Fifteenth Int. Conf. on IFSs, Burgas, 11-12 May 2011
NIFS 17 (2011), 2, 17-24

On intuitionistic fuzzy tree-interpretations by index matrices

Rangasamy Parvathi[1], Anthony Shannon[2], Panagiotis Chountas[3] and Krassimir Atanassov[4]

[1] Department of Mathematics,
Vellalar College for Women, Erode - 638 012, Tamilnadu, India
E-mail: paarvathis@rediffmail.com
[2] Faculty of Engineering & IT, University of Technology,
Sydney, NSW 2007, Australia
E-mails: tshannon38@gmail.com, anthony.Shannon@uts.edu.au
[3] HSCS - University of Westminster, London, HA1 3TP, UK
E-mail: chountp@wmin.ac.uk
[4] Dept. of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 105, 1113 Sofia, Bulgaria
E-mail: krat@bas.bg

Abstract: Using the apparatus of index matrices, representations of (graph) trees and of operations over them, are given. Three examples illustrate the graph-operations are discussed here.
Keywords: Index matrix, Intuitionistic fuzzy graph
AMS Classification: 03E72, 05C50

1 Introduction and preliminary remarks on index matrices

The concept of Index Matrix (IM) was introduced in [1] and discussed in more details in [2, 3]. Here, following [1, 2, 3], the basic definitions and properties related to IMs are given.

Let I be a fixed set of indices and $\mathcal{R}$ be the set of all real numbers. By IM with index sets K and L $(K, L \subset I)$, we mean the object,

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\},$ for $1 \le i \le m$, and $1 \le j \le n : a_{k_i,l_j} \in \mathcal{R}$.

On the basis of the above definition, in [3] the new object of the Intuitionistic Fuzzy IM (IFIM) was introduced, so that it has the form

$$[K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$$

$$\equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \langle \mu_{k_1,l_2}, \nu_{k_1,l_2} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\ k_2 & \langle \mu_{k_2,l_1}, \nu_{k_2,l_1} \rangle & \langle \mu_{k_2,l_2}, \nu_{k_2,l_2} \rangle & \dots & \langle \mu_{k_2,l_n}, \nu_{k_2,l_n} \rangle \\ \vdots & & & & \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \langle \mu_{k_m,l_2}, \nu_{k_m,l_2} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle \end{array},$$

where for every $1 \le i \le m, 1 \le j \le n$: $0 \le \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \le 1$.

For the IMs $A = [K, L, \{a_{k_i,l_j}\}]$, $B = [P, Q, \{b_{p_r,q_s}\}]$, six operations that are analogous to the usual matrix operations of addition and multiplication are defined, as well as other, specific ones. Here we give only operations "addition" and "structural subtraction".

$$A \oplus B = [K \cup P, L \cup Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} a_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ b_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ a_{k_i,l_j} + b_{p_r,q_s}, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q; \\ 0, & \text{otherwise} \end{cases}.$$

If the IMs $A_1, A_2, ..., A_n$ are given, then

$$\sum_{i=1}^{r}{}^{\oplus} A_i = A_1 \oplus A_2 \oplus ... \oplus A_n.$$

$$A \ominus B = [K - P, L - Q, \{c_{t_u,v_w}\}],$$

where "$-$" is the set–theoretic difference operation and

$$c_{t_u,v_w} = a_{k_i,l_j}, \text{ for } t_u = k_i \in K - P \text{ and } v_w = l_j \in L - Q.$$

Let IM $A = [K, L, \{a_{k,l}\}]$ be given.

First, local substitution over the IM is defined for the couples of indices (p, k) and/or (q, l), respectively, by

$$[\frac{p}{k}]A = [(K - \{k\}) \cup \{p\}, L, \{a_{k,l}\}],$$

$$[\frac{q}{l}]A = [K, (L - \{l\}) \cup \{q\}, \{a_{k,l}\}],$$

Secondly,

$$[\frac{p}{k}\frac{q}{l}]A = [\frac{p}{k}][\frac{q}{l}]A,$$

i.e.

$$[\frac{p}{k}\frac{q}{l}]A = [(K - \{k\}) \cup \{p\}, (L - \{l\}) \cup \{q\}, \{a_{k,l}\}].$$

Obviously, for the above indices k, l, p, q,

$$[\frac{k}{p}]([\frac{p}{k}]A) = [\frac{l}{q}]([\frac{q}{l}]A) = [\frac{k}{p}\frac{l}{q}]([\frac{p}{k}\frac{q}{l}]A) = A,$$

Let the sets of indices $P = \{p_1, p_2, ..., p_m\}$, $Q = \{q_1, q_2, ..., q_n\}$ be given.
Third, for them define sequentially,

$$[\frac{P}{K}]A = [\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_n}{k_n}]A,$$

$$[\frac{Q}{L}]A = ([\frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n}]A),$$

$$[\frac{P}{K}\frac{Q}{L}]A = [\frac{P}{K}][\frac{Q}{L}]A,$$

i.e.,

$$[\frac{P}{K}\frac{Q}{L}]A = [\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_m}{k_m}\frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_n}{l_n}]A = [P, Q, \{a_{k,l}\}]$$

Let A be an ordinary IM, and let its element a_{k_f,l_g} be an IM by itself,

$$a_{k_f,l_g} = [P, Q, \{b_{p_r,q_s}\}],$$

where

$$K \cap P = L \cap Q = \emptyset.$$

Here, introduce the following hierarchical operation

$$A|(a_{k_f,l_g}) = [(K - \{k_f\}) \cup P, (L - \{l_g\}) \cup Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} a_{k_i,l_j}, & \text{if } t_u = k_i \in K - \{k_f\} \text{ and } v_w = l_j \in L - \{l_g\} \\ b_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q \\ 0, & \text{otherwise} \end{cases}$$

If a_{k_f,l_g} is not an element of IM A, then

$$A|(a_{k_f,l_g}) = A.$$

2 Main results

Let us have a (fixed) set of vertices $\mathcal{V}$. An IFTree T (over $\mathcal{V}$) will be the ordered pair $T = (V^*, A^*)$ (see [5, 6, 7]), where for $V \subset \mathcal{V}$,

$$V^* = \{\langle v, \mu_V(v), \nu_V(v)\rangle | v \in V\},$$

and for $A \subset V \times V$,

$$A^* = \{\langle g, \mu_A(g), \nu_A(g)\rangle | (\exists v, w \in V)(g = \langle v, w\rangle \in A)\},$$

where $\mu_V(v)$ and $\nu_V(v)$ are degrees of membership and non-membership of the element $v \in \mathcal{V}$ to V and

$$0 \leq \mu_V(v) + \nu_V(v) \leq 1.$$

Let the IFTree $T = [V, A]$ be given, where V is the set of its vertices and A is the set of its arcs, and let it have the following IM-form

$$T = [V, V, \{a_{k_i,l_j}\}].$$

Let its vertex w be fixed and let the subtree with source vertex w be

$$U = [W, W, \{b_{k_i,l_j}\}],$$

where

$$W = \{w, w_1, w_2, ..., w_s\} \subseteq V.$$

Let P be the new IFTree to be inserted at the vertex w of the IFTree T and let it have the IM-form

$$P = [Q, Q, \{c_{k_i,l_j}\}],$$

for which $w \in Q$ and $\{q_1, q_2, ..., q_r\} \subset Q$ are destination vertices.

Then, the IM-form of the new IFTree T^* will be

$$T^* = ([V, V, \{a_{k_i,l_j}\}] \ominus [W, W, \{\overline{a}_{k_i,l_j}\}]) \oplus [Q, Q, \{c_{k_i,l_j}\}] \oplus \bigoplus_{i=1}^{r} \left[\frac{q_i}{w}\right] \left[\frac{w_{i,1}}{w_1}\right] ... \left[\frac{w_{i,s}}{w_s}\right] [W, W, \{b_{k_i,l_j}\}].$$

We illustrate these definitions by some examples.

Example 1. Let the ordered IFTree T_1, in Fig. 1, be given, and let P (see Fig. 2) be the new ordered IFTree to be inserted at vertex w of T_1. The resultant IFTree T_1^* is given in Fig. 3.

Let the two IFTrees have representations, respectively

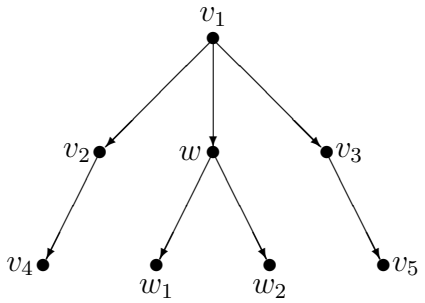

Fig. 1.

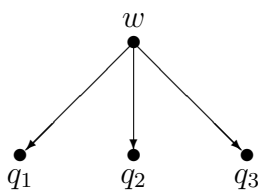

Fig. 2

$$T_1 = \begin{array}{c|cccccccc} & v_1 & v_2 & w & v_3 & v_4 & w_1 & w_2 & v_5 \\ \hline v_1 & \alpha_{v_1,v_1} & \alpha_{v_1,v_2} & \alpha_{v_1,w} & \alpha_{v_1,v_3} & \alpha_{v_1,v_4} & \alpha_{v_1,w_1} & \alpha_{v_1,w_2} & \alpha_{v_1,v_5} \\ v_2 & \alpha_{v_2,v_1} & \alpha_{v_2,v_2} & \alpha_{v_2,w} & \alpha_{v_2,v_3} & \alpha_{v_2,v_4} & \alpha_{v_2,w_1} & \alpha_{v_2,w_2} & \alpha_{v_2,v_5} \\ w & \alpha_{w,v_1} & \alpha_{w,v_2} & \alpha_{w,w} & \alpha_{w,v_3} & \alpha_{w,v_4} & \alpha_{w,w_1} & \alpha_{w,w_2} & \alpha_{w,v_5} \\ v_3 & \alpha_{v_3,v_1} & \alpha_{v_3,v_2} & \alpha_{v_3,w} & \alpha_{v_3,v_3} & \alpha_{v_3,v_4} & \alpha_{v_3,w_1} & \alpha_{v_3,w_2} & \alpha_{v_3,v_5} \\ v_4 & \alpha_{v_4,v_1} & \alpha_{v_4,v_2} & \alpha_{v_4,w} & \alpha_{v_4,v_3} & \alpha_{v_4,v_4} & \alpha_{v_4,w_1} & \alpha_{v_4,w_2} & \alpha_{v_4,v_5} \\ w_1 & \alpha_{w_1,v_1} & \alpha_{w_1,v_2} & \alpha_{w_1,w} & \alpha_{w_1,v_3} & \alpha_{w_1,v_4} & \alpha_{w_1,w_1} & \alpha_{w_1,w_2} & \alpha_{w_1,v_5} \\ w_2 & \alpha_{w_2,v_1} & \alpha_{w_2,v_2} & \alpha_{w_2,w} & \alpha_{w_2,v_3} & \alpha_{w_2,v_4} & \alpha_{w_2,w_1} & \alpha_{w_2,w_2} & \alpha_{w_2,v_5} \\ v_5 & \alpha_{v_5,v_1} & \alpha_{v_5,v_2} & \alpha_{v_5,w} & \alpha_{v_5,v_3} & \alpha_{v_5,v_4} & \alpha_{v_5,w_1} & \alpha_{v_5,w_2} & \alpha_{v_5,v_5} \end{array},$$

where $\alpha_{a,b} = \langle \mu_{a,b}, \nu_{a,b} \rangle$ for $a, b \in \{v_1, v_2, w, v_3, v_4, w_1, w_2, v_5\}$ and

$$P = \begin{array}{c|cccc} & w & q_1 & q_2 & q_3 \\ \hline w & \beta_{w,w} & \beta_{w,q_1} & \beta_{w,q_2} & \beta_{w,q_3} \\ q_1 & \beta_{q_1,w} & \beta_{q_1,q_1} & \beta_{q_1,q_2} & \beta_{q_1,q_3} \\ q_2 & \beta_{q_2,w} & \beta_{q_2,q_1} & \beta_{q_2,q_2} & \beta_{q_2,q_3} \\ q_3 & \beta_{q_3,w} & \beta_{q_3,q_1} & \beta_{q_3,q_2} & \beta_{q_3,q_3} \end{array},$$

where $\beta_{a,b} = \langle \mu_{a,b}, \nu_{a,b} \rangle$ for $a, b \in \{w, q_1, q_2, q_3\}$.

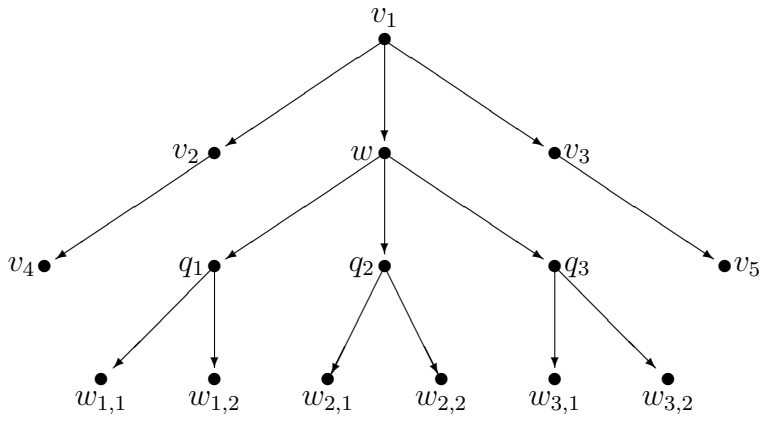

Fig. 3

Hawing in mind [4] and the fact that the IFTree is ordered, we can rewrite the IMs T_1 and P to the (equivalent) forms

$$T_1 = \begin{array}{c|ccccccc} & v_2 & w & v_3 & v_4 & w_1 & w_2 & v_5 \\ \hline v_1 & \alpha_{v_1,v_2} & \alpha_{v_1,w} & \alpha_{v_1,v_3} & \overline{O} & \overline{O} & \overline{O} & \overline{O} \\ v_2 & \overline{O} & \overline{O} & \overline{O} & \alpha_{v_2,v_4} & \overline{O} & \overline{O} & \overline{O} \\ w & \overline{O} & \overline{O} & \overline{O} & \overline{O} & \alpha_{w,w_1} & \alpha_{w,w_2} & \overline{O} \\ v_3 & \overline{O} & \overline{O} & \overline{O} & \overline{O} & \overline{O} & \overline{O} & \alpha_{v_3,v_5} \end{array}$$

and

$$P = \begin{array}{c|ccc} & q_1 & q_2 & q_3 \\ \hline w & \beta_{w,q_1} & \beta_{w,q_2} & \beta_{w,q_3} \end{array},$$

where $\overline{O} = \langle 0, 1 \rangle$.

The IM-form of the IFTree T_1^* is

$$\begin{aligned} T_1^* = ([&\{v_1, v_2, w, v_3, v_4, w_1, w_2, v_5\}, \{v_1, v_2, w, v_3, v_4, w_1, w_2, v_5\}, \{a_{k_i,l_j}\}] \\ &\ominus[\{w, w_1, w_2\}, \{w, w_1, w_2\}, \{\overline{a}_{k_i,l_j}\}]) \\ &\oplus[\{w, q_1, q_2, q_3\}, \{w, q_1, q_2, q_3\}, \{c_{k_i,l_j}\}] \\ \oplus \bigoplus_{i=1}^{3} &\left[\frac{q_i}{w}\right] \left[\frac{w_{i,1}}{w_1}\right] \left[\frac{w_{i,2}}{w_2}\right] [\{w, w_1, w_2\}, \{w, w_1, w_2\}, \{b_{k_i,l_j}\}] \\ = [&\{v_1, v_2, w, v_3, v_4, q_1, q_2, q_3, v_5, w_{1,1}, w_{1,2}, w_{2,1}, w_{2,2}, w_{3,1}, w_{3,2}\}, \\ &\{v_1, v_2, w, v_3, v_4, q_1, q_2, q_3, v_5, w_{1,1}, w_{1,2}, w_{2,1}, w_{2,2}, w_{3,1}, w_{3,2}\}, \{d_{a,b}\}] \\ = [&\{v_1, v_2, w, v_3, q_1, q_2, q_3\}, \\ &\{v_2, w, v_3, v_4, q_1, q_2, q_3, v_5, w_{1,1}, w_{1,2}, w_{2,1}, w_{2,2}, w_{3,1}, w_{3,2}, \}, \{d_{a,b}\}], \end{aligned}$$

where the values of the elements $d_{a,b}$ are determined as above.

Example 2. Let the ordered IFTree T_2, in Fig. 4, be given, and let P (see Fig. 2) be the new ordered IFTree to be inserted at vertex w of T_2. The resultant IFTree T_2^* is given in Fig. 5.

The IFTree T_2 has representation

$$T_2 = \begin{array}{c|cccc} & w & w_1 & w_2 & w_3 \\ \hline w & \beta_{w,w} & \beta_{w,w_1} & \beta_{w,w_2} & \beta_{w,w_3} \\ w_1 & \beta_{w_1,w} & \beta_{w_1,w_1} & \beta_{w_1,w_2} & \beta_{w_1,w_3} \\ w_2 & \beta_{w_2,w} & \beta_{w_2,w_1} & \beta_{w_2,w_2} & \beta_{w_2,w_3} \\ w_3 & \beta_{w_3,w} & \beta_{w_3,w_1} & \beta_{w_3,w_2} & \beta_{w_3,w_3} \end{array}$$

$$= \begin{array}{c|ccc} & w_1 & w_2 & w_3 \\ \hline w & \beta_{w,w_1} & \beta_{w,w_2} & \beta_{w,w_3} \end{array}.$$

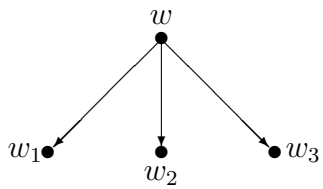

Fig. 4

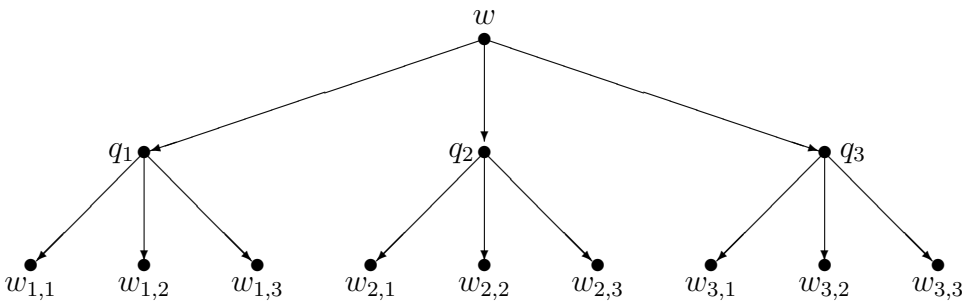

Fig. 5

Then

$$\begin{aligned} T_2^* = ([\{w, w_1, w_2, w_3\}, \{w, w_1, w_2, w_3\}, \{a_{k_i,l_j}\}] \ominus [\{w, w_1, w_2, w_3\}, \{w, w_1, w_2, w_3\}, \{a_{k_i,l_j}\}]) \\ \oplus [\{w, q_1, q_2, q_3\}, \{w, q_1, q_2, q_3\}, \{c_{k_i,l_j}\}] \\ \oplus \sum_{i=1}^{3} \left[\frac{q_i}{w}\right] \left[\frac{w_{i,1}}{w_1}\right] \left[\frac{w_{i,2}}{w_2}\right] \left[\frac{w_{i,3}}{w_3}\right] [\{w, w_1, w_2, w_3\}, \{w, w_1, w_2, w_3\}, \{b_{k_i,l_j}\}] \\ = [\{w, q_1, q_2, q_3\}, \{w, q_1, q_2, q_3\}, \{c_{k_i,l_j}\}] \\ \oplus \sum_{i=1}^{3} \left[\frac{q_i}{w}\right] \left[\frac{w_{i,1}}{w_1}\right] \left[\frac{w_{i,2}}{w_2}\right] \left[\frac{w_{i,3}}{w_3}\right] [\{w, w_1, w_2, w_3\}, \{w, w_1, w_2, w_3\}, \{b_{k_i,l_j}\}] \\ = [\{w, q_1, q_2, q_3, w_{1,1}, w_{1,2}, w_{1,3}, w_{2,1}, w_{2,2}, w_{2,3}, w_{3,1}, w_{3,2}, w_{3,3}\}, \\ \{w, q_1, q_2, q_3, w_{1,1}, w_{1,2}, w_{1,3}, w_{2,1}, w_{2,2}, w_{2,3}, w_{3,1}, w_{3,2}, w_{3,3}\}, \{c_{k_i,l_j}\}]. \\ = [\{w, q_1, q_2, q_3\}, \{q_1, q_2, q_3, w_{1,1}, w_{1,2}, w_{1,3}, w_{2,1}, w_{2,2}, w_{2,3}, w_{3,1}, w_{3,2}, w_{3,3}\}, \{c_{k_i,l_j}\}]. \end{aligned}$$

Example 3. Let the ordered IFTree T_3, in Fig. 6, be given, and let P (see Fig. 2) be the new ordered IFTree to be inserted at vertex w of T_3. The resultant IFTree T_2^* is given in Fig. 7.

The IFTree T_3 has representation

$$T_3 = \begin{array}{c|cccc} & v_1 & v_2 & w & v_3 \\ \hline v_1 & \beta_{v_1,v_1} & \beta_{v_1,v_2} & \beta_{v_1,w} & \beta_{v_1,v_3} \\ v_2 & \beta_{v_2,v_1} & \beta_{v_2,v_2} & \beta_{v_2,w} & \beta_{v_2,v_3} \\ w & \beta_{w,v_1} & \beta_{w,v_2} & \beta_{w,w} & \beta_{w,v_3} \\ v_3 & \beta_{v_3,v_1} & \beta_{v_3,v_2} & \beta_{v_3,w} & \beta_{v_3,v_3} \end{array}$$

$$= \begin{array}{c|ccc} & v_2 & w & v_3 \\ \hline v_1 & \beta_{v,v_2} & \beta_{v_1,w} & \beta_{v_1,v_3} \end{array}.$$

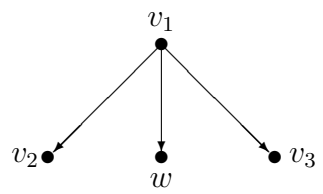

Fig. 6

The IM-form of the IFTree T_3^* is

$$\begin{aligned} T_3^* &= ([\{v_1, v_2, w, v_3\}, \{v_1, v_2, w, v_3\}, \{a_{k_i,l_j}\}] \ominus [\{w\}, \{w\}, \{0\}]) \\ &\oplus [\{w, q_1, q_2, q_3\}, \{w, q_1, q_2, q_3\}, \{c_{k_i,l_j}\}] \\ &= [\{v_1, v_2, w, v_3, q_1, q_2, q_3\}, \{v_1, v_2, w, v_3, q_1, q_2, q_3\}, \{d_{a,b}\}] \\ &= [\{v_1, w, \}, \{v_2, w, v_3, q_1, q_2, q_3\}, \{d_{a,b}\}]. \end{aligned}$$

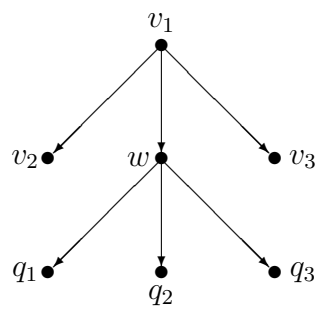

Fig. 7

3 Acknowledgement

The first and fourth authors would like to thank for the support provided by Department of Science and Technology, New Delhi, India and Ministry of Education and Science, Sofia, Bulgaria, Grant Ref. No. INT/Bulgaria/B-2/08 and BIn-02-09, “Design and development of intuitionistic fuzzy logic tools in information technologies”.

The third and fourth authors would like to thank for the support provided by Royal Society, Grant Ref. No. JP100372, “Generalised net models and intuitionistic fuzzy sets in intelligent systems”.

References

[1] Atanassov, K., Generalized index matrices, Comptes rendus de l'Academie Bulgare des Sciences, Vol. 40, 1987, No.11, 15–18.

[2] Atanassov, K., On index matrices, Part 1: Standard cases. Advanced Studies in Contemporary Mathematics, Vol. 20, 2010, No. 2, 291–302.

[3] Atanassov, K., On index matrices, Part 2: Intuitionistic fuzzy case. Proceedings of the Jangjeon Mathematical Society, Vol. 13, 2010, No. 2, 121–126.

[4] Atanassov K. On index matrix interpretations of intuitionistic fuzzy graphs. Notes on Intuitionistic Fuzzy Sets, Vol. 8 (2002), No. 4, 73–78.
http://ifigenia.org/wiki/issue:nifs/8/4/73-78

[5] Chountas, P., M. Alzebdi, A. Shannon, K. Atanassov, On intuitionistic fuzzy trees, Notes on Intuitionistic Fuzzy Sets, Vol. 15 (2009), No. 2, 30–32.
http://ifigenia.org/wiki/issue:nifs/15/2/30-32

[6] Chountas, P., A. Shannon, P. Rangasamy, K. Atanassov, On intuitionistic fuzzy trees and their index matrix interpretation. Notes on Intuitionistic Fuzzy Sets, Vol. 15 (2009), No. 4, 52–56.
http://ifigenia.org/wiki/issue:nifs/15/4/52-56

[7] Shannon, A., P. Rangasamy, K. Atanassov, P. Chountas, On intuitionistic fuzzy trees. Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics, Vol. I: Foundations, 2010, 177–184.