Fifth International Workshop on IFSs
Banska Bystrica, Slovakia, 19 Oct. 2009
NIFS 15 (2009), 4, 9-14

On Intuitionistic Fuzzy Subtraction, Related to Intuitionistic Fuzzy Negation $\neg_{11}$

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1 On intuitionistic fuzzy versions of operation "negation"

During the last four years 34 different versions of operation "negation" were introduced over the Intuitionistic Fuzzy Sets (IFS, see [1]). First, following [4], we will give the definitions of these "negation" operations.

In some of these definitions we shall use functions sg and $\overline{\text{sg}}$:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

The negations have the following forms (see, e.g., [4]):

$$\neg_1 A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_2 A = \{\langle x, \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_3 A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^2\rangle | x \in E\},$$

$$\neg_4 A = \{\langle x, \nu_A(x), 1 - \nu_A(x)\rangle | x \in E\},$$

$$\neg_5 A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), \mathrm{sg}(1 - \nu_A(x))\rangle | x \in E\},$$

$$\neg_6 A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), \mathrm{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_7 A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_8 A = \{\langle x, 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_9 A = \{\langle x, \overline{\mathrm{sg}}(\mu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_{10} A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), 1 - \nu_A(x)\rangle | x \in E\},$$

$$\neg_{11} A = \{\langle x, \mathrm{sg}(\nu_A(x)), \overline{\mathrm{sg}}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{12} A = \{\langle x, \nu_A(x).(\mu_A(x) + \nu_A(x)), \mu_A(x).(\mu_A(x) + \nu_A(x)^2)\rangle | x \in E\},$$

$$\neg_{13} A = \{\langle x, \mathrm{sg}(1 - \nu_A(x)), \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{14} A = \{\langle x, \mathrm{sg}(\nu_A(x)), \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{15} A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{16} A = \{\langle x, \overline{\mathrm{sg}}(\mu_A(x)), \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{17} A = \{\langle x, \overline{\mathrm{sg}}(1 - \nu_A(x)), \overline{\mathrm{sg}}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{18} A = \{\langle x, \nu_A(x).\mathrm{sg}(\mu_A(x)), \mu_A(x).\mathrm{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{19} A = \{\langle x, \nu_A(x).\mathrm{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{20} A = \{\langle x, \nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{21} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^n\rangle | x \in E\},$$

where real number $n \in [2, \infty)$,

$$\neg_{22} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{23} A = \{\langle x, (1 - \mu_A(x)).\mathrm{sg}(\mu_A(x)), \mu_A(x).\mathrm{sg}(1 - \nu_A(x))\rangle | x \in E\},$$

$$\neg_{24} A = \{\langle x, (1 - \mu_A(x)).\mathrm{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{25} A = \{\langle x, 1 - \nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{26} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{27} A = \{\langle x, 1 - \mu_A(x), \mu_A(x).(1 - \mu_A(x)) + \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{28} A = \{\langle x, \nu_A(x), (1 - \nu_A(x)).\nu_A(x) + \overline{\mathrm{sg}}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{29} A = \{\langle x, \max(0, \nu_A(x).\mu_A(x) + \overline{\mathrm{sg}}(1 - \nu_A(x))),$$
$$\min(1, \mu_A(x).(\nu_A(x).\mu_A(x) + \overline{\mathrm{sg}}(1 - \nu_A(x))) + \overline{\mathrm{sg}}(1 - \mu_A(x)))\rangle | x \in E\},$$

$$\neg_{30} A = \{\langle x, \nu_A(x).\mu_A(x),$$
$$\mu_A(x).(\nu_A(x).\mu_A(x) + \overline{\mathrm{sg}}(1 - \nu_A(x))) + \overline{\mathrm{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{31}A = \{\langle x, \max(0, (1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))),$$
$$(\min(1, \mu_A(x).((1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1-\mu_A(x)))\rangle | x \in E\},$$
$$\neg_{32}A = \{\langle x, (1-\mu_A(x)).\mu_A(x),$$
$$\mu_A(x).((1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$
$$\neg_{33}A = \{\langle x, \max(0, ((\nu_A(x).(1-\nu_A(x))) + \overline{\text{sg}}(1-\nu_A(x)))),$$
$$(\min(1, (((1-\nu_A(x)).((\nu_A(x).(1-\nu_A(x))) + \overline{\text{sg}}(1-\nu_A(x)))) + \overline{\text{sg}}(\nu_A(x)))))\rangle | x \in E\},$$
$$\neg_{34}A = \{\langle x, \nu_A(x).(1-\nu_A(x)),$$
$$(1-\nu_A(x)).(\nu_A(x).(1-\nu_A(x)) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(\nu_A(x))\rangle | x \in E\}.$$

Second, we will mention that in [5, 6] two versions of operation "subtraction" were defined.

Let the IFSs

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

be given (for the description of their components see [1]). Then

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}.$$

In [3] a series of new versions of operation "subtraction" was introduced. As a basis, the well-known formula from set theory:

$$A - B = A \cap \neg B$$

was used. In the IFS-case, we also can define the operation "subtraction" by:

$$A -'_i B = A \cap \neg_i B, \tag{1}$$

where $i = 1, 2, ..., 34$. On the other hand, as we discussed in [2], the Law for Excluding Middle is not always valid in IFS theory. By this reason we can introduce a new series of "subtraction" operations, that will have the form:

$$A -''_i B = \neg\neg A \cap \neg_i B, \tag{2}$$

where $i = 1, 2, ..., 34$.

Of course, for every two IFSs A and B it will be valid that:

$$A -'_1 B = A -''_1 B,$$

because the first negation will satisfy the Law for Excluding Middle, but in the rest cases this equality will not be valid.

In [3] the properties of negation $\neg_2$ and the generated by it two IF-subtractions were studied.

Below we will make a new step of the research on the new IF-operations, discussing the properties of two new IF-subtractions: $-'_{11}$ and $-''_{11}$, that are related to the IF-subtractions: $-'_2$ and $-''_2$.

2 Basic properties of operation $-'_{11}$

Using (1), we obrain the following form of the operation $-'_{11}$:

$$A -'_{11} B = \{\langle x, \min(\mu_A(x), \text{sg}(\nu_B(x))), \max(\nu_A(x), \overline{\text{sg}}(\nu_B(x)))\rangle | x \in E\}.$$

First, we have to check that in a result of the operation we obtain an IFS. Really, for two given IFSs A and B and for each $x \in E$ we obtain that if $\nu_B(x) = 0$, then

$$\min(\mu_A(x), \text{sg}(\nu_B(x))) + \max(\nu_A(x), \overline{\text{sg}}(\nu_B(x)))$$

$$= \min(\mu_A(x), 0) + \max(\nu_A(x), 0) \leq 1;$$

if $\nu_B(x) \geq 0$, then

$$\min(\mu_A(x), \text{sg}(\nu_B(x))) + \max(\nu_A(x), \overline{\text{sg}}(\nu_B(x)))$$

$$= \min(\mu_A(x), 1) + \max(\nu_A(x), 0) = \mu_A(x) + \nu_A(x) \leq 1.$$

Let us define the *empty IFS*, the *totally uncertain IFS*, and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

By analogy, we can prove the following assertions.

Theorem 1: For every two IFSs A and B:

(a) $A -'_{11} E^* = O^*$,

(b) $A -'_{11} O^* = A$,

(c) $E^* -'_{11} A = \neg_{11} A$,

(d) $O^* -'_{11} A = O^*$,

(e) $(A -'_{11} B) \cap C = (A \cap C) -'_{11} B = A \cap (C -'_{11} B)$,

(f) $(A \cap B) -'_{11} C = (A -'_{11} C) \cap (B -'_{11} C)$,

(g) $(A \cup B) -'_{11} C = (A -'_{11} C) \cup (B -'_{11} C)$,

(h) $(A -'_{11} B) -'_{11} C = (A -'_{11} C) -'_{11} B$.

Obviously

$$O^* -'_{11} U^* = O^*,\ O^* -'_{11} E^* = O^*,\ U^* -'_{11} O^* = U^*,$$

$$U^* -'_{11} E^* = O^*,\ E^* -'_{11} O^* = E^*,\ E^* -'_{11} U^* = O^*.$$

3 Basic properties of operation $-''_{11}$

First, we shall note that for each real number x the equalities:

$$\overline{\text{sg}}(\overline{\text{sg}}(x)) = \text{sg}(x) \text{ and } \text{sg}(\overline{\text{sg}}(x)) = \overline{\text{sg}}(x)$$

hold. Now, using (2) and having in mind that

$$\neg_{11}\neg_{11}A = \neg_{11}\{\langle x, \mathrm{sg}(\nu_A(x)), \overline{\mathrm{sg}}(\nu_A(x))\rangle | x \in E\}$$
$$= \{\langle x, \mathrm{sg}(\overline{\mathrm{sg}}(\nu_A(x))), \overline{\mathrm{sg}}(\overline{\mathrm{sg}}(\nu_A(x)))\rangle | x \in E\}$$
$$= \{\langle x, \overline{\mathrm{sg}}(\nu_A(x)), \mathrm{sg}(\nu_A(x))\rangle | x \in E\},$$

we obtain the following form of the operation $-''_{11}$:

$$A -''_{11} B = \{\langle x, \min(\overline{\mathrm{sg}}(\nu_A(x)), \mathrm{sg}(\nu_B(x))), \max(\mathrm{sg}(\nu_A(x)), \overline{\mathrm{sg}}(\nu_B(x)))\rangle | x \in E\}.$$

The check that the result of the operation is an IFS and the proofs of the next assertions are similar to above ones.

Theorem 2: For every IFS A:
(a) $A -''_{11} E^* = O^*$,
(b) $A -''_{11} O^* = \neg_{11}\neg_{11}A$,
(c) $E^* -''_{11} A = \neg_{11}A$,
(d) $O^* -''_{11} A = O^*$,
(e) $(A \cap B) -''_{11} \neg_{11}\neg_{11}C = (A -''_{11} C) \cap (B -''_{11} C)$,
(f) $(A \cap B) -'_{11} C = (A -'_{11} C) \cap (B -'_{11} C)$,
(g) $(A \cup B) -'_{11} C = (A -'_{11} C) \cup (B -'_{11} C)$,
(h) $(A -''_{11} B) -''_{11} \neg_{11}\neg_{11}C = (A -''_{11} C) -''_{11} \neg_{11}\neg_{11}B$.

Obviously,

$$O^* -''_{11} U^* = O^*,\ O^* -''_{11} E^* = O^*,\ U^* -''_{11} O^* = U^*,$$
$$U^* -''_{11} E^* = O^*,\ E^* -''_{11} O^* = E^*,\ E^* -''_{11} U^* = O^*.$$

4 Conclusion

In the next author's research the properties of the separate versions of the operations "subtraction" will be discussed by the above manner.

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