NIFS 7 (2001), 1, 1-3

REMARK ON A SPECIAL CLASS OF INTUITIONISTIC FUZZY SETS

Eulalia Szmidt[1], Janusz Kacprzyk [1] and Krassimir Atanassov[2]

[1] Systems Research Institute – Polish Academy of Sciences,
ul. Newelska 6, 01-447 Warsaw, POLAND
e-mails: {szmidt, kacprzyk}@ibspan.waw.pl

[2] Centre for Biomedical Engineering – Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, BULGARIA
e-mail: krat@bgcict.acad.bg

Let the Intuitionistic Fuzzy Set (IFS) A over the universe E be given (for the IFSs see, e.g. [1,2]). One of its geometrical interpretations is given on Fig. 1.

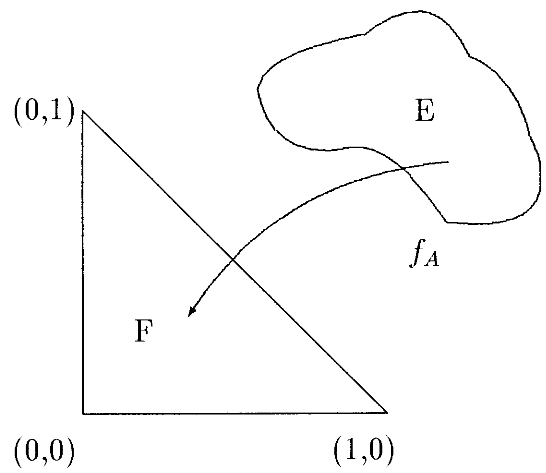

Fig. 1.

Let the following condition be valid for the given IFS

$$(\forall a \in [0,1])(\forall x \in E)(\forall y \in E)(\nu_A(y) - \nu_A(x) \neq a.(\mu_A(x) - \mu_A(y))). \qquad (*)$$

This IFS we shall call One-Strateficating IFS (OSIFS).
THEOREM For every OSIFS there exists an ordinary fuzzy set which can be juxtaposed bijectively to it.
Proof: Let an OSIFS A be given. Let $\alpha \in (0.5, 1)$ be a fixed number. Then

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1-\alpha).\pi_A(x)\rangle \mid x \in E\}$$

is an ordinary fuzzy set, where

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

The geometrical interpretation of the IFS $D_\alpha(A)$ is shown on Fig. 2.
From (*) it follows that for every two $x, y \in E$:

$$\mu_A(x) + \alpha.\pi_A(x) - \mu_A(y) - \alpha.\pi_A(y)$$
$$= \mu_A(x) - \mu_A(y) + \alpha.(\pi_A(x) - \pi_A(y))$$
$$= (1-\alpha).(\mu_A(x) - \mu_A(y)) + \alpha.(\nu_A(y) - \nu_A(x)).$$

From the fact that $0.5 < \alpha < 1$ it follows that $0 < 1 - \alpha < 0.5$. Therefore

$$0 < \frac{1-\alpha}{\alpha} < 1.$$

From here and from (*) it follows that for every two $x, y \in E$:

$$\mu_A(x) + \alpha.\pi_A(x) \neq \mu_A(y) + \alpha.\pi_A(y).$$

Hence, the OSUFS A can be juxtaposed bijectively to the ordinary fuzzy set $D_\alpha(A)$.

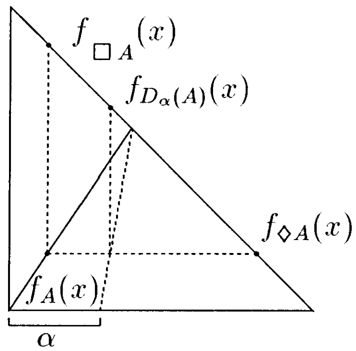

Fig. 2.

Easily it can be seen that the opposite direction is not valid. For example, let us have the OSIFS A and let for a fixed real number $b \in (0, \mu_A(x))$ us define the IFS B with the form:

$$B = (A - \{\langle x, \mu_A(x), \nu_A(x)\rangle\}) \cup \{\langle x, \mu_A(x) - b, \nu_A(x) - \frac{1-\alpha}{\alpha}.b\rangle\},$$

i.e., for all $y \in E - \{x\} : \mu_B(y) = \mu_A(y)$ and $\nu_B(y) = \nu_A(y)$; and

$$\mu_B(x) = \mu_A(x) - b,$$

$$\nu_B(x) = \nu_A(x) - \frac{1-\alpha}{\alpha}.b.$$

Obviously, the IFSs A and B are different. On the other hand, the sets $D_\alpha(A)$ and $D_\alpha(B)$ coincide for all elements of the universe E without x, and, finally, the operator D_α will juxtapose to the degrees $\mu_B(x)$ and $\nu_B(x)$ of the element x the degrees

$$\mu_A(x) - b + \alpha.(1 - \mu_A(x) + b - \nu_A(x) + \frac{1-\alpha}{\alpha}.b)$$

and

$$\nu_A(x) - \frac{1-\alpha}{\alpha}.b) + (1-\alpha).(1 - \mu_A(x) + b - \nu_A(x) + \frac{1-\alpha}{\alpha}.b).$$

Now, directly it can be seen that

$$\mu_A(x) - b + \alpha.(1 - \mu_A(x) + b - \nu_A(x) + \frac{1-\alpha}{\alpha}.b)$$

$$= \mu_A(x) + \alpha.(1 - \mu_A(x) - \nu_A(x))$$

$$\nu_A(x) - \frac{1-\alpha}{\alpha}.b + (1-\alpha).(1 - \mu_A(x) + b - \nu_A(x) + \frac{1-\alpha}{\alpha}.b)$$

$$= \nu_A(x) + (1-\alpha).(1 - \mu_A(x) - \nu_A(x)),$$

i.e., the sets $D_\alpha(A)$ and $D_\alpha(B)$ coincide everywhere.

Therefore, we cannot assume that the OSIFS A is the unique one, which can be juxtaposed bijectively to a given ordinary fuzzy set.

Anyway, in some situations it can be useful to convert any intuitionistic fuzzy set into a fuzzy set. The idea is a result of the fact that the development of models based on intuitionistic fuzzy sets is far less advanced than that for fuzzy models. It may be therefore an interesting idea to try to somehow find a fuzzy set corresponding to models based on intuitionistic fuzzy sets, and then use a huge array of tools and techniques available for fuzzy models. Obviously, it can not be done bijectively but a (not-bijective) method fulfilling a commonsence conditions (from an application point of view) was proposed in [3].

REFERENCES:

[1] Atanassov K., Intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 20 (1986), No. 1, 87-96.

[2] Atanassov K., Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag, 1999.

[3] Szmidt E. and Kacprzyk J. (1998) A Fuzzy Set corresponding to an intuitionistic fuzzy set. Int. Journal of Uncertainty Fuzziness and Knowledge Based Systems, Vol.6, No.5, 427-435.