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Inequalities with intuitionistic fuzzy topological and Gökhan Cuvalcioĝlu's operators

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Abstract

In the present article the author will discuss some inequations with operators C_μ, C_ν, I_μ, I_ν, and G. Cuvalcioĝlu's operator $E_{\alpha,\beta}$.

Keywords: Inequality, Intuitionistic fuzzy set, Operator

Basic Concepts

Let set E be given. An intuitionistic fuzzy set (IFS, [1]) A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x) + \nu_A(x)\rangle | \ x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu A(x) + \nu A(x) \leq 1.$$

Gökhan Cuvalcioĝlu's operator [2] $E_{\alpha,\beta}$ is defined as follows:

$$E_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta.\nu_A(x) + 1 - \beta)\rangle | x \in E\},$$

Where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

In [3] Krassimir Atanassov introduced "closure" (C, C_μ and C_ν) and "intersection" (I, I_μ and I_ν) operators:

$$C(A) = \{\langle x, K, L\rangle | x \in E\},$$

$$I(A) = \{x, k, l\rangle | x \in E\},$$

$$C_\mu (A) = \{\langle x, K, \min(1 - K, \nu_A(x))\rangle | x \in E\},$$

$$C_\nu(A) = \{\langle x, \mu_A(x), L\rangle | x \in E\},$$

$$I_\mu(A) = \{\langle x, k, \nu_A(x)\rangle | x \in E\},$$

$$I_\nu(A) = \{\langle x, \min(1 - l, \mu_A(x)), l\rangle | x \in E\},$$

where

$$K = \sup_{y \in E} \mu_A(y), \quad L = \inf_{y \in E} \nu_A(y)$$

and

$$k = \inf_{y \in E} (y), \quad l = \sup_{y \in E} (y).$$

Main Results

Theorem 1: For every IFS A and for every $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$:

$$I_\nu E_{\alpha,\beta}(A) \supseteq E_{\alpha,\beta} I_\nu(A)$$

Proof: Let A be a given IFS. Then

$I_\nu E_{\alpha,\beta}(A) = I_\nu\{\langle x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \min(1 - \sup_{y\in E} (\beta(\alpha\nu_A(y) + 1 - \alpha)), \beta(\alpha\mu_A(x) + 1 - \alpha)), \sup_{y\in E} \alpha(\beta\nu_A(y) + 1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \min(1 - \sup_{y\in E} \alpha\beta\nu_A(y) - \beta(1 - \alpha), \alpha\beta\mu_A(x) + \beta(1 - \alpha)), \sup_{y\in E} \alpha\beta\nu_A(y) + \alpha(1 - \beta)\rangle | x \in E\}$

$=\{\langle x, \min(1 - \alpha\beta l - \alpha(1 - \beta), \alpha\beta\mu_A(x) + \beta(1 - \alpha)), \alpha\beta l + \alpha(1 - \beta)\rangle | x \in E\}$

and

$E_{\alpha,\beta} I_\nu(A) = E_{\alpha,\beta}\{\langle x, \min(1 - \sup_{y\in E} \nu_A(y), \mu_A(x)), \sup_{y\in E} \nu_A(y)\rangle | x \in E\}$

$= \{\langle x, \beta(\alpha\min(1 - \sup_{y\in E} \nu_A(y), \mu_A(x)) + 1 - \alpha), \alpha(\beta \sup_{y\in E} \nu_A(y) + 1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \min(\alpha\beta(1 - \sup_{y\in E} \nu_A(y)), \alpha\beta\mu_A(x)) + \beta(1 - \alpha), \alpha\beta \sup_{y\in E} \nu_A(y) + \alpha(1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \min(\alpha\beta(1 - l), \alpha\beta\mu_A(x) + \beta(1 - \alpha), \alpha\beta l + \alpha(1 - \beta)\rangle | x \in E\}$

To further study the relation between $I_\nu E_{\alpha,\beta}(A)$ and $E_{\alpha,\beta} I_\nu(A)$ their degrees of membership and non-membership should be compared, as it was defined in [1] that:

For every two IFSs A and B:

$$A \subset B \text{ iff } (\forall x \in E)\ (\mu_A(x) \leq \mu_B(x)\ \&\ \nu_A(x) \geq \nu_B(x).$$

It is obvious that the degree of non-membership of $I_\nu E_{\alpha,\beta}(A)$ is equal to that of $E_{\alpha,\beta} I_\nu(A)$.

Now we will compare the degrees of membership.

First, let us mention that for

$$X = \min(a,b) - \min(c,b).$$

If $a \geq b$, then $X = b - \min(b,c) \geq 0$.

If $a \leq b$, then $X = a - \min(b,c)$;

if $a \geq c$, then $X = a - \min(b,c) \geq 0$;

if $a \leq c$, then $X = a - \min(b,c) \leq 0$.

Therefore

$1 - \alpha\beta l - \beta(1 - \alpha) - \alpha\beta\mu - \beta(1 - \alpha)$

$= 1 - \alpha\beta l - \beta + \alpha\beta - \alpha\beta\mu - \beta + \alpha\beta$

$= 1 - \alpha\beta(1 - \inf\mu) - \alpha + \alpha\beta - \alpha\beta\mu - \beta + \alpha\beta$

$= 1 - \alpha\beta + \alpha\beta\inf\mu - \alpha + 2\alpha\beta - \alpha\beta\mu - \beta$

$= 1 - \alpha\beta(1 + \inf\mu - \mu) - \alpha - \beta$

$1 - \mu + \inf\mu \geq 0$, so

$1 + \alpha\beta(1 + \inf\mu - \mu) - \alpha - \beta \geq 0$

and this leads to the conclusion that

$I_\nu E_{\alpha,\beta}(A) \supseteq E_{\alpha,\beta} I_\nu$.

Theorem 2: For every IFS A and for every $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$:

$$I_\mu E_{\alpha,\beta}(A) = E_{\alpha,\beta} I_\mu(A).$$

Proof: Let A be a given IFS. Then:

$I_\mu E_{\alpha,\beta}(A) = I_\mu\{\langle x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \sup_{y\in E} \beta(\alpha\mu_A(y) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta)\rangle | x\in E\}$

$= \{\langle x, \alpha\beta \sup_{y\in E} \mu_A(y) + \beta(1 - \alpha), \alpha\beta\nu_A(x) + \alpha(1 - \beta)\rangle | x \in E\}$

$= \{\langle x, \beta(\alpha \sup_{y\in E} \mu_A(y) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta)\rangle | x \in E\}$

$= E_{\alpha,\beta}I_\mu(A) = E_{\alpha,\beta}\{\langle x, \sup_{y\in E} \mu_A(y), \nu_A(x)\rangle | x \in E\}$

Analogically we can prove:

Theorem 3: For every IFS A and for every $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$:

$$C_\mu E_{\alpha,\beta}(A) \subseteq E_{\alpha,\beta}C_\mu(A)$$

and

Theorem 4: For every IFS A and for every $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$:

$$C_\nu E_{\alpha,\beta}(A) = E_{\alpha,\beta}C_\nu(A).$$

References

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