ individuals who vote for/against building of nuclear power plant (judges voting for/against acquittal, electors voting for/against a given candidate or his opponent, consumers expressing/not expressing interest in buying a product). Let us assume that each individual $x_i$ belongs to

- a set of individuals (judges, electors) voting for – to the extent $\mu(x_i)$

- a set of individuals voting against – to the extent $\nu(x_i)$

It is worth noticing that by means of the fuzzy set theory we cannot consider the situation in more details. By means of intuitionistic fuzzy set theory we can also point out

- a set of individuals who did not answer neither "yes" nor "no"– to the extent $\pi(x_i)$ whereas: $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$; $\pi(x_i)$ – an intuitionistic fuzzy index.

From the point of view of e.g. market analysts (election committees) it could be tempting to assess the above data in terms of the possible final results of voting giving intervals containing

- probability of voting for
$$Pr_{for} \in [\mu, \mu + \pi]$$
where:
$$\mu = \frac{1}{n} \sum_{i=1}^{n} \mu(x_i)$$
$$\pi = \frac{1}{n} \sum_{i=1}^{n} \pi(x_i)$$
- probability of voting against
$$Pr_{against} \in [\nu, \nu + \pi]$$
where:
$$\nu = \frac{1}{n} \sum_{i=1}^{n} \nu(x_i)$$
with the condition $Pr_{for} + Pr_{against} = 1$.

In terms of mass assignment (see Section 3) we could say that necessary support for is equal to $\mu$, necessary support against is equal to $\nu$, whereas possible support for (the best possible result) $Pos^+$ is equal to $\mu + \pi$, possible support against (the worst possible result) $Pos^-$ is equal to $\nu + \pi$.

**Remark**
In the above example we made a simplifying assumption assigning a sing of equality to probabilities and memberships/non-memberships. This assumption is valid under the condition that each value of membership/non-membership occurs with the same probability for each $x_i$. In this paper, for the sake of simpler notation, we follow this assumption. However, in general, probabilities for intuitionistic fuzzy sets are calculated in the following way (Szmidt and Kacprzyk [24, 25]):

**Definition 3** *Let us assign to every element of an intuitionistic fuzzy event* $A \subset E = \{x_1, ..., x_n\}$ *(where* $E$ *is the elementary event space) its probability of occurrence, i.e.* $p(x_1), ...$ $..., p(x_n)$.
*Minimal probability* $p_{\min}(A)$ *of an intuitionistic fuzzy event* $A$ *is equal to*

$$p_{\min}(A) = \sum_{i=1}^{n} p(x_i)\mu(x_i)$$

Table 1: The questions considered when changing a job

| No | Questions | +/?/− |
|---|---|---|
| 1 | Is the job interesting | + |
| 2 | Salaries | − |
| 3 | Possibilities of promotion | ? |
| 4 | Expected pension | − |
| 5 | Number of hours spent in work | ? |
| 6 | Holidays – how long | + |
| 7 | Is the work safe | + |
| 8 | Responsibility | + |
| 9 | Time of the travel: home–work | − |
| 10 | Social reputation | + |
| 11 | Necessary creativity | + |
| 12 | Connected stress | + |

*Maximal probability of an intuitionistic fuzzy event A is equal to*

$$p_{\max}(A) = p_{min}(A) + \sum_{i=1}^{n} p(x_i)\pi(x_i)$$

*so probability of an event A is a number from the interval* $[p_{min}(A), p_{max}(A)]$, *or*

$$p(A) \in [\sum_{i=1}^{n} p_A(x_i)\mu_A(x_i), \sum_{i=1}^{n} p_A(x_i)\mu_A(x_i) + \sum_{i=1}^{n} p_A(x_i)\pi_A(x_i)] \quad (6)$$

*and probability of a complement event* $A^C$ *is a number from the interval* $[p_{min}(A^C), p_{max}(A^C)]$, *or*

$$p(A^C) \in [\sum_{i=1}^{n} p_A(x_i)\nu_A(x_i), \sum_{i=1}^{n} p_A(x_i)\nu_A(x_i) + \sum_{i=1}^{n} p_A(x_i)\pi_A(x_i)] \quad (7)$$

Applications of intuitionistic fuzzy sets to group decision making, negotiations and other real situations are presented in (Szmidt and Kacprzyk [21, 22, 23, 26, 27, 28, 29, 30]).

The question arises how to assign the parameters.

## 2.1 Assigning the parameters by experts

Here we will illustrate the problem in the simplest case – when one person considers one decision only (this simple case can be easily extended to more complicated situations - with more persons and more decisions). Let us imagine that somebody considers a problem of changing his/her job. To decide if a new job is interesting enough to give up a previous one it seems reasonable to prepare a whole list of questions. The list would depend on the personal preferences but in general the following questions presented in Table 1 seem to be important.

The immediate conclusion from Table 1 is how to evaluate the considered case (under the condition that all the questions are equally important) - it is necessary to sum up

- all positive answers (7/12) - it is the value of the membership for the considered option,

- all negative answers (3/12) - it is the value of the non-membership for the considered option,
- all answers to which it was impossible to say "yes" or "not" (2/12) - it is the value of the intuitionistic fuzzy index for the considered option.

It is important that employing of intuitionistic fuzzy sets just forces an individual to consider both advantages (memberships) and disadvantages (non-memberships) of a considered solution. Next, the imprecise area is taken into account as well. The importance of such an approach lies in the fact that most people concentrate usually on one or two "most visible" aspects of a problem. They do not try to find out the contrary arguments or to consider uncertain (in wide sense, i.e. not restricted to randomness) aspects of a situation (cf. Sutherland [18]). Intuitionistic fuzzy sets with their structure make us consider a situation/problem more properly. We refer again an interested reader to (Szmidt and Kacprzyk [21, 22, 23, 26, 27, 28, 29, 30]) where we exploit this fact - using intuitionistic fuzzy sets to group decision making. In short, the problem boils down to selecting an option or a set of options which are best accepted by most of the individuals. The options are considered in pairs. Employing intuitionistic fuzzy sets forces each individual to look at each pair (i,j) of the options considering: advantages of the first option over the second one (membership function), disadvantages of the first option over the second one (non-membership function), and taking into account lack of knowledge (intuitionistic fuzzy index) as far as the two options are concerned. In other words, intuitionistic fuzzy sets force a user to explore a problem from different points of view – including all important aspects which should be taken into account but, unfortunately, are often omitted by people making decisions. This fact, strongly connected with a phenomenon called by the Nobel Prize winner Kahneman (cf. Kahneman [17]) "bounded rationality", caused among others by framing effect (explained in terms of salience and anchoring playing a central role in treatments of judgements and choice) places intuitionistic fuzzy sets among the up-to-date means of knowledge representation and processing.

## 3 Mass Assignment Theory and assigning the parameters from relative frequency distributions (histograms)

The theory of mass assignment has been developed by Baldwin (Baldwin [7], Baldwin et al. [10, 11]) to provide a formal framework for manipulating both probabilistic and fuzzy uncertainty.

A fuzzy set can be converted into a mass assignment (Baldwin [6]). This mass assignment represents a family of probability distributions.

**Definition 4 (Mass Assignment)**

*Let $A'$ be a fuzzy subset of a finite universe $\Omega$ such that the range of the membership function of $A'$, is $\{\mu_1, ..., \mu_n\}$ where $\mu_i > \mu_{i+1}$. Then the mass assignment of $A'$ denoted $m_{A'}$, is a probability distribution on $2^{\Omega}$ satisfying*

$$m_{A'}(F_i) = \mu_i - \mu_{i+1} \text{ where } F_i = \{x \in \Omega | \mu(x) \geq \mu_i\} \text{ for } i = 1, ..., n \quad (8)$$

The sets $F_1, ..., F_n$ are called the focal elements of $m_{A'}$. The detailed introduction to mass assignment theory is given by Baldwin at al. [10].

Table 2: Equality of the the parameters for Baldwin's voting model and IFS voting model

| | Baldwin's voting model | IFS voting model |
|---|---|---|
| voting in favour | $n$ | $\mu$ |
| voting against | $1-p$ | $\nu$ |
| abstaining | $p-n$ | $\pi$ |

**Example 2** (Baldwin [8])
Let $X = \{x_1,\ x_2,\ x_3,\ x_4\}$
If $A' = x_1/1 + x_2/0.7 + x_3/0.4 + x_4/0.3$
then the associated mass assignment is
$m_{A'} = x_1 : 0.3, \quad \{x_1,\ x_2\} = 0.3, \quad \{x_1,\ x_2,\ x_3\} = 0.1, \quad \{x_1,\ x_2,\ x_3,\ x_4\} = 0.3$

Support Pairs (the basic representation of uncertainty in the language FRIL [Baldwin at al. [10, 13]) are associated with mass assignments and represent an interval containing an unknown probability. Support Pairs are used to characterize uncertainty in facts and conditional probabilities in rules. A Support Pair $(n, p)$ comprises a necessary and possible support and can be interpreted as an interval in which the unknown probability lies. A voting interpretation is also useful (Baldwin and Pilsworth [9]): the lower (necessary) support $n$ represents the proportions of a sample population voting in favour of a proposition, whereas $(1-p)$ represents the proportion voting against; $(p-n)$ represents the proportion abstaining.

For intuitionistic fuzzy sets (cf. Section 2) we have

- the proportion of a sample population voting in favour of a proposition is equal to $\mu$ (membership function),
- the proportion voting against is equal to $\nu$ (non-membership function),
- $\pi$ represents the proportion abstaining.

In Table 2 equality of parameters from Baldwin's voting model and from intuitionistic fuzzy set (IFS) voting model is presented.

So we can represent a Support Pair $(n, p)$ using notation of intuitionistic fuzzy sets in the following way

$$(n, p) = (n, n + p - n) = (\mu, \mu + \pi) \tag{9}$$

i.e.: a Support Pair in Baldwin's voting model can be expressed by using notation of intuitionistic fuzzy sets.

It should be noted as well that the necessary support for the statement not being true is one minus the possibility of the support for the statement being true, i.e. $1 - p$. Similarly, the possible support for the statement being not true is one minus the necessary support for the statement being true i.e. $1 - n$. Taking into account the counterparts of the parameters, we can express this fact using notation of intuitionistic fuzzy sets as

$$(1 - p, 1 - n) = (\nu, \nu + \pi)$$

Let us look at three Support Pairs $(n, p)$ of special interests (Baldwin and Pilsworth [9])

- (1, 1) which represents total support for the associated statement,
- (0, 0) which represents total support against and
- (0, 1) which characterizes complete uncertainty in the support.

Of course the above Support Pairs have exactly the same meaning in intuitionistic fuzzy set model (under the assumption that we consider probabilities for intuitionistic fuzzy memberships/non-memberships as it was explained in Section 2):

- (1, 1) means that $\mu = 1$ and $\pi = 0$, i.e. total support,
- (0, 0) means $\mu = 0$ and $\pi = 0$ what involves $\nu = 1$, i.e. total support against,
- (0, 1) means $\mu = 0$ and $\pi = 1$ i.e.: complete uncertainty in the support.

In other words both Support Pairs and intuitionistic fuzzy set models give the same intervals containing the probability of the fact being true, and the difference between the upper and lower values of intervals is a measure of the uncertainty associated with the fact [20], [19].

The mass assignment structure is best used to represent knowledge that is statistically based such that the values can be measured, even if the measurements themselves are approximate or uncertain (Baldwin [12]).

**Definition 5 (Least Prejudiced Distribution)** *[10]*
*For $A'$ a fuzzy subset of a finite universe $\Omega$ such that $A'$ is normalized, the least prejudiced distribution of $A'$, denoted $lp_{A'}$, is a probability distribution on $\Omega$ given by*

$$lp_{A'}(x) = \sum_{F_i : x \in F_i} \frac{m_{A'}(F_i)}{|F_i|} \tag{10}$$

**Theorem 1** *[14] Let $P$ be a probability distribution on a finite universe $\Omega$ taking as a range of values $\{p_1, ..., p_n\}$ where $0 \le p_{i+1} < p_i \le 1$ and $\sum_{i=1}^{n} p_i = 1$. Then $P$ is the least prejudiced distribution of a fuzzy set $A'$ if and only if $A'$ has a mass assignment given by*

$$\begin{aligned} m_{A'}(F_i) &= \mu_i - \mu_{i+1} \quad for\ i = 1, ..., n-1 \\ m_{A'}(F_n) &= \mu_n \end{aligned}$$

*where*

$$\begin{aligned} F_i &= \{x \in \Omega | P(x) \ge p_i\} \\ \mu_i &= |F_i| p_i + \sum_{j=i+1}^{n} (|F_j| - |F_{j+1}|) p_j \end{aligned} \tag{11}$$

Proof (see Baldwin at al. [14] )
It is worth mentioning that the above algorithm is identical to the bijection method proposed by Dubois and Prade [15] although the motivation in [14] is quite different. Also Yager [31] considered a similar approach to mapping between probability and possibility. A further justification for the transformation was given by Yamada [32].

In other words, Theorem 1 gives a general procedure converting a relative frequency distribution into a fuzzy set, i.e. gives us means for generating fuzzy sets from data. As non-memberships for a fuzzy set are univocally assigned by memberships, Theorem 1 gives a full description of a fuzzy set.

**Example 3** [14] Let $\Omega = \{a, b, c, d\}$ and $P$ be a probability distribution on $\Omega$ such that

$$P(a) = 0.15, \ \ P(b) = 0.6, \ \ P(c) = 0.2, \ \ P(d) = 0.05$$

so that $p_1 = 0.6$, $p_2 = 0.2$, $p_3 = 0.15$, $p_4 = 0.05$.

Hence if $A'$ is a normalized fuzzy subset of $\Omega$ such that $lp_{A'} = P$ then we can determine $A'$ as follows: Given the ordering constraint imposed by $P$ we have that

$$m_{A'}(F_4) = \mu_4, \ \ m_{A'}(F_3) = \mu_3 - \mu_4, \ \ m_{A'}(F_2) = \mu_2 - \mu_3, \ \ m_{A'}(F_1) = 1 - \mu_2$$

where

$$F_4 = \{a, b, c, d\}, F_3 = \{a, b, c\}, F_2 = \{b, c\}, F_1 = \{b\}$$

This implies that $A' = b/1 + c/\mu_2 + a/\mu_3 + d/\mu_4$ and using the formula from Theorem 1 we obtain

$$\mu_4 = 4p_4 = 4(0.05) = 0.2$$

$$\mu_3 = 3p_3 + p_4 = 3(0.15) + 0.05 = 0.5$$

$$\mu_2 = 2p_2 + p_3 + p_4 = 2(0.2) + 0.15 + 0.05 = 0.6$$

Therefore, $A' = b/1 + c/0.6 + a/0.5 + d/0.2$

This way it was shown that using Theorem 1 we can convert the relative frequency distributions into a fuzzy set.

But Theorem 1 gives also an idea how to convert the relative frequency distributions into an intuitionistic fuzzy set.

When discussing intuitionistic fuzzy sets we consider memberships and independent non-memberships (3)–(4), so Theorem 1 gives only a part of the description we look for. To receive the full description of an intuitionistic fuzzy set (with independent memberships and non-memberships), it is necessary to repeat the procedure as in Theorem 1 two times. In result we obtain two fuzzy sets. To interpret them properly in terms of intuitionistic fuzzy sets we recall first a semantic for membership functions.

Dubois and Prade [16] have explored three main semantics for membership functions - depending on the particular applications. Here we apply the interpretation proposed by Zadeh [34] when he introduced the possibility theory. Membership $\mu(x)$ is there the degree of possibility that a parameter $x$ has value $\mu$.

In effect of repeating the procedure as in Theorem 1 two times (first – for data representing memberships, second – for data representing non-memberships), and taking into account interpretation that the obtain values are the degrees of possibility we receive the following results.

- First time we perform the steps from Theorem 1 for the relative frequencies connected to memberships. In effect we obtain (fuzzy) possibilities $Pos^+(x) = \mu(x) + \pi(x)$ that $x$ has value $Pos^+$.

  $Pos^+(x)$ (left side of the above equation) mean the values of a membership function for a fuzzy set (possibilities). In terms of intuitionistic fuzzy sets (right side of the above equation) these possibilities are equal to possible (maximal) memberships of an intuitionistic fuzzy set, i.e.

  $\mu(x) + \pi(x)$, where $\mu(x)$ – the values of the membership function for an intuitionistic fuzzy set, and $\mu(x) \in [\mu(x), \mu(x) + \pi(x)]$.

- Second time we perform the steps from Theorem 1 for the (independent) relative frequencies connected to non-memberships. In effect we obtain (fuzzy) possibilities $Pos^{-}(x) = \nu(x) + \pi(x)$ that $x$ has not value $Pos^{-}$.

  $Pos^{-}(x)$ (left side of the above equation) mean the values of a membership function for another (than in the previous step) fuzzy set (possibilities). In terms of intuitionistic fuzzy sets (right side of the above equation) these possibilities are equal to possible (maximal) non-memberships, i.e.

  $\nu(x)+\pi(x)$, where $\nu(x)$– the values of the non-membership function for an intuitionistic fuzzy set, and $\nu(x) \in [\nu(x), \nu(x) + \pi(x)]$

**The algorithm of assigning the parameters of intuitionistic fuzzy sets**

1. From Theorem 1 we calculate the values of the left sides of the equations:

$$Pos^{+}(x) = \mu(x) + \pi(x) \tag{12}$$

   and

$$Pos^{-}(x) = \nu(x) + \pi(x) \tag{13}$$

2. From (12)–(13), and taking into account (4) we obtain the values $\pi(x)$

$$Pos^{+}(x) + Pos^{-}(x) = \mu(x) + \pi(x) + \nu(x) + \pi(x) = 1 + \pi(x) \tag{14}$$

$$\pi(x) = Pos^{+}(x) + Pos^{-}(x) - 1 \tag{15}$$

3. Having the values $\pi(x)$, from (12) and (13) we obtain for each $x$: $\mu(x)$, and $\nu(x)$.

This way, starting from relative frequency distributions, and using Theorem 1, we receive full description of an intuitionistic fuzzy set.

**Example 4** The problem consists in classifying products (taking into account presence of 10 different levels of an element) as legal and illegal. The data descibing relative frequencies for legal and illegal products are respectively

- relative frequencies $p^{+}(i)$ for legal productss (for each $i$-th level of the presence of the conidered element), $i = 1, \ldots, 10$

$$\begin{aligned} &p^{+}(1) = 0., \quad p^{+}(2) = 0., \quad p^{+}(3) = 0.034, p^{+}(4) = 0.165, p^{+}(5) = 0.301, \\ &p^{+}(6) = 0.301, p^{+}(7) = 0.165, p^{+}(8) = 0.034, p^{+}(9) = 0., \quad p^{+}(10) = 0. \end{aligned} \tag{16}$$

- relative frequencies $p^{-}(i)$ for illegal products (for each $i$-th level of the presence of the conidered element), $i = 1, \ldots, 10$

$$\begin{aligned} &p^{-}(1) = 0.125, p^{-}(2) = 0.128, p^{-}(3) = 0.117, p^{-}(4) = 0.08, \quad p^{-}(5) = 0.05, \\ &p^{-}(6) = 0.05, \quad p^{-}(7) = 0.08, \quad p^{-}(8) = 0.117, p^{-}(9) = 0.128, p^{-}(10) = 0.125 \end{aligned} \tag{17}$$

From Theorem 1 and the data (16) we obtain possibilities $Pos^+(i)$ for legal products

$$Pos^+(1) = 0., Pos^+(2) = 0., \quad Pos^+(3) = 0.205, Pos^+(4) = 0.727, Pos^+(5) = 1.,$$
$$Pos^+(6) = 1., Pos^+(7) = 0.727, Pos^+(8) = 0.205, Pos^+(9) = 0., \quad Pos^+(10) = 0. \quad (18)$$

From Theorem 1 and the data (17) we obtain possibilities $Pos^-(i)$ for illegal products

$$Pos^-(1) = 1., \quad Pos^-(2) = 1., \quad Pos^-(3) = 0.961, Pos^-(4) = 0.737, Pos^-(5) = 0.503,$$
$$Pos^-(6) = 0.503, Pos^-(7) = 0.737, Pos^-(8) = 0.961, Pos^-(9) = 1., \quad Pos^-(10) = 1. \quad (19)$$

From (18), (19), and (15), we obtain the following values $\pi(i)$

$$\pi(1) = 0., \quad \pi(2) = 0., \quad \pi(3) = 0.166, \pi(4) = 0.464, \pi(5) = 0.503,$$
$$\pi(6) = 0.503, \pi(7) = 0.464, \pi(8) = 0.166, \pi(9) = 0., \quad \pi(10) = 0. \quad (20)$$

Finally, from (18) and (20) we obtain $\mu(i)$

$$\mu(1) = 0., \quad \mu(2) = 0., \quad \mu(3) = 0.039, \mu(4) = 0.263, \mu(5) = 0.497,$$
$$\mu(6) = 0.497, \mu(7) = 0.263, \mu(8) = 0.039, \mu(9) = 0., \quad \mu(10) = 0. \quad (21)$$

and from (19) and (20) we obtain $\nu(i)$

$$\nu(1) = 1., \quad \nu(2) = 1., \quad \nu(3) = 0.795, \nu(4) = 0.273, \nu(5) = 0.,$$
$$\nu(6) = 0., \quad \nu(7) = 0.273, \nu(8) = 0.795, \nu(9) = 1., \quad \nu(10) = 1. \quad (22)$$

This way starting from relative frequencies we have obtained the values $\mu$ (21), $\nu$ (22), and $\pi$ (20) characterizing the counterpart intuitionistic fuzzy set.

It is worth noticing a strong difference as far as our approach is concern, and the incorrect method of expressing an intuitinistic fuzzy set via two fuzzy sets constructed in a such way that memberships of the first fuzzy set are treated as the memberships of the intuitionistic fuzzy set, and memberships of the second fuzzy set are treated as the non-memberships of the same intuitionistic fuzzy set.

# 4 Conclusions

Two approaches to assigning parameters for intuitionistic fuzzy sets were presented. The first approach is via asking the experts. The second approach is automatic - starting from relative frequency distributions.

Both approaches seems to be useful. But the second one – the automatic, and mathematically justified method assigning the functions describing intuitinistic fuzzy sets seems to be especially important in the context of analysing information in big data bases.

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