21st ICIFS, 22–23 May 2017, Burgas, Bulgaria
Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 23, 2017, No. 2, 79–87

A method for graphical representation of membership functions for intuitionistic fuzzy inference systems

Oscar Castillo*, Amaury Hernandez-Aguila and Mario Garcia-Valdez

Tijuana Institute of Technology, Tijuana BC México
* e-mail: ocastillo@tectijuana.mx

Received: 23 January 2017 **Revised:** 11 February 2017 **Accepted:** 21 February 2017

Abstract: This work proposes an approach for graphically representing intuitionistic fuzzy sets for their use in Mamdani fuzzy inference systems. The proposed approach is used, and plots for several membership and non-membership functions are presented, including: triangular, Gaussian, trapezoidal, generalized bell, sigmoidal, and left-right functions. Plots of some operators used in fuzzy logic are also presented, i.e., union, intersection, implication and alpha-cut operators. The proposed approach should produce plots that are clear to understand in the design of an intuitionistic fuzzy inference system, as the membership and non-membership functions are clearly separated and can be plotted in the same figure and still be recognized with ease.
Keywords: Fuzzy inference systems, Intuitionistic fuzzy logic, Membership function.
AMS Classification: 03E72.

1 Introduction

Fuzzy sets have been used as the building blocks of many other areas and applications since they were conceived by L. A. Zadeh in 1965 [23]. One of the most prominent areas is fuzzy logic and its wide area of application of control. As a consequence of its usefulness in the industry, many works have been dedicated to improving the architectures and theory used in the construction of control systems. A remarkable example of this is the extension of fuzzy sets to the concept of type-2 fuzzy sets by L. A. Zadeh in 1975 [24] and the extension from traditional fuzzy sets to intuitionistic fuzzy sets (IFS) by K. T. Atanassov in 1986 [3]. These extensions focus on increasing the uncertainty a fuzzy set can model, and as a consequence these fuzzy sets can help in the creation of better controls where the input data has high levels of noise. Specifically, type-2 fuzzy sets enable the membership of an element in a fuzzy set to be described with another fuzzy set, and IFSs enable the description of an element in terms of both membership and non-membership.

A common application of fuzzy sets is in inference systems, and these systems are called fuzzy inference systems (FIS). FISs use fuzzy sets to associate different degrees of membership to the inputs of the system and work as the antecedents of the inference system. Other fuzzy sets are used to model the consequents of the inference system (as in a Mamdani type FIS). Traditional fuzzy sets (also called type-1 fuzzy sets), are commonly defuzzified in order to obtain a scalar value as the output of the system, instead of a fuzzy set. When a FIS uses type-2 fuzzy sets as its antecedents and consequents, the output of the system needs to be reduced to a type-1 fuzzy set first, and then this type-1 fuzzy set is defuzzified to a scalar value.

One of the drawbacks of working with type-2 fuzzy sets in a FIS is that the type reduction procedure is very time consuming, due to the high quantity of steps involved in the process, and the use of a type-2 FIS (T2-FIS) is slower than the use of a type-1 FIS (T1-FIS). The type reduction and defuzzification processes are described by N. N. Karnik and J. M. Mendel in [14]. This drawback is a possible explanation of why many controllers still rely on the use of T1-FIS instead of T2-FIS. Furthermore, most of the T2-FIS use a special case of type-2 fuzzy sets called interval type-2 fuzzy sets, as these fuzzy sets require less steps in their type reduction procedure, and thus, interval type-2 FIS (IT2-FIS) are faster than a general type-2 FIS (GT2-FIS).

IFSs can also be used to construct a Mamdani type FIS, as in [13]. In contrast to a T2-FIS, an intuitionistic FIS (IFIS) does not require a type reduction procedure, and an implementation of an IFIS should work nearly as fast as a T1-FIS. The advantage of an IFIS over a T2-FIS is that the inference system can handle more uncertainty without a high penalty in time.

At the time of writing this paper, there aren't many works involving Mamdani type intuitionistic inference systems yet, and the authors of this work are only aware of the work by O. Castillo et al. [7], and A. Hernandez-Aguila and M. Garcia-Valdez [13]. As a consequence of this lack of works involving Mamdani FISs, there is not a common way to graphically represent IFS for its use as membership and non-membership functions for an IFIS. Additionally, there is not a common way to graphically represent the architecture of an IFIS, as the one presented in Matlab’s Fuzzy Logic Toolbox. Having a standardized way of representing these components of an IFIS should ease the description of such systems in future research.

This work proposes an approach to construct graphical representations of IFSs for their use in IFISs, which is described in detail in Section 4. Some preliminaries can be found in Section 2, which are needed in order to understand the proposed approach. In Section 3, one can find a number of related works, which describe other ways of representing IFSs. Finally, in Sections 5 and 6 one can find the conclusions and the future work.

2 Preliminaries

An IFS, as defined by K. T. Atanassov in [4], is represented by a capital letter with superscript star. An example of this notation is $A*$. The definition of an IFS is also defined in [4], and is described in Equation (1). In this equation, x is an element in set $A*$, $\mu_{A*}(x)$ is the membership of x, and $\nu_{A*}(x)$ is the non-membership of x. For every triplet in A^*, (2) must be satisfied.

$$A* = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E\} \tag{1}$$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{2}$$

An IFS is a generalization of a traditional fuzzy set, meaning that a traditional fuzzy set can be expressed using the terminology of an IFS. An example of this is found in (3):

$$A* = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in E\}. \tag{3}$$

If $0 \leq \mu_A(x) + \nu_A(x) < 1$ is true for an IFS, it is said that indeterminacy exists in the set. This concept of indeterminacy can also be found in the literature as hesitancy or non-determinacy, and it is described in equation (4):

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x). \tag{4}$$

3 Related works

The most common approach to graphically representing an IFS is by lattices. Examples of this type of representation can be found in the works by I. Despi et al. [12], and G. Deschrijver et al. [11]. This is a popular approach to graphically represent an IFS as it enables more compact and concise mathematical expressions. Another representation that is suitable for mathematical processes is that of a matrix, and is discussed in detail in the works by R. Parvathi et al. [16], G. Çuvalcioglu et al. [9], and S. Yilmaz et al. [22].

IFSs have been graphically represented like membership functions are ussually represented in Mamdani FISs, and some example works are the ones by P. Angelov [2], K. T. Atanassov [4], and H. Davarzani and M. A. Khorheh [10]. This notation can be suitable for representing an architecture of an IFIS, but if the plot is in black and white, or in grayscale, the reader can get confused by the membership and non-membership plots. This problem can be alleviated by plotting the membership and non-membership functions in separate plots, is in the works by O. Castillo et al. [7], and M. Akram et al. [1].

There are several other graphical representations of IFSs, such as by radar charts, as in the work by V. Atanassova [5], and by geometrical representations, orthogonal projections and three-dimensional representations, as can be found in the work by E Szmidt and J. Kacprzyk [19].

Some applications of IFSs in the area of medical sciences can be found in the works by E. Szmidt and J. Kacprzyk [20], C. M. Own [15], and D. D. Chakarska and L. S. Antonov [8]. In the area of group decision making, we have an example in the work by Z. Xu [21]. IFSs have also been used in word recognition, in the area of artificial vision, as in the example work of L. Baccour et al. [6].

This work proposes that IFSs, in a Mamdani IFIS, should follow an approach similar to that found in the work by K. T. Atanassov [4], where the membership is plotted as is commonly done in a traditional FIS, but the non-membership function should be plotted as $1 - \nu_A$. The reason behind this decision is that the non-membership function should be easily differentiated from the membership function, while seeing both functions in the same plot. An implementation of an IFIS that uses this approach for representing IFSs for a Mamdani IFIS can be found in the work by A. Hernandez-Aguila and M. Garcia-Valdez [13].

4 Proposed approach

What follows is a series of graphical representations of several commonly used membership functions in FISs, as well as graphical representations of common operators used in the construction of these systems, such as the union, intersection, and implication between two IFSs.

In Figure 1, one can see how a traditional fuzzy set can be constructed using the proposed approach. A Gaussian membership function with mean of 50 and a standard deviation of 15 is depicted.

Figure 2 is the first case of an IFS that cannot be considered a traditional fuzzy set. The red line represents the membership function, while the blue line represents the non-membership function. As can be seen, the Gaussian membership function does not have a kernel, meaning that its highest valued member does not equal to 1. In this case, its highest valued member equals to 0.7, and for the non-membership function, its highest valued member equals to 0.3. The Gaussian membership function is constructed with a mean of 50 and a standard deviation of 15. For the non-membership function, it is constructed with a mean of 30 and standard deviation of 30.

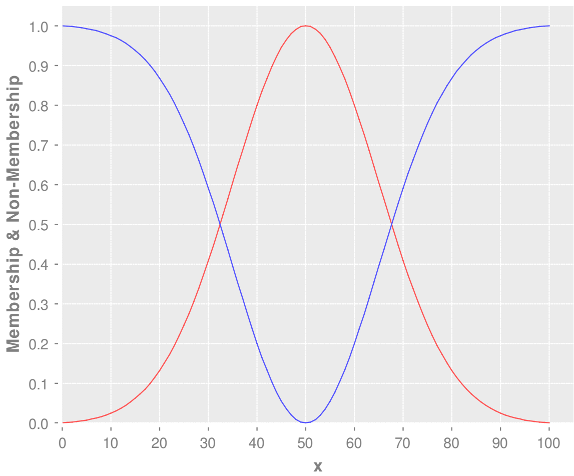

Figure 1. A traditional fuzzy set represented as an intuitionistic fuzzy set

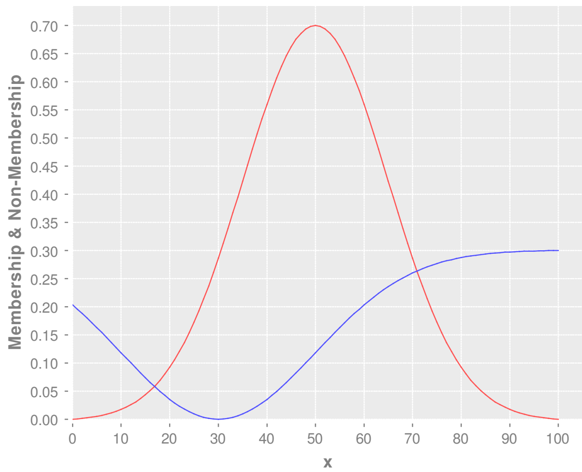

Figure 2. Example of an intuitionistic fuzzy set

The triangular membership function is depicted in Figure 3. The membership function is constructed with the following points: 30, 50, and 80, meaning that, from left to right, the last 0 valued member is at 30, the first and only 1 valued member is at 50, and the first 0 valued member after the previous series of non-0 valued members is at 80. In the same fashion, the non-membership function is constructed with the following points: 40, 60, and 80. The highest valued member for the membership function is equal to 0.8, and for the non-membership function it equals to 0.2.

The trapezoidal membership function is shown in Figure 4. The membership function is constructed using the following points: 20, 40, 60, and 80. These points mean that, from left to right, the first non-0 valued member will be at 20, and a line will be drawn from 20 to 40. All members from 40 to 60 will have a value of 1, and then a line will be drawn to 80. In the same fashion, the non-membership function is drawn using the following points: 40, 60, 80, and 100. The highest valued members in the membership function equal to 0.6, while in the non-membership function these members equal to 0.4.

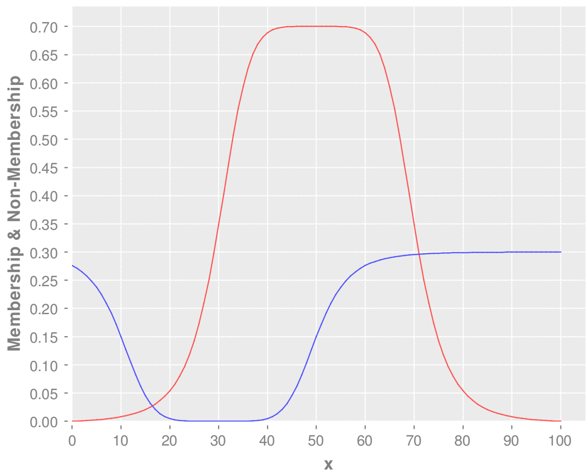

Figure 3. Example of triangular membership and non-membership functions

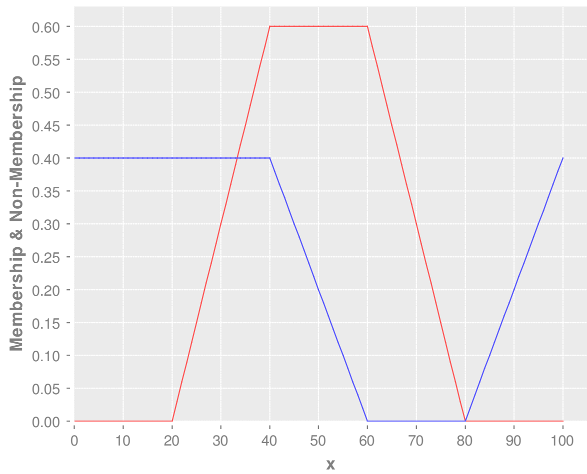

Figure 4. Example of trapezoidal membership and non-membership functions

In Figure 5 a generalized bell membership function is plotted. The membership function is constructed with a center of 50, a width of 20, and the parameter that determines the roundness of the corners of the bell is set at 3. Considering (5), which is the equation to generate a generalized bell, the membership function would be constructed with a = 20, b = 3, c = 50. In the case of the non-membership function, it would be constructed with a = 20, b = 3, c = 30. The highest valued member in the membership function is equal to 0.7, while the highest valued member in the non-membership function is equal to 0.3.

$$f(x;a,b,c)=\frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}. \tag{5}$$

Figure 6 shows an example of a left-right membership function and a non-membership function. Considering (6), the membership function is constructed with the following parameters: c = 50, α = 10, β = 65, and the non-membership function is constructed with the same parameters. Both the membership and the non-membership functions have highest valued members that equal to 1, and in this case we are depicting a traditional fuzzy set represented as an IFS.

$$LF(x;a,b,c) = \begin{cases} F_L\left(\dfrac{c-x}{\alpha}\right), & x \le c \\ \hline F_L\left(\dfrac{x-c}{\beta}\right), & x \ge c \end{cases}. \tag{6}$$

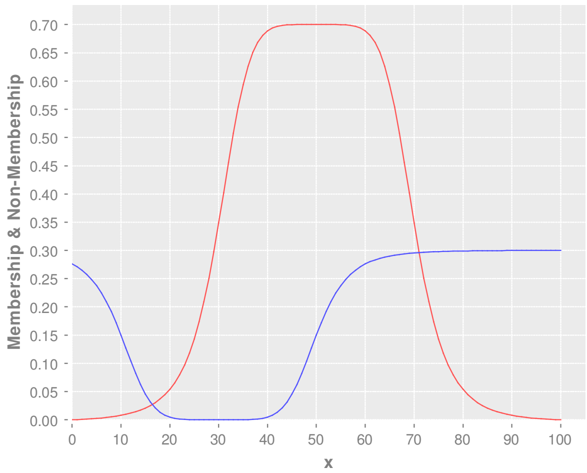

Figure 5. Example of generalized bell membership and non-membership functions

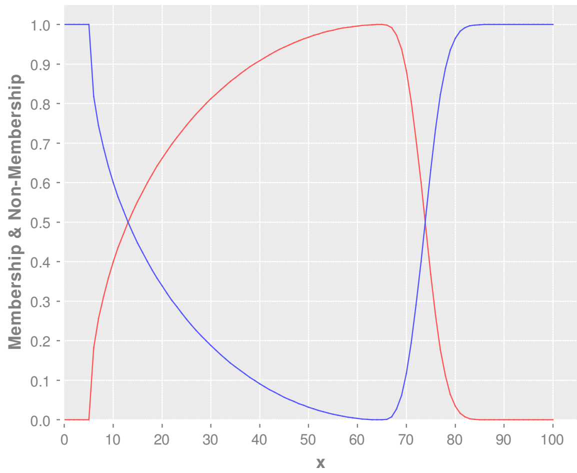

Figure 6. Example of left-right membership and non-membership functions

The last membership function presented in this Section is the sigmoidal membership function. Figure 7 presents a sigmoidal membership function that is constructed with the equation presented in (7). The membership and non-membership functions are constructed by using the same parameters, which are a = 0.3, b = 50. As in the example of the left-right membership and non-membership functions, we are depicting a traditional fuzzy set represented as an intuitionistic fuzzy set.

$$\mathrm{sig}(x;a,c) = \frac{1}{1+\exp[-a(x-c)]}. \tag{7}$$

Figure 8 shows the result of applying the union operator between two IFSs. Figure 9 shows the result of the intersection operator, and Figure 10 the result of the implication operator, both applied to two IFSs.

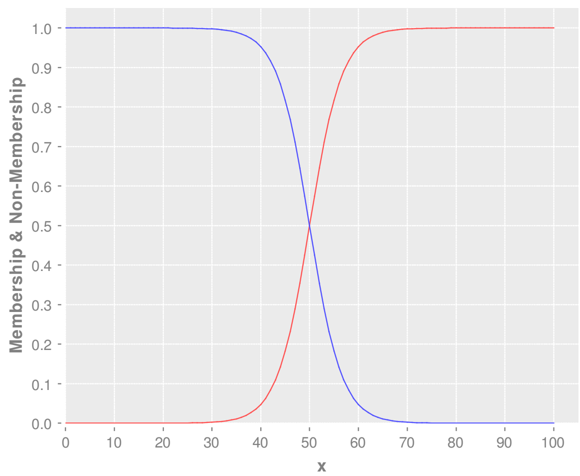

Figure 7. Example of sigmoidal membership and non-membership functions

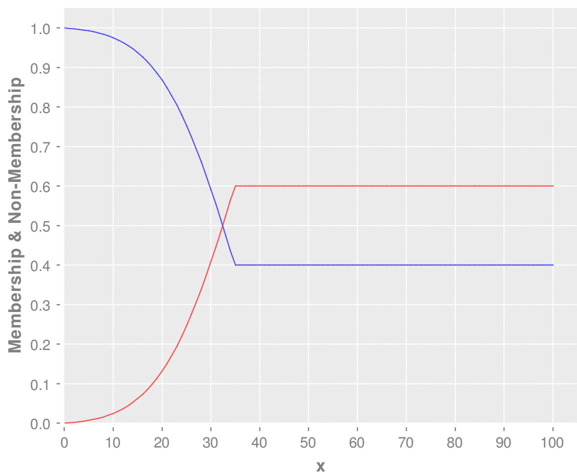

Figure 8. Example of the union of two intuitionistic fuzzy sets

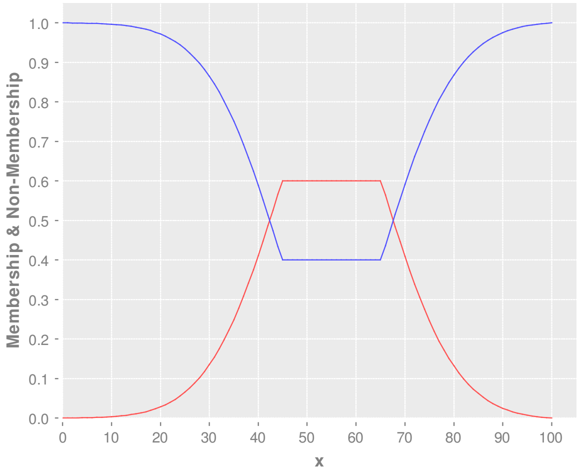

Figure 9. Example of the intersection of two intuitionistic fuzzy sets

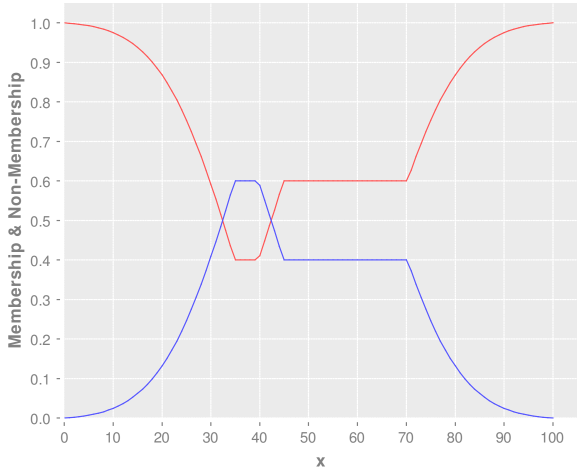

Figure 10. Example of implication of two intuitionistic fuzzy sets

Lastly, Figure 11 shows an example of an alpha-cut performed over Gaussian membership and non-membership functions. Some other works, as in the ones by P. K. Sharma [17, 18], describe procedures for performing an alpha cut to an IFS. Nevertheless, the authors of the present work could not find works that show a graphical representation of an alpha cut IFS.

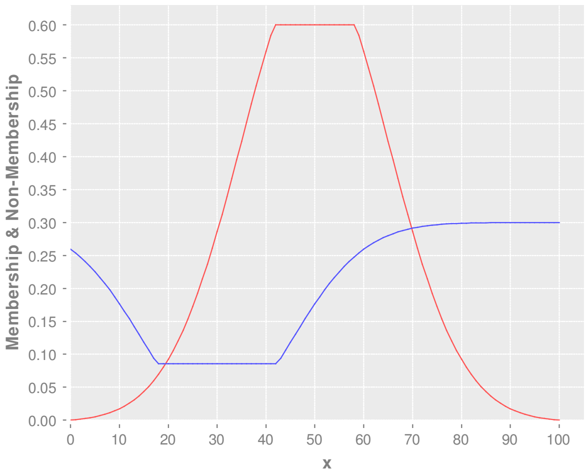

Figure 11. Example of an alpha cut of an intuitionistic fuzzy set

5 Conclusion

This work proposes an approach to graphically represent IFSs. The approach is focused on providing plots where the membership and the non-membership functions are easily recognized, and they can conveniently be used in IFISs. The work presents plots of the most common membership functions, along with their non-membership functions, and this way the reader can then decide if the use of this approach is convenient to represent the antecedents and consequents in the construction of an IFIS.

References

[1] Akram, M., Habib, S., & Javed, I. (2014). Intuitionistic fuzzy logic control for washing machines. *Indian Journal of Science and Technology*, 7(5), 654–661.

[2] Angelov, P. (1995). Crispification: defuzzification over intuitionistic fuzzy sets. *Bulletin for Studies and Exchanges on Fuzziness and its Applications (BUSEFAL)*, 64, 51–55.

[3] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.

[4] Atanassov K. T. (2003). Intuitionistic fuzzy sets: past, present and future. In *EUSFLAT Conf.*, 12–19.

[5] Atanassova, V. (2010). Representation of fuzzy and intuitionistic fuzzy data by Radar charts. *Notes on Intuitionistic Fuzzy Sets*, 16(1), 21–26.

[6] Baccour, L., Kanoun, S., Maergner, V., & Alimi, A. M. (2008). An application of intuitionistic fuzzy information for handwritten Arabic word recognition. Notes *on Intuitionistic Fuzzy Sets*, 14(2), 67–72.

[7] Castillo, O., Alanis, A., Garcia, M., & Arias, H. (2007). An intuitionistic fuzzy system for time series analysis in plant monitoring and diagnosis. *Applied Soft Computing*, 7(4), 1227–1233.

[8] Chakarska, D. D., & Antonov, L. S. (1995). Application of intuitionistic fuzzy sets in plant tissue culture and in invitro selection. *Notes on Intuitionistic Fuzzy Sets*, 16(1), 70–73.

[9] Cuvalcioglu G., Yilmaz S., & Bal, A. (2015). Some algebraic properties of the matrix representation of intuitionistic fuzzy modal operators. *Notes on Intuitionistic Fuzzy Sets*, 21(2), 6–18.

[10] Davarzani, H., & Khorheh, M. A. (2013). A novel application of intuitionistic fuzzy sets theory in medical science: Bacillus colonies recognition. *Artificial Intelligence Research*, 2(2), 1–17.

[11] Deschrijver, G., Cornelis, C., & Kerre, E. E. (2004). On the representation of intuitionistic fuzzy t-norms and t-conorms. *IEEE Transactions on Fuzzy Systems*, 12(1), 45–61.

[12] Despi, I., Opris, D., & Yalcin, E. (2013). Generalised Atanassov intuitionistic fuzzy sets. *Proceeding of the Fifth International Conference on Information, Process, and Knowledge Management*, France, 51–56.

[13] Hernandez-Aguila, A., & Garcia-Valdez, M. (2016). A proposal for an intuitionistic fuzzy inference system. *IEEE World Congress on Computational Intelligence*. (To be published).

[14] Karnik, N. N., & Mendel, J. M. (2001). Centroid of a type-2 fuzzy set. *Information Sciences*, 132(1), 195–220.

[15] Own, C. M. (2009). Switching between type-2 fuzzy sets and intuitionistic fuzzy sets: an application in medical diagnosis. *Applied Intelligence*, 31, 283–291.

[16] Parvathi, R., Thilagavathi, S., Thamizhendhi, G., & Karunambigai, M. G. (2014). Index matrix representation of intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets*, 20(2), 100–108.

[17] Sharma, P. K. (2011). Cut of intuitionistic fuzzy groups. *International Mathematical Forum*, 6(53), 2605–2614.

[18] Sharma, P. K. (2011). Cut of intuitionistic fuzzy modules. *International Journal of Mathematical Sciences and Applications*, 1(3), 1489–1492.

[19] Szmidt, E., & Kacprzyk, J. (2000). Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 114(3), 505–518.

[20] Szmidt, E., & Kacprzyk, J. (2001). Intuitionistic fuzzy sets in some medical applications. In Computational Intelligence. *Theory and Applications* (148–151). Springer, Berlin, Heidelberg.

[21] Xu, Z. (2007). Intuitionistic preference relations and their application in group decision making. *Information Sciences*, 177(11), 2363–2379.

[22] Yilmaz, S., Citil, M., & Cuvalcioglu, G. (2015). Some properties of the matrix representation of the intuitionistic fuzzy modal operators. *Notes on Intuitionistic Fuzzy Sets*, 21(2), 19–24.

[23] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.

[24] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, 8(3), 199–249.