Notes on Intuitionistic Fuzzy Sets
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A new type of intuitionistic fuzzy modal operators over intuitionistic fuzzy pairs

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Abstract: In the paper, two intuitionistic fuzzy modal operators from a new type are introduced over intuitionistic fuzzy pairs. Some of the basic properties of the new operators are formulated and checked.
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1 Introduction

The concept of an Intuitionistic Fuzzy Pair (IFP) was introduced in [4] independently with Z. Xu's paper [6], where he called the same object an intuitionistic fuzzy value. In a series of papers, a lot of operations, relations and operators are defined over IFPs (see, e.g., [2]). A part of them are given in Section 2 and they are used in Section 3, where two modal operators from a new type are introduced and some of their basic properties are given.

2 Preliminaries

In [4], the pair $\langle a, b\rangle$ so that $a, b, a + b \in [0, 1]$ is called an IFP. For two pairs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$, the following relations, operations and operators (and a lot of others) are introduced:

$$
\begin{aligned}
x \leq y &\text{ iff } \ a \leq c \ \text{ and } \ b \geq d, \\
x = y &\text{ iff } x \leq y \ \text{ and } \ y \leq x, \\
x \wedge y &= \langle \min(a,c), \max(b,d) \rangle, \\
x \vee y &= \langle \max(a,c), \min(b,d) \rangle, \\
x + y &= \langle a + c - ac, bd \rangle, \\
x.y &= \langle ac, b + d - bd \rangle, \\
x@y &= \langle \frac{a+c}{2}, \frac{b+d}{2} \rangle, \\
\neg x &= \langle b, a \rangle; \\
\Box x &= \langle a, 1-a \rangle, \\
\Diamond x &= \langle 1-b, b \rangle, \\
D_\alpha x &= \langle a + \alpha(1-a-b), b + (1-\alpha)(1-a-b) \rangle, \\
F_{\alpha,\beta} x &= \langle a + \alpha(1-a-b), b + \beta(1-a-b) \rangle, \\
G_{\alpha,\beta} x &= \langle \alpha a, \beta b \rangle, \\
H_{\alpha,\beta} x &= \langle \alpha a, b + \beta(1-a-b) \rangle, \\
H^*_{\alpha,\beta} x &= \langle \alpha a, b + \beta(1-\alpha a-b) \rangle, \\
J_{\alpha,\beta} x &= \langle a + \alpha(1-a-b), \beta b \rangle, \\
J^*_{\alpha,\beta} x &= \langle a + \alpha(1-a-\beta b), \beta b \rangle; \\
\boxdot_{\alpha,\beta,\gamma,\delta} x &= \langle \alpha a + \gamma, \beta b + \delta \rangle, \\
&\quad \text{where } \alpha,\beta,\gamma,\delta \in [0,1] \text{ and } \max(\alpha,\beta) + \gamma + \delta \leq 1, \\
\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} x &= \langle \alpha a - \varepsilon b + \gamma, \beta b - \zeta a + \delta \rangle, \\
&\quad \text{where } \alpha,\beta,\gamma,\delta,\varepsilon,\zeta \in [0,1], \max(\alpha-\zeta,\beta-\varepsilon) + \gamma + \delta \leq 1, \\
&\quad \text{and } \min(\alpha-\zeta,\beta-\varepsilon) + \gamma + \delta \geq 0, \\
\otimes_{\alpha,\beta,\gamma,\delta} x &= \langle \alpha a + \gamma b, \beta a + \delta b \rangle, \\
&\quad \text{where } \alpha,\beta,\gamma,\delta \in [0,1] \text{ and } \alpha + \beta \leq 1, \gamma + \delta \leq 1.
\end{aligned}
$$

3 Main results

Everywhere below we will use:

$$
\begin{aligned}
x &= \langle a, b \rangle, \\
y &= \langle c, d \rangle.
\end{aligned}
$$

Let us define the following two new operators:

$$
\begin{aligned}
\triangle x &= \triangle \langle a, b \rangle = \langle a + b, 0 \rangle, \\
\triangledown x &= \triangledown \langle a, b \rangle = \langle 0, a + b \rangle.
\end{aligned}
$$

The geometrical interpretation of both operators is shown on Fig. 1.

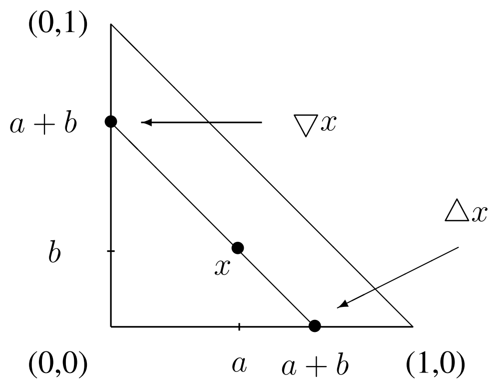

Figure 1. Geometrical interpretation of an element of an IFS.

We see immediately that

$$\bigtriangleup\langle 1,0\rangle = \bigtriangleup\langle 0,1\rangle = \langle 1,0\rangle,$$

$$\bigtriangledown\langle 1,0\rangle = \bigtriangledown\langle 0,1\rangle = \langle 0,1\rangle,$$

$$\bigtriangledown x \leq x \leq \bigtriangleup x.$$

The new operators have the following properties.

Proposition 1. *For each IFP x:*

(a) $\bigtriangleup \bigtriangledown x = \bigtriangleup x$,

(b) $\bigtriangledown \bigtriangleup x = \bigtriangledown x$.

Proof. (a) Let x be an IFP. Then

$$\bigtriangleup \bigtriangledown x = \bigtriangleup \bigtriangledown \langle a,b\rangle = \bigtriangleup\langle 0, a+b\rangle = \langle a+b, 0\rangle = \bigtriangleup x.$$

(b) The assertion is proved analogically. $\square$

Therefore, there is an important difference in the behaviour of the new modal operators and the intuitionistic fuzzy forms of the classical modal operators, that satisfy equalities $\square \lozenge x = \lozenge x$ and $\lozenge \square x = \square x$ (cf [2, 5]).

Between the two types of modal operators there are the following relationships.

Proposition 2. *For each IFP x:*

(a) $\square \bigtriangleup x \leq \bigtriangleup \square x$,

(b) $\square \bigtriangledown x = \bigtriangledown \square x$,

(c) $\lozenge \bigtriangleup x = \bigtriangleup \lozenge x$,

(d) $\lozenge \bigtriangledown x \geq \bigtriangledown \lozenge x$.

Proof. We check directly that:

$$\Box \bigtriangleup x = \Box\langle a+b,0\rangle = \langle a+b,1-a-b\rangle \leq \langle 1,0\rangle = \bigtriangleup\langle a,1-a\rangle = \bigtriangleup \Box x,$$
$$\Box \bigtriangledown x = \Box\langle 0,a+b\rangle = \langle 0,1\rangle = \bigtriangledown\langle a,1-a\rangle = \bigtriangledown \Box x,$$
$$\Diamond \bigtriangleup x = \Diamond\langle a+b,0\rangle = \langle 1,0\rangle = \bigtriangleup\langle a,1-a\rangle = \bigtriangleup \Box x,$$
$$\Diamond \bigtriangledown x = \Diamond\langle 0,a+b\rangle = \langle 1-a-b,a+b\rangle \geq \langle 0,1\rangle = \bigtriangledown\langle 1-b,b\rangle = \bigtriangledown \Diamond x.$$

This completes the proof. □

In [2] it is shown that operator D_α extends both operators $\Box$ and $\Diamond$ because for each IFP x: $\Box x = D_0$ and $\Diamond x = D_1 x$, while operator $F_{\alpha,\beta}$ extends operator D_α, because for each IFP x: $D_\alpha = f_{\alpha,1-\alpha}$. Now, an extension of Proposition 2 has the following form.

Proposition 3. *For each IFP x and for every $\alpha, \beta \in [0,1]$ so that $a+b \in [0,1]$:*

(a) $D_\alpha \bigtriangleup x \leq \bigtriangleup D_{al} x$,

(b) $D_\alpha \bigtriangledown x \geq \bigtriangledown D_{al} x$,

(c) $F_{\alpha,\beta} \bigtriangleup x \leq \bigtriangleup F_{al,\beta} x$,

(d) $F_{\alpha,\beta} \bigtriangledown x \geq \bigtriangledown F_{al,\beta} x$.

More interesting is the case of the relationships between the new operators and operator $G_{\alpha,\beta}$.

Proposition 4. *For each IFP x and for every $\alpha, \beta \in [0,1]$:*

(a) *if $\alpha \leq \beta$, then*

$$G_{\alpha,\beta} \bigtriangleup x \leq \bigtriangleup G_{al,\beta} x,$$
$$G_{\alpha,\beta} \bigtriangledown x \geq \bigtriangledown G_{al,\beta} x,$$

(b) *if $\alpha \geq \beta$, then*

$$G_{\alpha,\beta} \bigtriangleup x \geq \bigtriangleup G_{al,\beta} x,$$
$$G_{\alpha,\beta} \bigtriangledown x \leq \bigtriangledown G_{al,\beta} x.$$

Proposition 5. *For each IFP x and for every $\alpha, \beta \in [0,1]$ so that $a+b \in [0,1]$:*

(a) $H_{\alpha,\beta} \bigtriangleup x \leq \bigtriangleup H_{al,\beta} x$,

(b) $H_{\alpha,\beta} \bigtriangledown x \geq \bigtriangledown H_{al,\beta} x$,

(c) $J_{\alpha,\beta} \bigtriangleup x \geq \bigtriangleup J_{al,\beta} x$,

(d) $J_{\alpha,\beta} \bigtriangledown x \leq \bigtriangledown J_{al,\beta} x$,

(e) $H^*_{\alpha,\beta} \bigtriangleup x \leq \bigtriangleup H^*_{al,\beta} x$,

(f) $H^*_{\alpha,\beta} \bigtriangledown x \geq \bigtriangledown H^*_{al,\beta} x$,

(g) $J^*_{\alpha,\beta} \triangle x \geq \triangle J^*_{al,\beta} x$,

(h) $J^*_{\alpha,\beta} \bigtriangledown x \leq \bigtriangledown J^*_{al,\beta} x$.

Proposition 6. *For each IFP x:*

(a) $\neg \triangle \neg x = \bigtriangledown x$,

(b) $\neg \bigtriangledown \neg x = \triangle x$,

The relationships between the new operators and some of the operations over IFPs are the following.

Proposition 7. *For every two IFPs x and y:*

(a) $\triangle(x + y) \leq \triangle x + \triangle y$,

(b) $\triangle(x.y) \geq \triangle x \,.\, \triangle y$,

(c) $\triangle(x@y) = \triangle x \,@\, \triangle y$,

(d) $\bigtriangledown(x + y) \geq \bigtriangledown x + \bigtriangledown y$,

(e) $\bigtriangledown(x.y) \leq \bigtriangledown x \,.\, \bigtriangledown y$,

(f) $\bigtriangledown(x@y) = \bigtriangledown x \,@\, \bigtriangledown y$.

4 Conclusion

The so introduced intuitionistic fuzzy modal operators over IFPs can be modified (and this will be an object of a nexr research of the author) for the case of Intuitionistic Fuzzy Sets (IFSs, see, e.g., [1]) and interval-valued IFSs (see, e.g., [3]).

The new operators can have application in different areas as Data Mining, Decision making, intercriteria analysis and others.

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