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HANDLING UNCERTAINTY IN NATURAL SENTENCES VIA INTUITIONISTIC FUZZY SETS

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Abstract:

Natural language allows us to maintain the uncertainties we may in our knowledge and does not require us to impose an unjustified precision. Fuzzy sets and fuzzy logic provide (in contrast to classical logic) an effective conceptual framework for dealing with uncertainty and imprecision present in language sentences. Intuitionistic fuzzy sets seem even to be more important in the area due to the fact that they are generalisation of fuzzy sets with another degree of freedom letting us better model uncertainty. In this paper we show a new application (examples) of intuitionistic fuzzy sets – their ability to handling uncertainty in natural language sentences with imprecise predicates and modal operators (*necessity* "□"and *possibility* "◊" which have no counterparts in the ordinary fuzzy set theory).

INTRODUCTION

The natural language plays a fundamental role not only in human communication but even in human way of thinking and regarding the world. It allows us to maintain the uncertainties we may in our knowledge and does not require us to impose an unjustified precision.

Fuzzy sets and fuzzy logic provide (in contrast to classical logic) an effective conceptual framework for dealing with uncertainty and imperfect information present in language sentences. Fuzzy set theory was proposed by Zadeh in 1965 as a simple and efficient tool for the representation and processing of imprecise concepts and quantities exemplified by *tall men*, *large numbers* etc. which fall beyond the scope of conventional precise mathematics because their very essence is a gradual (not abrupt) transition between the membership and non-membership of elements in a set.

Intuitionistic fuzzy sets seem to be even more important in the area of handling uncertainty in natural sentences due to the fact that they are generalisation of fuzzy sets with another degree of freedom. This way we can better model imperfect information (examples of applications of intuitionistic fuzzy sets in negotiation processes are given e.g. in [12, 13], in widely understood decision making – in [14]).

The purpose of this paper is to show a new application of intuitionistic fuzzy sets – their ability to handling uncertainty in natural language sentences with imprecise predicates and modal operators ("necessity" and "possibility") which have no counterparts in the ordinary fuzzy set theory. The material is organised as follows: In Sections 1 and 2 we briefly present fuzzy sets and intuitionistic fuzzy sets, respectively. In Section 3 we give short description of the operators: "□" and "◊" applied for IFSs. In Section 4 we propose an original method which

1. FUZZY SETS

Fuzzy sets were introduced by Zadeh in 1965 [1] as a simple and effective tool for modelling vague and imprecise (but intuitively fully understood and familiar) concepts and quantities exemplified by "tall man", "large numbers" etc. A fuzzy set A' is just a set of ordered pairs as follows

$$A' =_{df}\{<x, \mu_{A'}(x)>: x \in X\} \quad (1)$$

where the function $\mu_{A'}: X \to [0;1]$ is called a *membership function*, and the value $\mu_{A'}(x)$ is interpreted as a *membership level* (*degree*) of element x to a fuzzy set A'. Such a fuzzy set can be also noted as an ordered pair $<A', \mu_A>$ with notions given by (1).

A concept, which plays a basic and dominant role in fuzzy logic and reasoning is the concept of a "linguistic variable". A linguistic variable is a variable the values of which are not numbers but words or sentences or propositions in a natural language or artificial language. A linguistic variable encapsulates the properties of approximate or imprecise concepts in a systematic and computationally useful way. It reduces the apparent complexity of describing a system by matching a semantic tag to the underlying concept. Yet a linguistic variable always represents a fuzzy space (another way of saying that when we evaluate a linguistic variable we come with a fuzzy set).

Example 1
Let X be a set of points belonging to a circle with r radius. Let all points which can be described as "close to the centre of the circle" belong to a fuzzy set $<A', \mu_A>$. The membership function is given as follows:

$$\mu_{A'}(x) = \min\{1, {}^4/_3\,(1 - d(x)/r)\} \qquad \forall x \in X \quad (2)$$

where: $d(x)$ – the distance between point x and the centre of X.

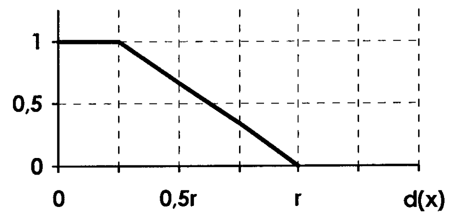

Figure 1: The membership function given by (2)

Basing on (2) we can define the linguistic variable *how_distant*, which takes its values in the set {FAR, QUITE FAR, QUITE CLOSE, CLOSE}. Obviously, all given adverbs describe the distance between any point x and the centre of X. Table (3) relates those adverbs to relevant intervals of $\mu_{A'}$ values:

HOW_DISTANT	$\mu_{A'}(x)$
FAR	$[0; {}^1/_3)$
QUITE FAR	$[{}^1/_3; {}^2/_3)$
QUITE CLOSE	$[{}^2/_3; {}^8/_9)$
CLOSE	$[{}^8/_9; 1]$

(3)

The *how_distant* variable can be used for sample sentences modelling as follows:

1. Point x such that: $d(x) = 0{,}2\,r$ ($\mu_{A'}(x) = 1$) is CLOSE to the centre of X.
2. Point x such that: $d(x) = 0{,}625\,r$ ($\mu_{A'}(x) = 0{,}5$) is QUITE FAR from the centre of X.

The fuzzy set A' discussed in our example is given in Figure 2, where the brightness level of a point is inversely related to the membership degree of that point (where: white – 0, black – 1):

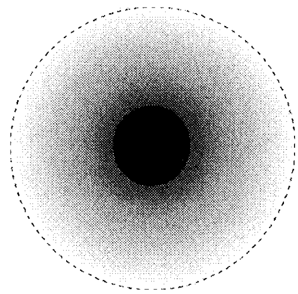

Figure 2: The fuzzy set A' with the membership function given by (2).

The membership function in the example above and – generally – in any model built via fuzzy sets is not strictly determined through the theory, but usually empirically found to make the model as close to our intuition as possible. The most frequently applied membership functions are described in [1-7].

2. INTUITIONISTIC FUZZY SETS

Intuitionistic fuzzy sets were introduced by Atanassov in 1983 (cf. e.g. [8, 9]) as a generalisation of Zadeh's fuzzy set theory. An intuitionistic fuzzy set A in space X is given as follows [9]:

$$A =_{\mathrm{df}} \{<x, \mu_A(x), \nu_A(x)>: x \in X\} \quad (4)$$

where: $\mu_A: X \rightarrow [0;1]$ – membership function
$\nu_A: X \rightarrow [0;1]$ – non-membership function

and μ_A, ν_A functions satisfy the condition :

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \qquad \forall x \in X \quad (5)$$

The difference between 1 and the sum of $\mu_A(x)$ and $\nu_A(x)$ is interpreted as a *hesitancy degree* (*hesitancy margin*) formally called *intuitionistic fuzzy index* of element x in an intuitionistic fuzzy set A:

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \qquad \forall x \in X \quad (6)$$

Obviously:

$$0 \leq \pi_A(x) \leq 1 \qquad \forall x \in X \quad (7)$$

Summing up: If we want to describe fully an intuitionistic fuzzy set, we must use any two functions from the triplet{*membership function, non-membership function, intuitionistic fuzzy index function*}.

Example 2 *Modelling of imprecise predicate „to be close to the centre of the circle" via intuitionistic fuzzy sets*
Let X be a set of points belonging to a circle with r radius. In order to model via intuitionistic fuzzy sets how close is a point to the centre of a circle we must give any two functions from the triplet $\{\mu_A, \nu_{A,}, \pi_A\}$. Let us assume that membership function μ_A is the same as $\mu_{A'}$ in Example 1, i.e. given by (2). Next, let non-membership function ν_A be given as follows:

$$\begin{array}{ll} 0 & \text{dla } x\text{: } 0 \leq d(x) \leq 0{,}5r \\ (2 \cdot d(x)/r)-1 & \text{dla } x\text{: } 0{,}5r \leq d(x) \leq r \end{array} \quad (8)$$

Thus, having in mind (2) and (8), from (6) we obtain intuitionistic fuzzy index in the form:

0	dla x: $0 \le d(x) \le 0{,}25\,r$
$^{1}/_{3}(4\cdot d(x)/r - 1)$	dla x: $0{,}25r \le d(x) \le 0{,}5\,r$
$^{2}/_{3}(1 - d(x)/r)$	dla x: $0{,}5r \le d(x) \le r$

(9)

Functions (2), (8) and (9) are given in Figure. 3:

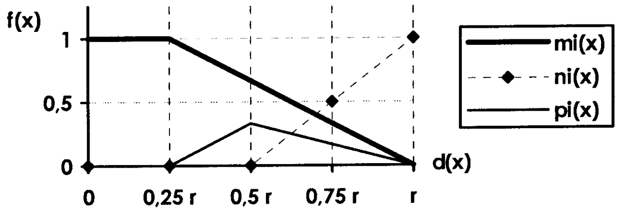

Figure 3: The membership function – $\mu(x)$, non-membership function – $\nu(x)$ and intuitionistic fuzzy index – $\pi(x)$ for the intuitionistic fuzzy set A from Example 2.

Now, let us define the linguistic variable *how_distant* as in Example 1 - by (3). Moreover, with the intuitionistic fuzzy index of the intuitionistic fuzzy set A we have a possibility to characterise how certain is the opinion modelled by (3): let us define linguistic variable *how_certain*, which takes its values in the set {SURE, QUITE SURE, QUESTIONABLE}. Relationships between this linguistic variable set and values of π_A are shown in Table (10):

HOW_CERTAIN	$\pi_A(x)$
SURE	$[0; ^{1}/_{9})$
QUITE SURE	$[^{1}/_{9}; ^{1}/_{4})$
QUESTIONABLE	$^{1}/_{4}$ and more than $^{1}/_{4}$

(10)

The *how_distant* (3) and *how_certain* (10) linguistic variables can be used for sample sentences modelling as follows:

1. Point x such that $d(x)=0{,}2r$ ($\mu_A(x)=1$) is CLOSE to the centre of X; this opinion is SURE ($\pi_A(x)=0$).
2. Point x such that $d(x)=0{,}625r$ ($\mu_A(x)=0{,}5$) is QUITE FAR form the centre of X; this opinion is QUESTIONABLE ($\pi_A(x)=^{1}/_{4}$).

It is worth noticing that in the sense of (1) any crisp set Z is a special case of a fuzzy set with the membership function taking values as follows:

1 – if $x \in Z$
0 – otherwise (11)

Analogously – in the sense of (4) – any fuzzy set $<A', \mu_A>$ is a special case of an intuitionistic fuzzy set with the non-membership function taking values as follows:

$$\nu_A(x) = 1 - \mu_A(x) \qquad \forall x \in X \tag{12}$$

Note, that for elements of an intuitionistic fuzzy set $<A, \mu_A, \nu_A>$ for which their intuitionistic fuzzy index is equal to 0, we can be **sure** as far as the membership degree of these elements are concerned. On the other hand: each intuitionistic fuzzy set $<A, \mu_A, \nu_A>$ with membership and non-membership functions satisfying condition (12) is a classical (Zadeh's) fuzzy set.

3. OPERATORS TRANSFORMING INTUITIONISTIC FUZZY SETS INTO FUZZY SETS

Operators transforming an intuitionistic fuzzy set into a fuzzy set are usually defined as operations on their membership or/and non-membership functions. The most typical example of such an operator is Atanassov's operator described in [9], considered also in [11]. In this article we are mainly interested in two operators: *necessity* "□" and *possibility* "◊" transforming an intuitionistic fuzzy set into a fuzzy set.

Let X – any classical non-empty space, $<A, \mu_A, \nu_A>$ – an intuitionistic fuzzy set in X. The two operators *necessity* "□" and *possibility* "◊" transforming an intuitionistic fuzzy set A into a fuzzy set are defined as follows [8, 9]:

$$\Box A = <A, \mu_A, 1-\mu_A> \tag{13}$$
$$\Diamond A = <A, 1-\nu_A, \nu_A> \tag{14}$$

Notice, that sets obtained *via* (13) and (14) from an intuitionistic fuzzy set A are typical Zadeh's fuzzy sets: they do satisfy condition (12).

The above definitions of "□" and "◊" indicate that these operators are meaningless in the case of fuzzy sets, and this is therefore a demonstration of the fact that intuitionistic fuzzy sets are proper extension of the ordinary fuzzy sets.

4. MODELLING OF NATURAL LANGUAGE SENTENCES CONTAINING MODAL OPERATORS

The original way of applying operators „□" and „◊" (which are typical for intuitionistic fuzzy sets and have no counterparts for fuzzy sets) is modelling of natural language sentences containing imprecise predicates and preceded by modal operators: *it is necessary, that...* ("box", "□") or/and *it is possible, that...* ("diamond", "◊"), for instance:

1. *It is necessary, that* Peter is a tall man.
2. *It is possible, that* Peter is a tall man.

Let us denote the sentence „Peter is a tall man" as p; thus, we must denote the sentences 1 and 2 as: $\Box p$ and $\Diamond p$, respectively. If sentence p, i.e: imprecise predicate „to be a tall man" is modelled by a linguistic variable, the values of which are related to intuitionistic fuzzy sets A_1, A_2, A_3...., etc., then sentences $\Box p$ and $\Diamond p$ can be modelled by linguistic variable the values of which are related – by (13) – (14) – to intuitionistic fuzzy sets $\Box A_1$, $\Box A_2$, $\Box A_3$.... and $\Diamond A_1$, $\Diamond A_2$, $\Diamond A_3$... respectively. Details of those operations are shown in Example 3:

Example 3

Let us define three intuitionistic fuzzy sets $<S, \mu_S, \nu_S>$, $<M, \mu_M, \nu_M>$ and $<L, \mu_L, \nu_L>$ in space W – the set of all words. Sets S, M, L represent SHORT, MEDIUM, and LONG words in W, respectively. Their membership and non-membership functions are given as follows:

$$\mu_S(w)=\begin{cases} 1 & if \quad s: N(w)\le 4 \\ \frac{8-N(w)}{4} & if \quad s: 4\le N(w)\le 8 \\ 0 & otherwise \end{cases} \tag{15}$$

$$\nu_S(w)=\sqrt{\mu_S(s)}-\mu_S(s) \tag{16}$$

$$\mu_M(w)=\begin{cases} \frac{N(w)-4}{4} & if \quad s: 4\le N(w)\le 8 \\ \frac{12-N(w)}{4} & if \quad s: 8\le N(w)\le 12 \\ 0 & otherwise \end{cases} \tag{17}$$

$$\nu_M(w)=\sqrt{\mu_M(w)}-\mu_M(w) \tag{18}$$

$$\mu_L(w)=\begin{cases} 0 \quad if \quad s:N(s)\leq 8 \\ \frac{N(s)-8}{4} \quad if \quad s:8\leq N(s)\leq 1 \\ 1 \quad otherwise \end{cases} \quad (19) \qquad \nu_L(w)=\sqrt{\mu_L(w)}-\mu_L(w) \quad (20)$$

$$\forall w \in \boldsymbol{W}$$

where: $N(s)$ – the number of letters in word w (the length of word w)

Membership functions, non-membership functions, and intuitionistic fuzzy indices for intuitionistic fuzzy sets ***S***, ***M***, ***L*** are given in Figures 5, 6, 7, respectively:

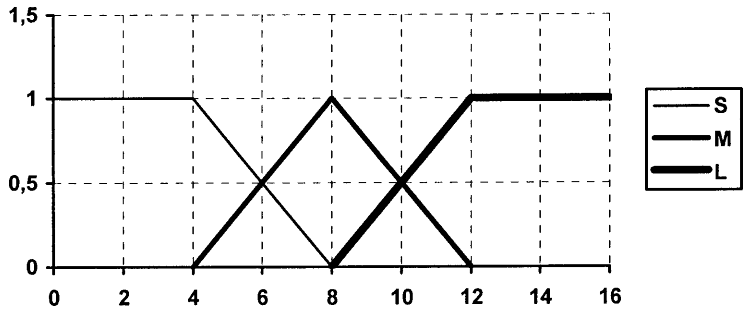

Figure 5: Memberships functions for SHORT, MEDIUM, and LONG words given in Example 3 (as functions of numbers of letters).

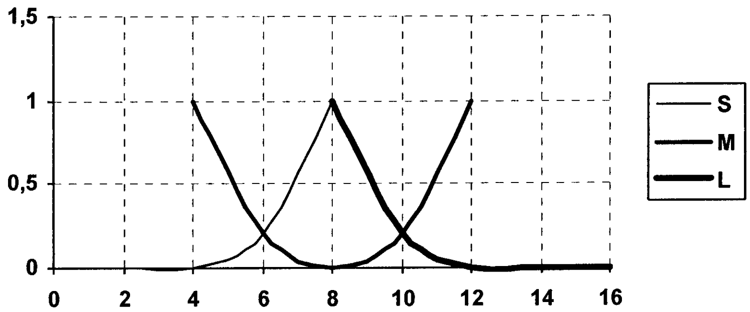

Figure 6: Non-membership functions for SHORT, MEDIUM, and LONG words given in Example 3 (as functions of numbers of letters).

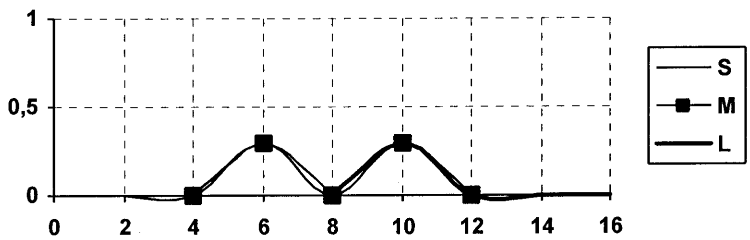

Figure 7: Intuitionistic fuzzy indices for SHORT, MEDIUM, and LONG words given in Example 3 (as functions of numbers of letters).

Now, we relate μ_S, μ_M, μ_L functions to the elements of the set {SHORT, MEDIUM, LONG} respectively; the given adjectives are the values of the linguistic variable *how_long_word*. Then, we relate π_S, π_M, π_L functions to the elements of the set {SURE, QUITE SURE, QUESTIONABLE}, respectively *via* Table (21):

HOW_CERTAIN	$\pi_S(x)$, $\pi_M(x)$, $\pi_L(x)$
SURE	$[0; {}^1/_8)$
QUITE SURE	$[{}^1/_8; {}^1/_4)$
QUESTIONABLE	${}^1/_4$ and more than ${}^1/_4$

(21)

The given adjectives are the values of the variable *how_certain*. (see also Example 2).
Now – with linguistic variables *how_long_word* and *how_certain* – it is possible to model the imprecise sentences describing the length of words, and containing the level of certainty for given opinion, for instance:

r: The word "cat" is SHORT (μ_S("cat")=1, (μ_M("cat")=0);
this opinion is SURE (π_S("cat")=0)

s: The word "furniture" is MEDIUM (μ_M("furniture")=0,75, μ_L("furniture")=0,25);
this opinion QUITE SURE (π_M("furniture")=0,133).

Thus, sentences □***r***, ◊***r***, □***s***, and ◊***s*** are now of the form:

□***r***	*It is necessary, that* the word "cat" is SHORT;
◊***r***	*It is possible, that* the word "cat" is SHORT;
□***s***	*It is necessary, that* the word "furniture" is MEDIUM;
◊***s***	*It is possible, that* the word "furniture" is MEDIUM.

Now, let us transform IFS <***S***, μ_S, ν_S>, which has just been used to model □***r***, ◊***r*** sentences, through operators „□" and „◊" given by (13), (14). We obtain two IFSs of the form <***S***, μ_S, 1-μ_S> and <***S***, 1-ν_S, ν_S> satisfying condition (12), so, in fact, they are two Zadeh's fuzzy sets. Analogously, we transform IFS <***M***, μ_M, ν_M>, used to model □***s***, ◊***s*** sentences. Notice, that intuitionistic fuzzy indices of <***S***, μ_S>, <***M***, μ_M>, and of <***L***, μ_L> now equals 0 for all words. It means, that after the transformation we lost the second part of the sentence, which was used to describe the certainty level of the given opinion. However, we can restore that measure, if we interpret the membership degree for any element of a fuzzy set as the level of the truth (in scale: 1 – true, 0 – false) for the obtained modal sentence, i.e. sentences □***r***, ◊***r***, □***s***, and ◊***s*** have the level of true: 1; 1; 0,75; 0,884, respectively. Especially, two last results are very close to human intuition: the possibility of any fact seems to be truer, than its necessity.

5. Final conclusions and further work suggestions

We have shown the concept of mathematical modelling of natural language sentences containing imprecise predicates and modal operators *necessity* "□" and *possibility* "◊". This is a new original application of intuitionistic fuzzy set theory. The procedures enabling the natural language modelling and analysis using fuzzy logic elements are essential and very popular in their applications in decision and expert systems. Any sentence obtained through the procedures described in Example 3, Section 4 can be a premise to fuzzy reasoning and inference. It is also possible to find some modal logic axioms to be satisfied by those sentences.

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