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# On decomposition of intuitionistic fuzzy prime submodules

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**Abstract:** This article is in continuation of the first author's previous paper on intuitionistic fuzzy prime submodules, [13]. In this paper, we explore the decomposition of intuitionistic fuzzy submodule as the intersection of finite many intuitionistic fuzzy prime submodules. Many other forms of decomposition like irredundant and normal decomposition are also investigated. **Keywords:** Intuitionistic fuzzy prime ideal (submodule), Residual quotient, Intuitionistic fuzzy prime decomposition, Irredundant and normal decomposition.

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# **1** Introduction

Prime ideals play a central role in commutative ring theory. One of the natural generalizations of prime ideals which have attracted the interest of several authors is the notion of prime submodules (see for example [1, 2, 7, 8] and [9]). These have led to more information on the structure of the R-module M. A proper submodule P of M is called *prime* if  $r \in R$  and  $x \in M$ , with  $rx \in P$  implies that  $r \in (P : M)$  or  $x \in P$ , where  $(P : M) = \{r \in R : rM \subseteq N\}$ , which is clearly an ideal of R. Also, if P is a prime submodule of M, then (P : M) is a prime ideal of R.

The decomposition of an ideal (submodule) into prime ideal (prime submodule) is a traditional pillar of ideal (module) theory. It provides the algebraic foundation for decomposing an algebraic variety into its irreducible components. From another point of view, prime decomposition provides a generalization of the factorization of an integer as a product of prime-powers. In this paper, we study intuitionistic fuzzy prime decomposition, irredundant intuitionistic fuzzy prime decomposition.

# 2 Preliminaries

Throughout the paper, R is a commutative ring with unity  $1, 1 \neq 0, M$  is a unitary R-module and  $\theta$  is the zero element of M.

**Definition 2.1** ([3]). Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ , where the functions  $\mu_A : X \to [0,1]$  and  $\nu_A : X \to [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set A, respectively, and  $0 \le \mu_A(x) + \nu_A(x) \le 1$  for each  $x \in X$ .

#### Remark 2.2.

(i) When  $\mu_A(x) + \nu_A(x) = 1$ , i.e.,  $\nu_A(x) = 1 - \mu_A(x) = \mu_{A^c}(x)$ . Then A is called a fuzzy set.

(ii) The class of intuitionistic fuzzy subsets of X is denoted by IFS(X).

For  $A, B \in IFS(X)$  we say  $A \subseteq B$  if and only if  $\mu_A(x) \le \mu_B(x)$  and  $\nu_A(x) \ge \nu_B(x)$  for all  $x \in X$ . Also,  $A \subset B$  if and only if  $A \subseteq B$  and  $A \neq B$ .

The following are two very basic definitions given in [5,6] and [11].

**Definition 2.3.** Let  $A \in IFS(R)$ . Then A is called *intuitionistic fuzzy ideal* (*IFI*) of R if for all  $x, y \in R$ , the followings are satisfied

- (i)  $\mu_A(x-y) \ge \mu_A(x) \land \mu_A(y);$
- (ii)  $\mu_A(xy) \ge \mu_A(x) \lor \mu_A(y);$
- (iii)  $\nu_A(x-y) \le \nu_A(x) \lor \nu_A(y);$
- (iv)  $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$ .

**Definition 2.4.** Let  $A \in IFS(M)$ . Then A is called *an intuitionistic fuzzy module* (IFM) of M if for all  $x, y \in M, r \in R$ , the followings are satisfied

- (i)  $\mu_A(x-y) \ge \mu_A(x) \land \mu_A(y);$
- (ii)  $\mu_A(rx) \ge \mu_A(x);$
- (iii)  $\mu_A(\theta) = 1;$
- (iv)  $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y);$

(v) 
$$\nu_A(rx) \leq \nu_A(x);$$

(vi)  $\nu_A(\theta) = 0.$ 

Let IFM(M) denote the set of all intuitionistic fuzzy R-modules of M and IFI(R) denote the set of all intuitionistic fuzzy ideals of R. We note that when R = M, then  $A \in IFM(M)$  if and only if  $\mu_A(\theta) = 1, \nu_A(\theta) = 0$  and  $A \in IFI(R)$ .

**Definition 2.5** ([11]). For  $A, B \in IFS(M)$  and  $C \in IFS(R)$ , define the residual quotient (A : B) and (A : C) as follows:

$$(A:B) = \bigcup \{ D: D \in IFS(R) \text{ such that } D \cdot B \subseteq A \}$$

and

$$(A:C) = \bigcup \{ E: E \in IFS(M) \text{ such that } C \cdot E \subseteq A \}$$

**Theorem 2.6** ([11]). For  $A, B \in IFS(M)$  and  $C \in IFS(R)$ . Then, we have:

(i)  $(A:B) \cdot B \subseteq A;$ (ii)  $C \cdot (A:C) \subseteq A;$ (iii)  $C \cdot B \subseteq A \Leftrightarrow C \subseteq (A:B) \Leftrightarrow B \subseteq (A:C).$ 

**Theorem 2.7** ([11]). For  $A_i(i \in J), B \in IFS(M)$  and  $C \in IFS(R)$ . Then, we have:

- (i)  $(\bigcap_{i\in J} A_i : B) = \bigcap_{i\in J} (A_i : B);$
- (ii)  $(\bigcap_{i \in J} A_i : C) = \bigcap_{i \in J} (A_i : C).$

**Theorem 2.8** ([11]). For  $A, B \in IFS(M)$  and  $C \in IFS(R)$ ,

- (i) if  $A \in IFM(M)$ , then  $(A : B) = \bigcup \{D : D \in IFI(R) \text{ such that } D \cdot B \subseteq A\}$ ;
- (ii) if  $C \in IFI(R)$ , then  $(A : C) = \bigcup \{E : E \in IFS(M) \text{ such that } C \cdot E \subseteq A\}$ .

**Theorem 2.9** ([11]). For  $A, B \in IFM(M)$  and  $C \in IFI(R)$ . Then  $(A : B) \in IFI(R)$  and  $(A : C) \in IFM(M)$ .

**Theorem 2.10** ([11]). For  $A, B_i \in IFS(M)$  and  $C_i \in IFS(R), (i \in J)$ . Then, we have:

- (i)  $(A:\bigcup_{i\in J}B_i)=\bigcap_{i\in J}(A:B_i);$
- (ii)  $(A:\bigcup_{i\in J}C_i)=\bigcap_{i\in J}(A:C_i).$

**Definition 2.11** ([4,12]). An intuitionistic fuzzy ideal P of a ring R, not necessarily nonconstant, is called *intuitionistic fuzzy prime ideal*, if for any intuitionistic fuzzy ideals A and B of R the condition  $AB \subseteq P$  implies that either  $A \subseteq P$  or  $B \subseteq P$ .

**Definition 2.12** ([13]). A non-constant intuitionistic fuzzy submodule A of M is said to be prime if for  $C \in IFI(R)$  and  $D \in IFM(M)$  such that  $C \cdot D \subseteq A$  then either  $D \subseteq A$  or  $C \subseteq (A : \chi_M)$ . The set of intuitionistic fuzzy prime submodules of M is denoted by IF - Spec(M).

**Theorem 2.13** ([13]). Let A be an intuitionistic fuzzy prime submodule of M. Then  $A^*$  is a prime submodule of M, where  $A^* = \{x \in M : \mu_A(x) > 0 \text{ and } \nu_A(x) < 1\}$  is the support of A.

**Theorem 2.14** ([13]). If M = R, then  $B \in IFM(M)$ , is an intuitionistic fuzzy prime submodule of M if and only if B is an intuitionistic fuzzy prime ideal.

**Theorem 2.15** ([13]). (a) Let N be a prime submodule of M and  $\alpha, \beta \in (0, 1)$  such that  $\alpha + \beta < 1$ . If A is an IFS of M defined by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in N \\ \alpha, & \text{if otherwise} \end{cases}; \quad \nu_A(x) = \begin{cases} 0, & \text{if } y \in N \\ \beta, & \text{otherwise} \end{cases}$$

for all  $x \in M$ . Then A is an intuitionistic fuzzy prime submodule of M. (b) Conversely, any intuitionistic fuzzy prime submodule can be obtained as in (a).

**Proposition 2.16** ([13]). Let A, B be two intuitionistic fuzzy prime submodules of M. Then  $A \cap B$  is also an intuitionistic fuzzy prime submodule of M.

**Corollary 2.17** ([13]). If  $A_i (i \in J)$  are intuitionistic fuzzy prime submodules of M. Then  $\bigcap_{i \in J} A_i$  is also an intuitionistic fuzzy prime submodule of M.

**Theorem 2.18** ([13]). If  $B \in IFS(M)$  and  $A \in IF - Spec(M)$ , then:

- (i) if  $B \subseteq A$ , then  $(A : B) = \chi_R$ ; and
- (ii) if  $B \nsubseteq A$ , then  $(A : B) = (A : \chi_M)$ .

**Theorem 2.19** ([13]). Let  $A \in IFM(M)$  and  $C \in IFI(R)$ . If A is an intuitionistic fuzzy prime submodule of M, then:

- (i) if  $C \nsubseteq (A : \chi_M)$ , then (A : C) = A; and
- (ii) if  $C \subseteq (A : \chi_M)$ , then  $(A : C) = \chi_M$ .

**Theorem 2.20** ([13]). Let A be an intuitionistic fuzzy prime submodule of M and  $B \in IFM(M)$ . If  $(A : B) \neq \chi_R$ , then (A : B) is an intuitionistic fuzzy prime ideal of R.

**Corollary 2.21** ([13]). If A is an intuitionistic fuzzy prime submodule of M, then  $(A : \chi_M)$  is an intuitionistic fuzzy prime ideal of R.

**Theorem 2.22** ([13]). Let  $A \in IFM(M)$  and  $C \in IFI(R)$ . If A is an intuitionistic fuzzy prime submodule of M and  $(A : C) \neq \chi_M$ , then (A : C) is an intuitionistic fuzzy prime submodule of M.

**Corollary 2.23** ([13]). If A is an intuitionistic fuzzy prime submodule of M. then  $(A : \chi_R)$  is an intuitionistic fuzzy prime submodule of M.

# **3** Decomposition of intuitionistic fuzzy prime submodules

**Definition 3.1.** Let  $A \in IFM(M)$ . A decomposition of A as a finite intersection  $A = \bigcap_{i=1}^{n} A_i$  of intuitionistic fuzzy prime submodules of M is called *intuitionistic fuzzy prime decomposition* of A and the intuitionistic fuzzy prime ideals  $\{(A_i : \chi_M) | i = 1, 2, ..., n\}$  are called the set of associated intuitionistic fuzzy prime ideals of A.

An intuitionistic fuzzy prime decomposition  $A = \bigcap_{i=1}^{n} A_i$  is called *irredundant* if no  $A_i$  contains  $\bigcap_{j=1, j\neq i}^{n} A_j$ , and an irredundant intuitionistic fuzzy prime decomposition of A is called *normal* if the distinct  $A_i$  have distinct associated intuitionistic fuzzy prime ideals.

**Definition 3.2.** An intuitionistic fuzzy prime submodule  $A_i$  in the normal prime decomposition  $A = \bigcap_{i=1}^{n} A_i$  is called *isolated* if the associated intuitionistic fuzzy prime ideal  $(A_i : \chi_M)$  is minimal in the set of associated intuitionistic fuzzy prime ideals of A.

**Theorem 3.3.** If  $A_i$ , (i = 1, 2, 3, ..., n) are intuitionistic fuzzy prime submodules of M, then  $(\bigcap_{i=1}^n A_i : \chi_M)$  is an intuitionistic fuzzy prime ideal of R.

*Proof.* Since  $A_i$ , (i = 1, 2, 3, ..., n) are intuitionistic fuzzy prime submodules of M, by Corollary 2.17,  $\bigcap_{i=1}^n A_i$  is also an intuitionistic fuzzy prime submodule of M. Also by Corollary 2.21,  $(\bigcap_{i=1}^n A_i : \chi_M)$  is an intuitionistic fuzzy prime ideal of R.

**Theorem 3.4.** Let  $A \in IFM(M)$ , and  $A = \bigcap_{i=1}^{n} A_i$  be an irredundant intuitionistic fuzzy prime decomposition of A, where  $A_i$  are intuitionistic fuzzy prime submodules of M. Then an intuitionistic fuzzy prime ideal  $C \in \{(A_i : \chi_M) | i = 1, 2, ..., n\}$  if and only if there exist  $B \in IFM(M)$  such that (A : B) = C. Hence the set of intuitionistic fuzzy prime ideals  $\{(A_i : \chi_M) | i = 1, 2, ..., n\}$  is independent of the particular irredundant intuitionistic fuzzy prime decomposition of A.

*Proof.* Let  $A \in IFM(M)$ , and  $A = \bigcap_{i=1}^{n} A_i$  be an irredundant intuitionistic fuzzy prime decomposition of A, where  $A_i$  are intuitionistic fuzzy prime submodules of M. Now for any  $B \in IFM(M)$ , we have

$$(A:B) = (\bigcap_{i=1}^{n} A_i:B) = \bigcap_{i=1}^{n} (A_i:B)$$

Then by Theorem 2.18. and Corollary 2.21., we have  $(A_i : B) = \chi_R$  if  $B \subseteq A_i$  and  $(A_i : B)$  is an intuitionistic fuzzy prime ideal of R if  $B \nsubseteq A_i$ . Hence  $(A_i : B) \in IFI(R)$ . Thus

$$(A:B) = \bigcap_{i=1}^{n} (A_i:B) = \bigcap_{j=1}^{m} (A_{s_j}:\chi_M),$$

where the intersection is taken over those  $s_j$  such that  $B \not\subseteq A_{s_j}$ .

Also (A : B) is an intuitionistic fuzzy prime ideal of R. We suppose that (A : B) = C. Then by [6, Theorem 3.18], we get

$$C = \bigcap_{j=1}^{m} (A_{s_j} : \chi_M) \supseteq (A_{s_1} : \chi_M) (A_{s_2} : \chi_M) \cdots (A_{s_m} : \chi_M)$$

As (A : B) = C is an intuitionistic fuzzy prime ideal of R, so  $C \supseteq (A_{s_j} : \chi_M)$  for some  $s_j$ . Also  $(A : B) \subseteq (A_{s_j} : \chi_M)$ . It follows that  $C = (A_{s_j} : \chi_M)$ .

Next consider any one of the associated intuitionistic fuzzy ideal  $(A_i : \chi_M)$  of  $A = \bigcap_{i=1}^n A_i$ . Let  $B = \bigcap_{j=1, j \neq i}^n A_j$ . Then, we have:

$$(A:B) = ((\bigcap_{k=1}^{n} A_k) : (\bigcap_{j=1, j \neq i}^{n} A_j)) = \bigcap_{k=1}^{n} (A_k : \bigcap_{j=1, j \neq i}^{n} A_j),$$
 by Theorem 2.7.

As  $\bigcap_{j=1, j\neq i}^{n} A_j \subseteq A_j, \forall j, j \neq i$  implies  $(A_j : \bigcap_{j=1, j\neq i}^{n} A_j) = \chi_R$ , by Theorem 2.18.

By the irredundancy of the set of  $A_i$ , we have  $\bigcap_{j=1, j\neq i}^n A_j \nsubseteq A_i$ . Thus by Theorem 2.18. we get  $(A_i : \bigcap_{j=1, j\neq i}^n A_j) = (A_i : \chi_M)$ . Therefore,  $(A : B) = (A_i : \chi_M)$ .

Hence the set of intuitionistic fuzzy prime ideals  $\{(A_i : \chi_M) | i = 1, 2, ..., n\}$  is independent of the particular irredundant intuitionistic fuzzy prime decomposition of A.

**Theorem 3.5.** Let  $A \in IFM(M)$ . If A has an intuitionistic fuzzy prime decomposition, then A has a normal intuitionistic fuzzy prime decomposition.

*Proof.* We assume that A has an intuitionistic fuzzy prime decomposition  $A = \bigcap_{i=1}^{n} A_i$ . If  $A_{i_1}, A_{i_2}, \ldots, A_{i_k} \in \{A_1, A_2, \ldots, A_n\}$  are such that  $(A_{i_1} : \chi_M) = (A_{i_2} : \chi_M) = \cdots = (A_{i_k} : \chi_M)$ , let  $A_i = \bigcap_{j=1}^{k} A_{i_j}$ . Then  $A_i$  is an intuitionistic fuzzy prime submodule of M and  $(A_i : \chi_M) = (A_i : \chi_M)$ , by Corollary 2.17. Thus  $A = A_1 \cap A_2 \cap \cdots \cap A_m$ , where the  $A_i$  have distinct associated intuitionistic fuzzy prime ideals. If  $A_i \supseteq \bigcap_{j=1, j \neq i}^{m} A_j$ , for some i, then  $A_i$  is deleted. Therefore A has a normal intuitionistic fuzzy prime decomposition.

**Theorem 3.6.** Let  $A \in IFM(M)$ . Suppose that  $A = \bigcap_{i=1}^{n} A_i$  is a normal intuitionistic fuzzy prime decomposition of A. Then there exists a finite set  $\{(A_i : \chi_M) | i = 1, 2, ..., m\}, m \leq n$ , where the  $(A_i : \chi_M)$  are minimal in the set of associated intuitionistic fuzzy prime ideals of  $A = \bigcap_{i=1}^{n} A_i$ , such that  $(A : \chi_M) = \bigcap_{i=1}^{m} (A_i : \chi_M)$  and  $(A : (\bigcup_{i=1}^{m} (A_i : \chi_M))) = A$  when  $m \geq 2$ .

*Proof.* Suppose that  $A = \bigcap_{i=1}^{n} A_i$  is a normal intuitionistic fuzzy prime decomposition of A. Then

$$(A:\chi_M) = (\bigcap_{i=1}^n A_i:\chi_M) = \bigcap_{i=1}^n (A_i:\chi_M)$$
, by Theorem 2.7.

Let C be an intuitionistic fuzzy prime ideal of R such that  $C \supseteq (A : \chi_M)$ . Then

$$C \supseteq \bigcap_{i=1}^{n} (A_i : \chi_M) \supseteq (A_1 : \chi_M) (A_2 : \chi_M) \cdots c(A_n : \chi_M)$$
, by [6, Theorem 3.18.]

So  $C \supseteq (A_i : \chi_M)$  for some *i*. Thus *C* contains some  $(A_i : \chi_M)$  that is minimal among  $(A_1 : \chi_M), (A_2 : \chi_M), \dots, (A_n : \chi_M)$ . Hence if we select those  $(A_i : \chi_M)$  in  $\{(A_1 : \chi_M), (A_2 : \chi_M), \dots, (A_n : \chi_M)\}$  that are minimal and reindex, then we have

$$(A:\chi_M) = \bigcap_{i=1}^m (A_i:\chi_M).$$

If  $m \ge 2$ , then  $(A : \bigcup_{i=1}^{m} (A_i : \chi_M)) = \bigcap_{i=1}^{m} (A_i : (A_i : \chi_M))$ , by Theorem 2.10.

As  $(A_i : \chi_M) \nsubseteq (A : \chi_M) = \bigcap_{i=1}^m (A_i : (A_i : \chi_M)), (A : (A_i : \chi_M)) = A, \forall i \in \{1, 2, ..., n\},$ by Theorem 2.19.

Hence  $A = (A : \bigcup_{i=1}^{m} (A_i : \chi_M)).$ 

**Theorem 3.7.** Let  $A = \bigcap_{i=1}^{n} A_i$  be a normal intuitionistic fuzzy prime decomposition of A and  $A_i$  be isolated intuitionistic fuzzy prime submodules of M. Then

$$A = (A: \bigcap_{j=1, j\neq i}^n (A_j: \chi_M)), \forall i = 1, 2, \dots, n.$$

*Proof.* Since  $(A_1 : \chi_M) \cdots (A_{i-1} : \chi_M) (A_{i+1} : \chi_M) \cdots (A_n : \chi_M) \subseteq \bigcap_{j=1, j \neq i}^n (A_j : \chi_M)$ , it follows from the normality of  $(A_i : \chi_M)$  that  $\bigcap_{j=1, j \neq i}^n (A_j : \chi_M) \notin (A_i : \chi_M)$  and hence

$$\bigcap_{j=1,j\neq i}^{n} (A_j:\chi_M) \not\subseteq \bigcap_{j=1}^{n} (A_j:\chi_M) = (A:\chi_M).$$

Thus by Theorem 2.18., we have  $(A : \bigcap_{j=1, j \neq i}^{n} (A_j : \chi_M)) = A, \forall i = 1, 2, ..., n.$ 

**Example 3.8.** Let G be any finite abelian group of order  $n = p_1 p_2 \dots p_k$ , where  $p_i$  are distinct primes. Then by the structure theorem of finitely generated group we have

$$G \cong Z_{p_1} \oplus Z_{p_2} \oplus \dots \oplus Z_{p_k}$$

Take M = G, then M is a  $\mathbb{Z}$ -module. Let  $M = \langle x_1, x_2, \dots, x_k \rangle$  such that  $o(x_i) = p_i$ , for  $1 \leq i \leq k$ . Let  $M_0 = \langle 0 \rangle$ ,  $M_1 = \langle x_1 \rangle$ ,  $M_2 = \langle x_1, x_2 \rangle, \dots, M_k = \langle x_1, x_2, \dots, x_k \rangle = M$  be the chain of maximal submodules of M such that  $M_0 \subset M_1 \subset \cdots \subset M_{k-1} \subset M_k$ .

Let A be any intuitionistic fuzzy submodule of M defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in M_0 \\ \alpha_1 & \text{if } x \in M_1 \backslash M_0 \\ \alpha_2 & \text{if } x \in M_2 \backslash M_1 \\ \vdots \\ \alpha_k & \text{if } x \in M_k \backslash M_{k-1} \end{cases}; \nu_A(x) = \begin{cases} 0 & \text{if } x \in M_0 \\ \beta_1 & \text{if } x \in M_1 \backslash M_0 \\ \beta_2 & \text{if } x \in M_2 \backslash M_1 \\ \vdots \\ \beta_k & \text{if } x \in M_k \backslash M_{k-1}. \end{cases}$$

where  $1 = \alpha_0 \ge \alpha_1 \ge \cdots \ge \alpha_k$  and  $0 = \beta_0 \le \beta_1 \le \cdots \le \beta_k$  and the pair  $(\alpha_i, \beta_i)$  are called double pins and the set  $\wedge(A) = \{(\alpha_0, \beta_0), (\alpha_1, \beta_1), \dots, (\alpha_k, \beta_k)\}$  is called the set of double pinned flags for the IFSM A of M (see [10, Theorem 3.4]).

Define IFSs  $A_i$  on M as follows:

$$\mu_{A_i}(x) = \begin{cases} 1, & \text{if } x \in  \\ \alpha_{i+1}, & \text{if otherwise} \end{cases}; \quad \nu_{A_i}(x) = \begin{cases} 0, & \text{if } x \in  \\ \beta_{i+1}, & \text{if otherwise} \end{cases}$$

where  $\alpha_i, \beta_i \in (0, 1)$  such that  $\alpha_i + \beta_i \leq 1$ , for  $1 \leq i \leq k$  and  $\alpha_{k+1} = \alpha_1, \beta_{k+1} = \beta_1$ . Clearly,  $A_i$  are intuitionistic fuzzy prime submodules of M. It can be easily checked that  $A = \bigcap_{i=1}^n A_i$  is an intuitionistic fuzzy prime decomposition of A.

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