Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy hollow submodules

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Abstract: In this paper, the notion of intuitionistic fuzzy hollow submodule of a module is defined. We attempt to investigate various properties of this module. A characterization of an intuitionistic fuzzy hollow module in terms of an intuitionistic fuzzy quotient modules is established. A relationship between a hollow submodule, and an indecomposable intuitionistic fuzzy submodule is also obtained. We also investigate the nature of equivalent conditions of intuitionistic fuzzy small submodules, and intuitionistic fuzzy hollow submodules.
Keywords: Intuitionistic fuzzy small submodule, Intuitionistic fuzzy quotient module, Intuitionistic fuzzy indecomposable module, Intuitionistic fuzzy hollow submodule.
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1 Introduction

One of the prominent extension of fuzzy sets theory given by Zadeh [16] in 1965 is the theory of "Intuitionistic Fuzzy Sets" coined by Atanassov [1, 2] in 1986. Biswas [4] was the first one to intuitionistic fuzzify the concept of subgroup of a group. Later on many mathematicians start intuitionistic fuzzifying the other concepts of algebraic structures, for example, the concept of intuitionistic fuzzy subring and ideals were introduced by Hur and others in [7, 8] and that of intuitionistic fuzzy submodule of a module by Davvan and others in [6, 9, 10, 12]. The notion of intuitionistic fuzzy essential (or large) submodules was introduced by Basnet in [12], whereas that of intuitionistic fuzzy superfluous (or small) submodules was introduced by the authors in [13]. In this paper, we will introduce and study the concept of intuitionistic fuzzy hollow submodules of a module and investigate various properties.

2 Preliminaries

For the sake of convenience we set our former concepts which will be used in this paper. Throughout the paper, R will be a commutative ring with unity $1, 1 \neq 0, M$ is a unitary R-module and θ is the zero element of M.

Definition 2.1 ([5]). A submodule S of a module M over a ring R is said to be a small submodule of M denoted by $S \ll M$, if for any submodule K of M, $S + K = M$
$\Rightarrow K = M$.

Proposition 2.2. *([5, 15]) Let M be a module and suppose that $K \leq N \leq M$ and $H \leq M$. Then*

(i) $N \ll M$ if and only if $K \ll M$ and $N/K \ll M/K$;

(ii) $H + K \ll M$ if and only if $H \ll M$ and $K \ll M$;

(iii) If $K \ll N$, then $K \ll M$;

(iv) If N is a direct summand of M, then $K \ll M$ if and only if $K \ll N$;

(v) If $M = M_1 \bigoplus M_2$ and $K_i \leq M_i$ for $i = 1, 2$, then $K_1 \bigoplus K_2 \ll M_1 \bigoplus M_2$ if and only if $K_1 \ll M_1$ and $K_2 \ll M_2$.

Definition 2.3 ([5]). A nonzero module M is said to be indecomposable if $\{\theta\}$ and M are the only direct summands of M.

Proposition 2.4 ([5, 15]). *If $K \ll M$ and M/K is indecomposable then M is indecomposable.*

Definition 2.5 ([15]). A R-module M is said to be hollow if, when N_1 and N_2 are submodules of M such that $N_1 + N_2 = M$, then either $N_1 = M$ or $N_2 = M$. Equivalently, M is called a hollow module if every proper submodule of M is a small submodule of M.

Definition 2.6 ([1]). Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Remark 2.7.

(i) When $\mu_A(x) + \nu_A(x) = 1$, i.e., when $\nu_A(x) = 1 - \mu_A(x) = \mu_{A^c}(x), \forall x \in X$. Then A is called a fuzzy set.

(ii) We denote the IFS $A = \{\langle x, , \mu_A(x), \nu_A(x)\rangle | x \in X\}$ by $A = (\mu_A, \nu_A)$.

Definition 2.8 ([1, 2]). Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs of X. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$.

(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in X\}$.

(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle | x \in X\}$.

Definition 2.9 ([6, 9, 10, 12])**.** Let M be a module over a ring R. An IFS $A = (\mu_A, \nu_A)$ of M is called an intuitionistic fuzzy submodule (IFSM) if

(i) $\mu_A(\theta) = 1, \nu_A(\theta) = 0$, where θ is the zero element of M;

(ii) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x + y) \leq \max\{\nu_A(x), \nu_A(y)\}$;

(iii) $\mu_A(rx) \geq \mu_A(x)$ and $\nu_A(rx) \leq \nu_A(x), \forall x, y \in M, r \in R$.

Condition (ii) and (iii) can be combined to a single condition
$\mu_A(rx+sy) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(rx+sy) \leq \max\{\nu_A(x), \nu_A(y)\}, \forall x, y \in M, r, s \in R$.

The set of intuitionistic fuzzy submodules of R-module M is denoted by $IFM(M)$.

Definition 2.10. ([13]) We define two IFSs $\chi_{\{\theta\}} = (\mu_{\chi_{\{\theta\}}}, \nu_{\chi_{\{\theta\}}})$ and $\chi_M = (\mu_{\chi_M}, \nu_{\chi_M})$ of R-module M as:

$$\mu_{\chi_{\{\theta\}}}(x) = \begin{cases} 1, & \text{if } x = \theta \\ 0, & \text{if } x \neq \theta \end{cases}; \ \nu_{\chi_{\{\theta\}}}(x) = \begin{cases} 0, & \text{if } x = \theta \\ 1, & \text{if } x \neq \theta \end{cases}, \text{ and } \mu_{\chi_M}(x) = 1; \nu_{\chi_M}(x) = 0, \forall x \in M.$$

Then it can be easily verified that $\chi_{\{\theta\}}, \chi_M \in IFM(M)$. These are called trivial IFSMs of module M. Any IFSM of module M other than these is called proper IFSM.

Definition 2.11 ([13])**.** Let $A = (\mu_A, \nu_A)$ be an IFS of X, then support of A is denoted by A^* and is defined as $A^* = \{x \in X : \mu_A(x) > 0 \text{ and } \nu_A(x) < 1\}$ and we denote the set $A_* = \{x \in X : \mu_A(x) = 1 \text{ and } \nu_A(x) = 0\}$.

By [13, Proposition (2.16)], if A is an IFSM of M, then A_* is a submodule of M.

Definition 2.12 ([13, 14])**.** Let $A, B \in IFM(M)$ be such that $A \subseteq B$. Then the quotient of B with respect to A is an IFSM of M/A^*, denoted by B/A, and is defined as $B/A(x + A^*) = (\mu_{B/A}(x + A^*), \nu_{B/A}(x + A^*))$, where
$\mu_{B/A}(x + A^*) = Sup\{\mu_B(x + y) : y \in A^*\}$ and $\nu_{B/A}(x + A^*) = Inf\{\nu_B(x + y) : y \in A^*\}$, where $x \in B^*$.

Lemma 2.13 ([13])**.** *Let $A \in IFM(M)$. Then $A_* = M$ if and only if $A = \chi_M$. Also, if $B \in IFM(M)$ such that $A \subseteq B$, then $A_* \subseteq B_*$.*

Lemma 2.14 ([13])**.** *Let $A, B \in IFM(M)$, then $(A \cap B)_* = A_* \cap B_*$, $(A \cup B)_* = A_* \cup B_*$. The results can be extended to infinite intersection and unions. Further, if A and B have finite pinned flag sets then $(A + B)_* = A_* + B_*$, where the sum of two IFSMs is defined as*
$\mu_{A+B}(x) = \vee_{x=a+b}\{\mu_A(a) \wedge \mu_B(b)\}$ and $\nu_{A+B}(x) = \wedge_{x=a+b}\{\nu_A(a) \vee \nu_B(b)\}$, where $x \in M$.

Lemma 2.15 ([13]). *If $A, B \in IFM(M)$. Then sum $A + B$ is called the direct sum of A and B if $A \cap B = \chi_{\{\theta\}}$ and is written as $A \oplus B$.*

Definition 2.16 ([13]). An IFSM $A(\neq \chi_{\{\theta\}})$ of M is said to be indecomposable IFSM if there does not exists IFSMs B and $C(\neq \chi_{\{\theta\}}, A)$ of M such that $A = B \oplus C$.

Definition 2.17. ([13], [14]) Let M be an M-module and $A \in IFM(M)$. Then A is said to be an intuitionistic fuzzy small (superfluous) submodule (IFSSM) of M, if for any $B \in IFM(M)$, $A + B = \chi_M \Rightarrow B = \chi_M$. It is denoted by the notation $A \ll_{IF} M$ or $A \ll_{IF} \chi_M$.

It is obvious that $\chi_{\{\theta\}}$ is always an IFSSM of M.

Let A and B be any two intuitionistic fuzzy submodules of M such that $A \subseteq B$, then A is called an intuitionistic fuzzy submodule of B. A is called an intuitionistic fuzzy small submodule in B, denoted by $A \ll_{IF} B$ or $A \ll_{IF} B^*$ in the sense that for every intuitionistic fuzzy submodule C of M satisfying $A|_{B^*} + C|_{B^*} = \chi_{B^*}$ implies that $C|_{B^*} = \chi_{B^*}$ (or $C|_{B^*} \neq \chi_{B^*}$ implies that $A|_{B^*} + C|_{B^*} \neq \chi_{B^*}$), where $A|_{B^*}, C|_{B^*}$ denote the restriction of A, C on B^* respectively.

Theorem 2.18 ([13]). *Let M be a module and $N \leq M$. Then $N \ll M$ if and only if $\chi_N \ll_{IF} M$.*

Theorem 2.19 ([13]). *Let $A \in IFM(M)$. Then $A \ll_{IF} M$ if and only if $A_* \ll M$.*

Theorem 2.20 ([13]). *Let $A, B \in IFM(M)$ with $A \subseteq B$. Then $A \ll_{IF} B$ if and only if $A_* \ll B_*$.*

Theorem 2.21 ([13]). *Let $A, B \in IFM(M)$ be such that $A \subseteq B$. Then $B \ll_{IF} M$ if and only if $A \ll_{IF} M$ and $B/A \ll_{IF} (\chi_M/A^*)$.*

3 Intuitionistic fuzzy hollow submodules

Definition 3.1. An intuitionistic fuzzy submodule B with $B_* \neq \{\theta\}$ of M is said to be an intuitionistic fuzzy hollow submodule, if for every submodule A of B with $A_* \neq B_*, A$ is an intuitionistic fuzzy small submodule of B. Also, a R-module $M \neq \{\theta\}$ is called an intuitionistic fuzzy hollow submodule if for every $A \in IFM(M)$ with $A_* \neq M$ implies that $A \ll_{IF} M$.

Example 3.2. Consider $M = \mathbf{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition modulo 8. Then M is a module over the ring $\mathbf{Z}$. Let $S = \{0, 2, 4, 6\}$. Define an IFS $B = (\mu_B, \nu_B)$ of M by

$$\mu_B(x) = \begin{cases} 1, & \text{if } x \in S \\ \alpha, & \text{otherwise} \end{cases}; \quad \nu_B(x) = \begin{cases} 0, & \text{if } x \in S \\ \beta, & \text{otherwise} \end{cases},$$

where $\alpha, \beta \in (0, 1]$ with $\alpha + \beta \leq 1$. Then B is an intuitionistic fuzzy submodule of M.

Let $K = \{0, 4\}$. Define an IFS $A = (\mu_A, \nu_A)$ of M by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in K \\ \alpha', & \text{otherwise} \end{cases}; \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in K \\ \beta', & \text{otherwise} \end{cases},$$

where $\alpha' \leq \alpha, \beta \leq \beta' \in [0, 1)$ with $\alpha' + \beta' \leq 1$.

Then clearly, B, A are the only intuitionistic fuzzy submodules of M with $A_*, B_* \neq M$, and A is the only intuitionistic fuzzy submodules of B with $A_* \neq B_*$. Also, it can be seen that B, A are intuitionistic fuzzy submodules of M, and A is an intuitionistic fuzzy small submodule of B. It follows that B is an intuitionistic fuzzy hollow submodule, and M is an intuitionistic fuzzy module.

Theorem 3.3. *A non-zero R-module M is an intuitionistic fuzzy hollow submodule if and only if for every $A, B \in IFM(M)$ with $A, B \neq \chi_M$ implies that $A + B \neq \chi_M$. .*

Proof. Straight forward, follows directly from the definition. □

Theorem 3.4. *A non-zero R-module M is a hollow module if and only if M is an intuitionistic fuzzy hollow module.*

Proof. Let M be a hollow module, and let A be an intuitionistic fuzzy submodule of with $A \neq \chi_M$. Then A_* is a submodule of M with $A_* \neq M$. Since M is a hollow module, $A_* \ll M$. Thus Theorem (2.19) implies that $A \ll_{IF} M$. Hence, M is an intuitionistic fuzzy hollow module.

Conversely, we assume that M is an intuitionistic fuzzy hollow module. Let N be a submodule of M such that $N \leq M$. Then χ_N ($\neq \chi_M$) is an intuitionistic fuzzy submodule of M. Since M is an intuitionistic fuzzy hollow module, $\chi_N \ll_{IF} M$ (it follows from Theorem (2.18)). This implies $N \ll M$ (Theorem (2.19)). □

Theorem 3.5. *Let $B \in IFM(M)$ be such that $B \neq \chi_{\{\theta\}}$. Then B is an intuitionistic fuzzy hollow submodule of M if and only if B_* is a hollow submodule of M.*

Proof. Let B be an intuitionistic fuzzy hollow submodule of M. To show that B_* is a hollow submodule of M. Let N be a proper submodule of B_*. Then $\chi_N \subset B$ with $(\chi_N)_* \neq B_*$. Since B is an intuitionistic fuzzy hollow submodule of M. So, we have $\chi_N \ll_{IF} B$ which is equivalent to $N \ll B_*$ (Theorem (2.20). Hence B_* is a hollow submodule of M.

Conversely, we assume that B_* is a hollow submodule of M. Let $A \in IFM(M)$ be such that $A \subseteq B$ and $A_* \neq B_*$. Then A_* is a proper submodule if B_* and so $A_* \ll B_*$. Therefore, we have $A \ll_{IF} B$ (Theorem (2.20)). Hence, B is an intuitionistic fuzzy hollow submodule of M. □

Theorem 3.6. *Every intuitionistic fuzzy hollow submodule is indecomposable.*

Proof. Let A be an intuitionistic fuzzy hollow submodule of a module M. If A is not indecomposable, then there exists $B, C \in IFM(M)$ with $B_*, C_*(\neq \chi_{\{\theta\}}, A)$ such that $A = B \bigoplus C$. This implies that $A_* = B_* \bigoplus C_*$ and $B_*, C_*(\neq A_*)$. This implies that A_* is not indecomposable. But, hollow submodules are indecomposable and by Theorem (3.5) A_* is a hollow submodule of M, and so A_* is indecomposable, contradiction. Therefore, A is an intuitionistic fuzzy hollow submodule of M. □

Corollary 3.7. *If M is an intuitionistic fuzzy hollow module, then χ_M is an indecomposable module.*

Theorem 3.8. *Let B be an intuitionistic fuzzy hollow submodule of M and let $A \in IFM(M)$ be such that $A \subseteq B$ with $A_* \subseteq B_*$. Then B/A is an intuitionistic fuzzy hollow submodule of M/A.*

Proof. Let $C \in IFM(M)$ be such that $A \subset C \subset B$ satisfying $(C/A)_* \neq (B/A)_*$. We claim that $C/A \ll_{IF} B/A$. Since $(C/A)_* \neq (B/A)_*$, $C_* \neq B_*$. Thus $C \subset B$ with $C_* \neq B_*$. Since B is an intuitionistic fuzzy hollow submodule, so we have $C \ll_{IF} B$. Thus $A \subset C \subset B$ with $C \ll_{IF} B$. So by Theorem (2.21) we get $C/A \ll_{IF} B/A$. Therefore, B/A is an intuitionistic fuzzy hollow submodule of M/A. □

Theorem 3.9. *Let $B \in IFM(M)$. Then B is an intuitionistic fuzzy hollow submodule of M if and only if every $A \in IFM(M)$ with $A \subset B$, and $A_* \neq B_*$, B/A is an intuitionistic fuzzy hollow submodule of M/A and $A \ll_{IF} B$.*

Proof. Let B be an intuitionistic fuzzy hollow submodule of M. Since $A \in IFM(M)$ with $A \subset B$ and $A_* \subset B_*$, $A \ll_{IF} B$ and so, by Theorem (3.8), we get B/A is an intuitionistic fuzzy hollow submodule of M/A.

Conversely, we assume that $A \ll_{IF} B$ and B/A is an intuitionistic fuzzy hollow submodule of M/A. Let $C \in IFM(M)$ with $C \subset B$ and $C_* \subset B_*$. Since $A \ll_{IF} B$ implies $A_* \ll B_*$ (Theorem (2.20)). Thus we have $A_* + C_* \neq B_*$. Since $A_* + C_* = (A+C)_* \neq B_*$. Thus $A + C \subset B$ with $(A+C)_* \neq B_*$. This implies that $((A+C)/A)_* \neq (B/A)_*$. Since B/A is an intuitionistic fuzzy hollow submodule and $(A+C)/A \subset B/A$. Therefore, we have

$$(A+C)/A \ll_{IF} B/A \tag{3.1}$$

Let $D \in IFM(M)$ be such that $C \mid_B^* + D \mid_B^* = \chi_{B^*}$. we have

$$(A + C \mid_B^* + D \mid_B^*)/A = \chi_{B^*}/A = \chi_{(B/A)^*} \tag{3.2}$$

Since $C \subset B$, so we have

$$(A+C)/A + (A + D \mid_B^*)/A = \chi_{(B/A)^*} \tag{3.3}$$

Also, since $A \subset B$, so we have

$$(A+C)/A + ((A+D)/A) \mid_{(B/A)^*} = \chi_{(B/A)^*} \tag{3.4}$$

Now, equation (3.1) and (3.4) together implies that $((A+D)/A) \mid_{(B/A)^*} = \chi_{(B/A)^*}$
$\Rightarrow (A + D \mid_B^*)/A = \chi_{(B/A)^*} = \chi_{(B)^*}/A \Rightarrow A + D \mid_B^* = \chi_{B^*} \Rightarrow D \mid_B^* = \chi_{B^*}$ (Since $A \ll_{IF} B$).

Thus, we have for $D \in IFM(M)$, with $C + D \mid_{B^*} = \chi_{B^*}$ implies that $D \mid_{B^*} = \chi_{B^*}$. Therefore, we have $C \ll_{IF} B$. This shows that B is an intuitionistic fuzzy hollow submodule of M. □

Theorem 3.10. *Let $B \in IFM(M)$. Then B is an intuitionistic fuzzy hollow submodule of M if and only if every $A \in IFM(M)$ with $A \subset B$ and $A_* \neq B_*, B/A$ is an intuitionistic fuzzy indecomposable submodule of M/A.*

Proof. Let B be an intuitionistic fuzzy hollow submodule of M. Let $A \in IFM(M)$ with $A \subset B$ and $A_* \neq B_*$. Then by Theorem (3.8) , B/A is an intuitionistic fuzzy hollow submodule of $M/A(= \chi_M/A_*)$. Thus, by Theorem (3.6), we have B/A is an intuitionistic fuzzy indecomposable submodule of M/A.

Conversely, we assume that for every $A \in IFM(M)$, with $A \subset B$ and $A_* \neq B_*$,
B/A is an intuitionistic fuzzy indecomposable submodule of M/A.
Let $C \in IFM(M)$ be such that $C \subset B$ with $C_* \neq B_*$. Since B/A is an intuitionistic fuzzy indecomposable submodule of M/A, and $C/A \subset B/A$ with $(C/A)_* \neq (B/A)_*$. So, for every $D \in IFM(M)$, $D \subset B$ with $D_* \neq B_*$ we have

$$C/A \bigoplus D/A \neq B/A \Rightarrow (C + D)/A \neq B/A \Rightarrow C + D \neq B \subset \chi_{B^*}.$$

Thus, $C + D \neq \chi_{B^*}$, and so $C \ll_{IF} B$. This implies that B is an intuitionistic fuzzy hollow submodule of M. □

As a consequence of Theorem (3.9) and (3.10), we have the following result.

Theorem 3.11. *Let $B \in IFM(M)$. Then the following statements are equivalent:*

1. *B is an intuitionistic fuzzy hollow submodule of M*
2. *For every $A \in IFM(M)$ with $A \subset B$ and $A_* \neq B_*$, B/A is an intuitionistic fuzzy hollow submodule of M/A and $A \ll_{IF} B$.*
3. *For every $A \in IFM(M)$ with $A \subset B$ and $A_* \neq B_*$, B/A is an intuitionistic fuzzy indecomposable submodule of M/A.*

4 Conclusions

In this paper, we have defined intuitionistic fuzzy hollow submodule of a module, and some of their properties were investigated. This may help toward the study of the intuitionistic fuzzy finite spanning, and intuitionistic fuzzy hollow dimension of a module which dualize the notion of Goldie, and uniform dimension of a module, respectively.

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