APPLICATIONS OF INTUITIONISTIC FUZZY OPERATORS IN IMAGE PROCESSING

Dr.(Mrs).R. PARVATHI
ASSISTANT PROFESSOR IN MATHEMATICS
VELLALAR COLLEGE FOR WOMEN
ERODE – 638 012
TAMILNADU, INDIA.

Приложения на операторите над интуиционистки размитите множества за обработка на изображения

- SETS
- FUZZY SETS
- INTUITIONISTIC FUZZY SETS (IFSs)
- IFSs OF SECOND TYPE
- OPERATORS ON IFSs
- APPLICATIONS

Множества / Размити множества / Интуиционистки размити множества (ИРМ) / ИРМ от втори тип / Оператори над ИРМ / Приложения

SETS

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{if } x \notin X \end{cases}$$

That is,

$$\chi_A : X \to \{0,1\}$$

Множества

FUZZY SETS

membership function

$$\mu_A : X \rightarrow [0,1]$$

Размити множества с функция за принадлежност

INTUITIONISTIC FUZZY SETS

membership function

$$\mu_A : X \to [0, 1]$$

non-membership function

$$\gamma_A : X \to [0, 1]$$

Интуиционистки размити множества
с функции за принадлежност и непринадлежност

INTUITIONISTIC FUZZY SETS

- IFS of First Type

$$0 \le \mu_A(x) + \gamma_A(x) \le 1$$

- IFS of Second Type

$$0 \le \mu_A(x)^2 + \gamma_A(x)^2 \le 1$$

Интуиционистки размити множества от първи и от втори тип

Degree of uncertainty

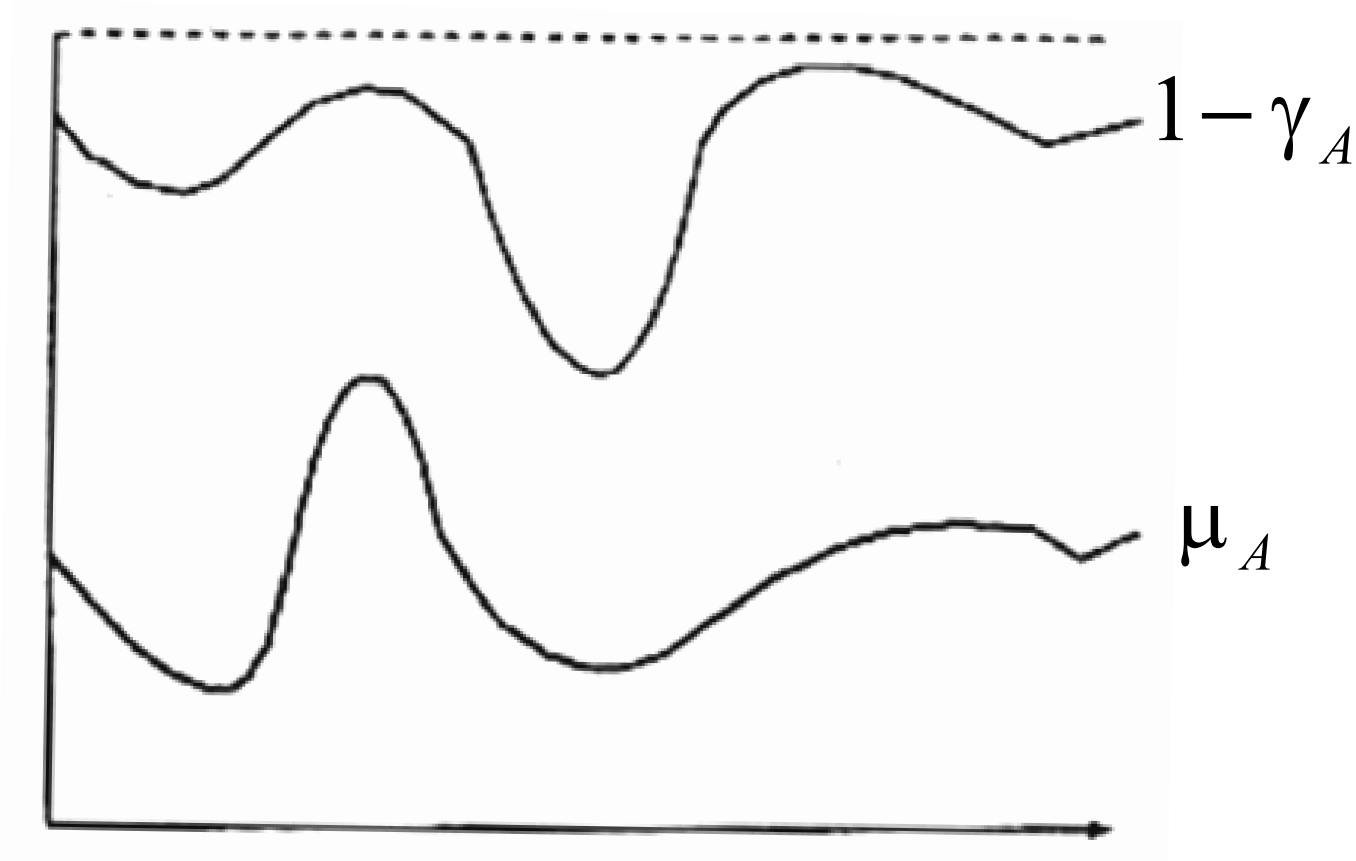
- **Definition** The value $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ is called the *degree of non- determinacy* (or the degree of uncertainty) of the element $x \in X$ to the intuitionistic fuzzy set A.

Степен на несигурност (неопределеност) на елемента *x* към ИРМ *A*

First (Standard) geometric interpretation of an IFS

Първа (стандартна) геометрична интерпретация на ИРМ

Analogue of the first (Standard) geometric interpretation



Аналог на първата геометрична интерпретация

SOME BASIC OPERATORS ON INTUITIONISTIC FUZZY SETS OF SECOND TYPE

Някои основни оператори над ИРМ от втори вид

OPERATORS ON IFSST

- Let X be a nonempty set. Let A and B be two IFSsST such that

$$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$$

$$B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$$

Нека *X* е непразно множество, а *A* и *B* са ИРМ2

Define the following operators on A and B:

$A \subseteq \mathrm{B}$ iff $\mu_{\mathrm{A}}(x) \leq \mu_{\mathrm{B}}(x)$ and $\gamma_{\mathrm{A}}(x) \geq \gamma_{\mathrm{B}}(x)$ for each $\mathrm{x} \in \mathrm{X}$.

$A \supseteq B$ iff $\mu_{\mathrm{A}}(x) \geq \mu_{\mathrm{B}}(x)$ and $\gamma_{\mathrm{A}}(x) \leq \gamma_{\mathrm{B}}(x)$ for each $\mathrm{x} \in \mathrm{X}$.

$A = B$ iff $\mu_{\mathrm{A}}(x) = \mu_{\mathrm{B}}(x)$ and $\gamma_{\mathrm{A}}(x) = \gamma_{\mathrm{B}}(x)$ for each $\mathrm{x} \in \mathrm{X}$.

$A \cup B = \{\langle \mathrm{x}, \max(\mu_{\mathrm{A}}(x), \mu_{\mathrm{B}}(x)), \min(\gamma_{\mathrm{A}}(x), \gamma_{\mathrm{B}}(x))\rangle : x \in X\}.$

$A \cap B = \{\langle \mathrm{x}, \min(\mu_{\mathrm{A}}(x), \mu_{\mathrm{B}}(x)), \max(\gamma_{\mathrm{A}}(x), \gamma_{\mathrm{B}}(x))\rangle : x \in X\}.$

$$Product \;\; A \cdot B = \left\{ \left\langle x,\ \mu_A(x)^2 \cdot \mu_B(x)^2,\ \gamma_A(x)^2 + \gamma_B(x)^2 - \gamma_A(x)^2 \cdot \gamma_B(x)^2 \right\rangle : x \in X \right\}.$$

$$A^{\mathrm{n}} = \left\{ \left\langle x,\ \mu_A(x)^{2\mathrm{n}}, 1 - (1 - \gamma_A(x)^2)^{\mathrm{n}} \right\rangle : x \in X \right\}.$$

$$\mathrm{nA} = \left\{ \left\langle x, 1 - (1 - \mu_A(x)^2)^{\mathrm{n}}, \gamma_A(x)^{2\mathrm{n}} \right\rangle : x \in \mathrm{X} \right\}.$$

SOME MORE OPERATORS ON INTUITIONISTIC FUZZY SETS OF SECOND TYPE

Други оператори над интуиционистки размитите множества от втори тип

- CONTRAST INTENSIFICATION
- CONCENTRATION
- DILATION
- NORMALIZATION
- FUZZIFICATION

Интензификация на контраста / Концентрация / Удебеляване / Нормализация / Размиване (Фъзификация)

CONTRAST INTENSIFICATION

Definition : **Let A be an IFSST. Then,**

INTEN(A) = $\{\langle x, \mu_{\text{INTENA}}(x), \gamma_{\text{INTENA}}(x)\rangle : x \in \text{X}\}$

where

$$\mu_{\text{INTENA}}(x) = 1 - (1 - \mu_{\text{A}}(x)^{8})^{2}$$

$$\gamma_{\text{INTENA}}(x) = [1 - (1 - \gamma_{\text{A}}(x)^{2})^{2}]^{4}$$

Интензификация на контраста

IMAGE PROCESSING

One of the pre-processing functions in pattern recognition.

An image is processed so that its quality is improved. This is done by

- Contrast intensification
- Smoothing
- Sharpening
- Edge detection etc.,

Обработка на изображения – една от функциите предшестващи разпознаването на образи. Прави се чрез интензификация на контраста, оглаждане или изостряне на формите, откриване на ръбове и др.

IMAGE DEFINITION

An image is represented mathematically by M×N array as follows:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1N} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{M1} & x_{M2} & x_{M3} & \cdots & x_{MN} \end{bmatrix}$$

whose row and column indices identify a point (m, n) in the image and the corresponding matrix element value x_{mn} denotes the grey level at that point.

- **For the purpose of processing, this image along with the coordinates of its pixels is stored in the computer in the form of an array of MN numbers.**

CONTRAST INTENSIFICATION - ALGORITHM

STEP 1: Input image – образ на входа

STEP 2: Gray levels – нива на сивото

STEP 3: Membership values – принадлежност

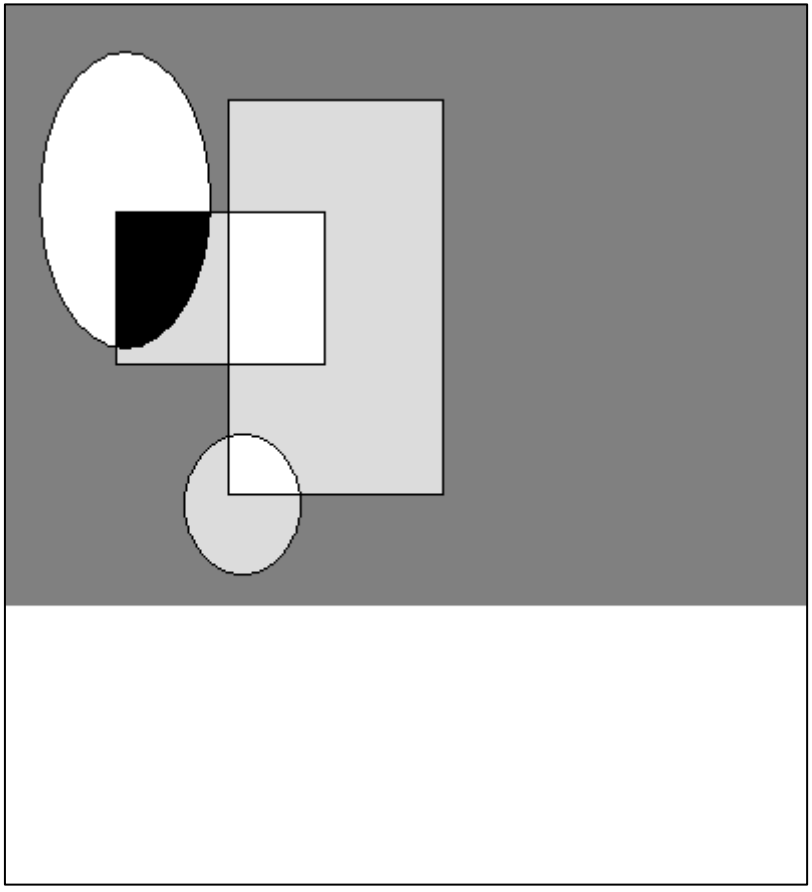
STEP 4: Non-membership values – непринадлежност

STEP 5: Modification using contrast intensification – модифициране чрез интензификация на контраста

STEP 6: New gray levels – нови нива на сивото

STEP 7: Output image – образ на изхода

STEP 1: INPUT IMAGE



Стъпка 1: Образ на входа

STEP 2: GRAY LEVELS

0	0	0
128	0	0
0	128	0
128	128	0
0	0	128
128	0	128
0	128	128
192	192	192
192	220	192
166	202	240
64	32	0

Стъпка 2: Нива на сивото

STEP 3: MEMBERSHIP VALUES

$$\mu_{mn} = \left\{ 1 + \frac{g_{\max} - x_{mn}}{F_d} \right\}^{-F_e}$$

where F_e and F_d are arbitrary.

0.012346	0.012346	0.012346
0.026087	0.012346	0.012346
0.012346	0.026087	0.012346
0.026087	0.026087	0.012346
0.012346	0.012346	0.026087
0.026087	0.012346	0.026087
0.012346	0.026087	0.026087
0.058824	0.058824	0.058824
0.058824	0.130435	0.058824
0.038961	0.073171	1
0.012346	0.012346	0.012346

Стъпка 3: Стойности на функцията за принадлежност

STEP 4: NON - MEMBERSHIP VALUES

$$\gamma_{mn} = \begin{cases} \frac{1}{2}\max\left[\left|1-\mu_{mn}\right|,\left|0-\mu_{mn}\right|\right], & 0 \le \mu_{mn} \le 0.5, \\ \frac{1}{2}\min\left[\left|1-\mu_{mn}\right|,\left|0-\mu_{mn}\right|\right], & 0.5 \le \mu_{mn} \le 1. \end{cases}$$

0	0	0
0 . 486957	0 . 493827	0 . 493827
0 . 493827	0 . 486957	0 . 493827
0 . 486957	0 . 486957	0 . 493827
0 . 493827	0 . 493827	0 . 486957
0 . 486957	0 . 493827	0 . 486957
0 . 493827	0 . 486957	0 . 486957
0 . 470588	0 . 470588	0 . 470588
0 . 470588	0 . 434783	0 . 470588
0 . 480519	0 . 463415	0
0 . 493827	0 . 493827	0 . 493827

Стъпка 4: Стойности на функцията за непринадлежност

STEP 5: MODIFICATION USING CONTRAST INTENSIFICATION

MODIFIED MEMBERSHIP VALUES

$$\mu_{mn}' = 1 - (1 - \mu_{mn}^{8})^{2}$$

0.033638	0.033638	0.033638
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	1
0	0	0

Стъпка 5: Модифициране на контраста чрез интензификация

STEP 5: MODIFICATION USING CONTRAST INTENSIFICATION

MODIFIED NON – MEMBERSHIP VALUES

$$\gamma_{mn}' = \left(1 - (1 - \gamma_{mn}^{2})^{2}\right)^{4}$$

0.030536	0.033638	0.033638
0.033638	0.030536	0.033638
0.030536	0.030536	0.033638
0.033638	0.033638	0.030536
0.030536	0.033638	0.030536
0.033638	0.030536	0.030536
0.024065	0.024065	0.024065
0.024065	0.013735	0.024065
0.027842	0.021605	0
0.033638	0.033638	0.033638
0	0	0

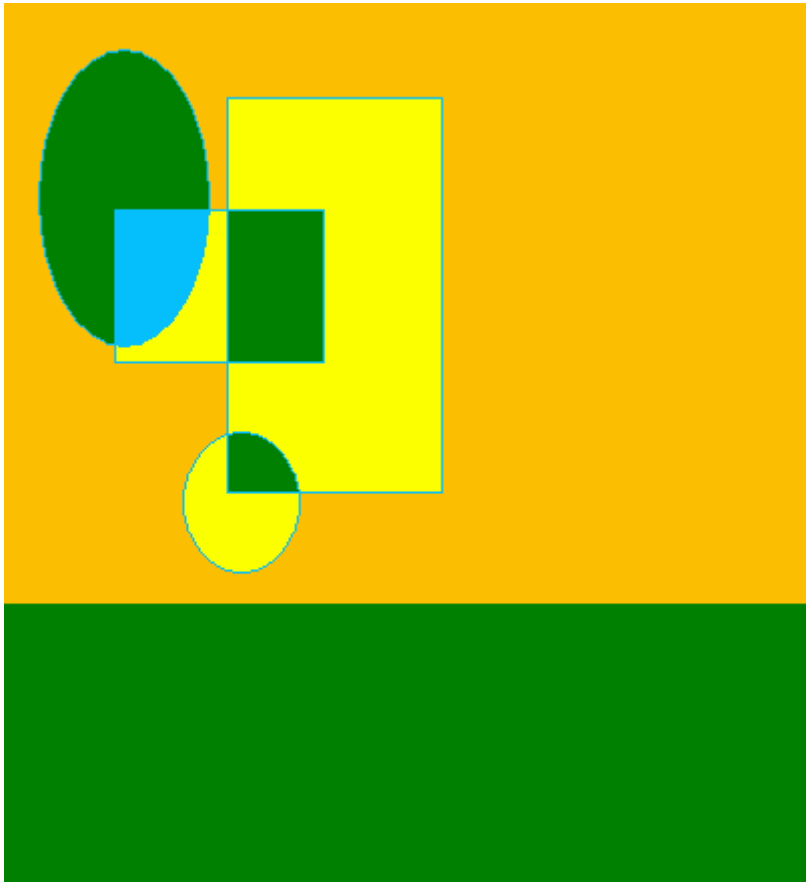
STEP 6: NEW GRAY LEVELS

$$g_{mn} = \frac{1}{2}\{[g_1 - F_d(\mu_{mn}')^{-1/F_e} - 1] + [1/\gamma_{mn}']\}$$

0.003087	0.003087	0.003087
64.006523	0.003087	0.003087
0.003087	64.006523	0.003087
64.006523	64.006523	0.003087
0.003087	0.003087	64.006523
64.006523	0.003087	64.006523
0.003087	64.006523	64.006523
96.014709	96.014709	96.014709
96.014709	110.032608	96.014709
83.009743	101.018295	120.25
0.003087	0.003087	0.003087

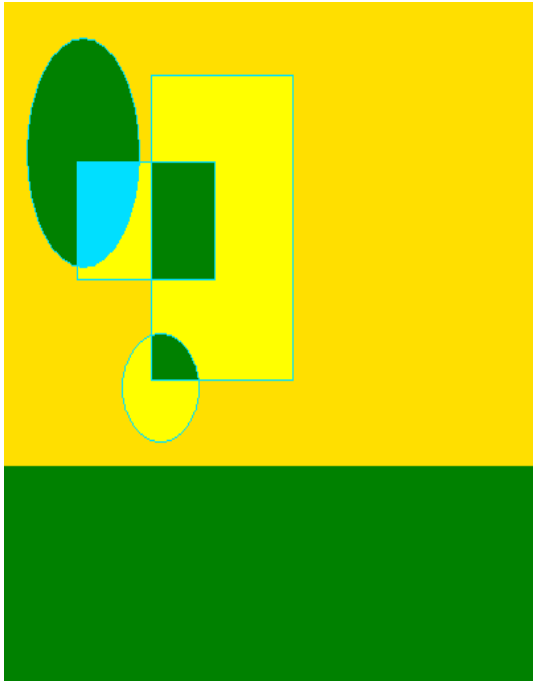
Стъпка 6: Нови нива на сивото

STEP 7: OUTPUT IMAGE

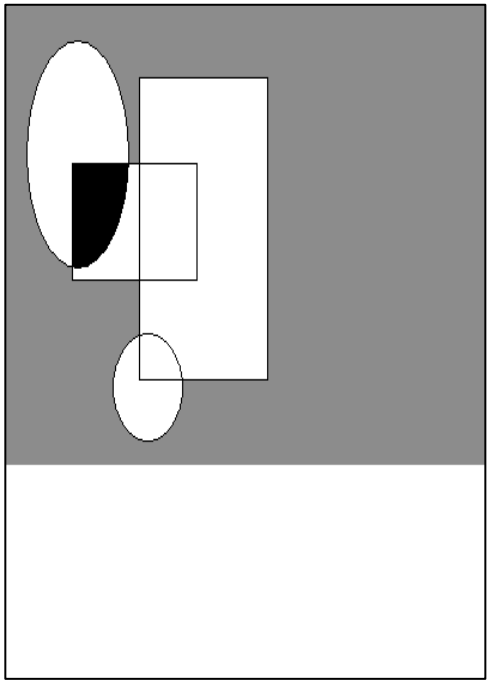


Стъпка 7: Образ на изхода

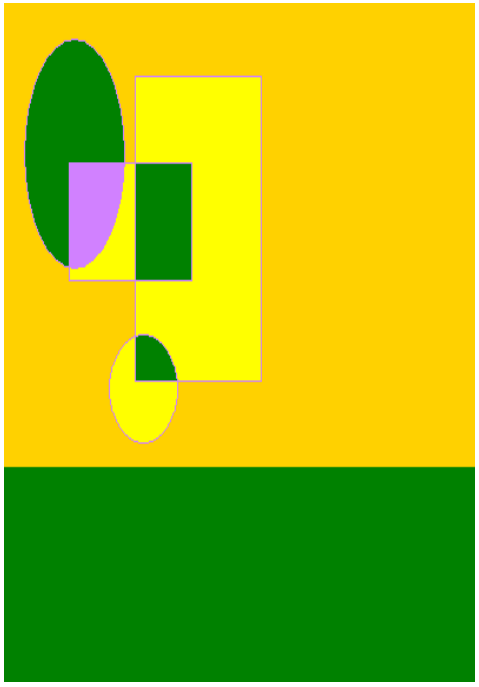
COMPARISON : INPUT / OUTPUT IMAGES



Fe = 1; Fd = 3



INPUT IMAGE



Fe = 2; Fd = 34

Сравнение на образите на изхода и входа

ROLE OF CONTRAST INTENSIFICATION OPERATOR IN FUZZY, FIRST AND SECOND TYPE INTUITIONISTIC FUZZY IMAGES

Роля на оператора за интензификация на контраста при размити изображения и изображения от ИРМ1 и ИРМ2 тип

A method based on intuitionistic fuzzy approach for image enhancement using contrast intensification operator

Метод, базиран на ИРМ подход за подобряване качеството на изображенията, използвайки оператор за интензификация на контраста

Original Image

Оригинално изображение

Enhancement Algorithm

STEP 1

- Set the parameters Fe , Fd and calculate X_{max}.
- Fe = 2;

$$F_d = \frac{X_{\max} - X_{mn}}{(0.5)^{-1/F_e} - 1}$$

Алгоритъм за подобряване на качеството на изображението

Стъпка 1: Приемане на стойности за параметрите

STEP 2

- Define the membership function

$$\mu_{mn} = G\left(X_{mn}\right) = \left[1 + \frac{X_{\max} - X_{mn}}{F_d}\right]^{-F_e}$$

Стъпка 2: Уточняване на функцията за принадлежност

STEP 3

Modify the membership values using contrast intensification operator on fuzzy sets

$$\mu_{mn}' = \begin{cases} 2\,[\mu_{mn}]^2\,, & 0 \le \mu_{mn} \le 0.5, \\ 1-2[1-\mu_{mn}]^2, & 0.5 \le \mu_{mn} \le 1. \end{cases}$$

Стъпка 3: Модифициране на стойностите за принадлежност, използвайки оператор за интентизификация на контраста

STEP 4

Generate new gray levels

$$g_{mn} = G^{-1}(\mu_{mn}') = X_{max} - F_d * ((\mu_{mn}')^{-\frac{1}{F_e}}) + F_d$$

Стъпка 4: Генериране на нови нива на сивото

Fuzzy contrast intensified image is obtained.

Получено е изображение с интензифициран размит контраст

STEP 5

Calculate the non-membership values in terms of membership values.

$$\gamma_{mn} = \begin{cases} \frac{1}{2} \max\left[\left|1-\mu_{mn}\right|, \left|0-\mu_{mn}\right|\right], & \text{if } 0 \le \mu_{mn} \le 0.5, \\ \frac{1}{2} \min\left[\left|1-\mu_{mn}\right|, \left|0-\mu_{mn}\right|\right], & \text{if } 0.5 \le \mu_{mn} \le 1, \end{cases}$$

such that $0 \le \mu_{mn} + \gamma_{mn} \le 1.$

Стъпка 5: Изчисляване на стойностите на функцията за непринадлежност на база стойностите на функцията за принадлежност

STEP 6

Modify membership and non-membership values using contrast intensification on first type IFS.

$$\mu_{mn}' = 1-(1-\mu_{mn}^{2})^{2}, \qquad 0 \le \mu_{mn} \le 1.$$

$$\gamma_{mn}' = (1-(1-\gamma_{mn})^{2})^{2}, \qquad 0 \le \gamma_{mn} \le 1.$$

Стъпка 6: Модифициране на стойностите на функциите за принадлежност и непринадлежност чрез оператор за интензификация на контраста (ИРМ1)

STEP 7

Calculate the new gray level using the modified membership and non-membership values.

$$g_{mn}' = G^{-1}(\mu_{mn}') = g_{max} - F_d * \left(\left(\sqrt{\mu_{mn}' * (c_1 - \gamma_{mn}')}\right)^{-\frac{1}{C_2 * F_e}}\right) + F_d$$

where c_1 and c_2 are arbitrary constants.

Стъпка 7: Изчисляване на новите нива на сивото, използвайки модифицираните стойности на функция за принадлежност и непринадлежност

Enhanced image using contrast intensification operator on first type IFS.

Образът е подобрен, използвайки оператор за интензификация на контраста на ИРМ1

STEP 8

Modify membership and non-membership values using contrast intensification on second type IFS.

$$\mu_{mn}' = 1-(1-\mu_{mn}^{8})^{2}, \qquad 0 \le \mu_{mn} \le 1.$$

$$\gamma_{mn}' = (1-(1-\gamma_{mn}^{2})^{2})^{4}, \qquad 0 \le \gamma_{mn} \le 1.$$

Стъпка 8: Модифициране на стойностите на функциите за принадлежност и непринадлежност, използвайки оператор за интензификация на контраста (ИРМ2)

STEP 9

Calculate the new gray level using the modified membership and non-membership values.

$$g_{mn}' = G^{-1}(\mu_{mn}') = g_{\max} - F_d * ((\sqrt{\mu_{mn}' * (c_3 - \gamma_{mn}')})^{-\frac{1}{C_4 * F_e}}) + F_d$$

where c_3 and c_4 are arbitrary constants.

Стъпка 9: Изчисляване на новите нива на сивото, използвайки модифицираните стойности на функцията за принадлежност и непринадлежност

Second type intuitionistic fuzzy contrast intensified image

Образът е подобрен, използвайки оператор за интензификация на контраста на ИРМ2

The overall view of the resultant images

Input image

Fuzzy

First type IFS

Second type IFS

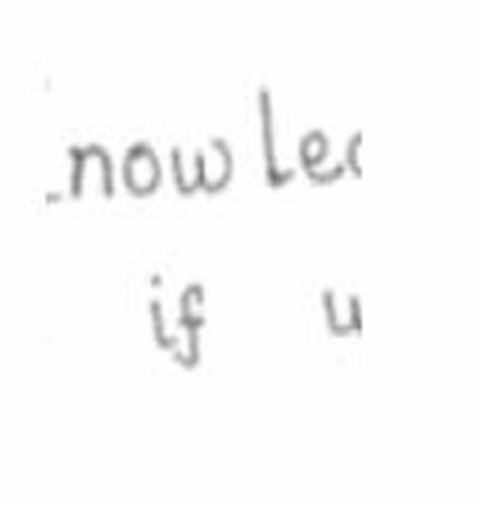
IEEE on Intelligent Systems 2006, p733-737.

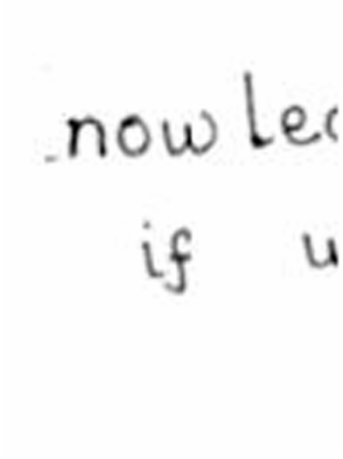
TEXT DOCUMENTS

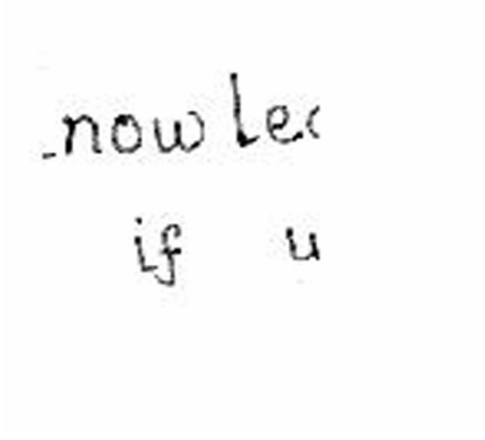
The algorithm is tested with a text document and the resultant images are shown below.

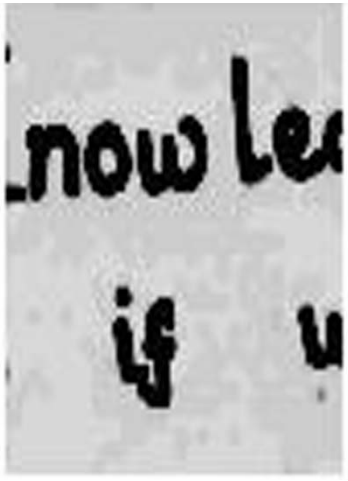
Текстови документи

Алгоритъмът е тестван и над текстови документи, резултатът е показан по-долу


now le
if u


now le
if u


now le
if u


now le
if u

Morphological Operators

…means scientific study of form and structure of animals and plants….

Морфологични оператори – изследват научно формата и структурата на растения и животни

- Morphological operators are based on the notion of ‘fitting’ a structuring element.
- It also involves the study of different ways in which a structuring element interacts with the image under study, modifies its shape to other image, which is more expressive than the initial image.

Морфологичните оператори са основани на идеята за напасване на структурни елементи. Включва изучаването на различни начини, по които структурният елемент взаимодейства с обработваното изображение, модифицирайки формата му към друга, по-изразителна от тази на оригиналното изображение.

Binary Vs Grayscale

- Binary image – Crisp – 0 or 1
- Grayscale ima – fuzzy – between 0 and 1
  - Both image and str. element are fuzzy membership functions

Монохромни изображения срещу такива от сивата гама

Structuring element

- The origin of the mathematical morphology operators were initially defined on the binary images. Mathematical morphological operators are based on simply transforming an input image with a specific *structuring element*. Structuring elements include 0's and 1's as their elements in a matrix form

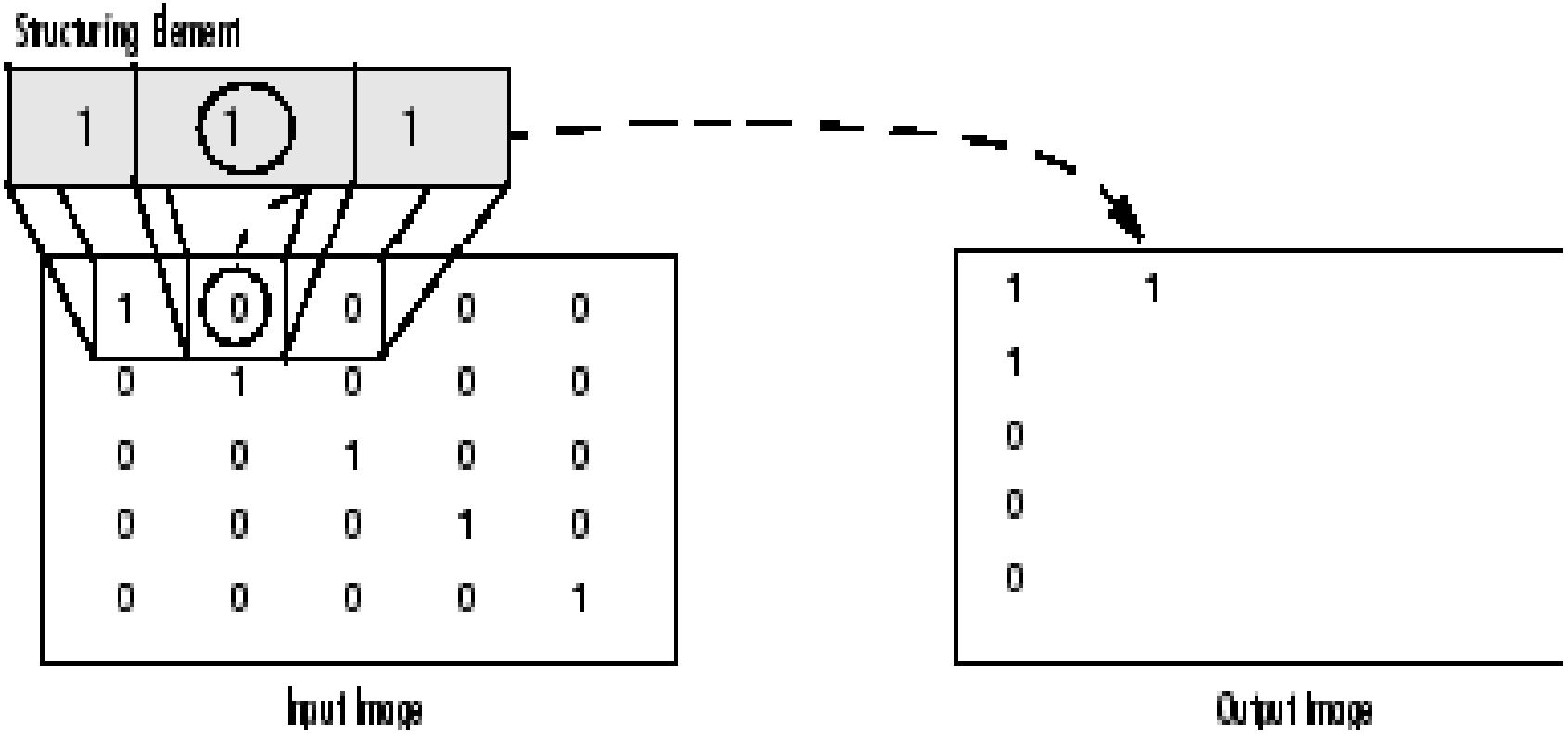
- A typical structuring element matrix is smaller than an image matrix in size. The center pixel of the structuring element called the origin, defines the pixel which is being processed.
- The pixels in the structuring element containing 1's define the *neighbourhood* of the structuring element. These pixels are also considered in dilation or erosion processing.

Dilation and Erosion

- Dilation and erosion are two fundamental morphological operations. Dilation(thickens) adds pixels to the boundaries of objects in an image, while erosion (thins) removes pixels on object boundaries. The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image.

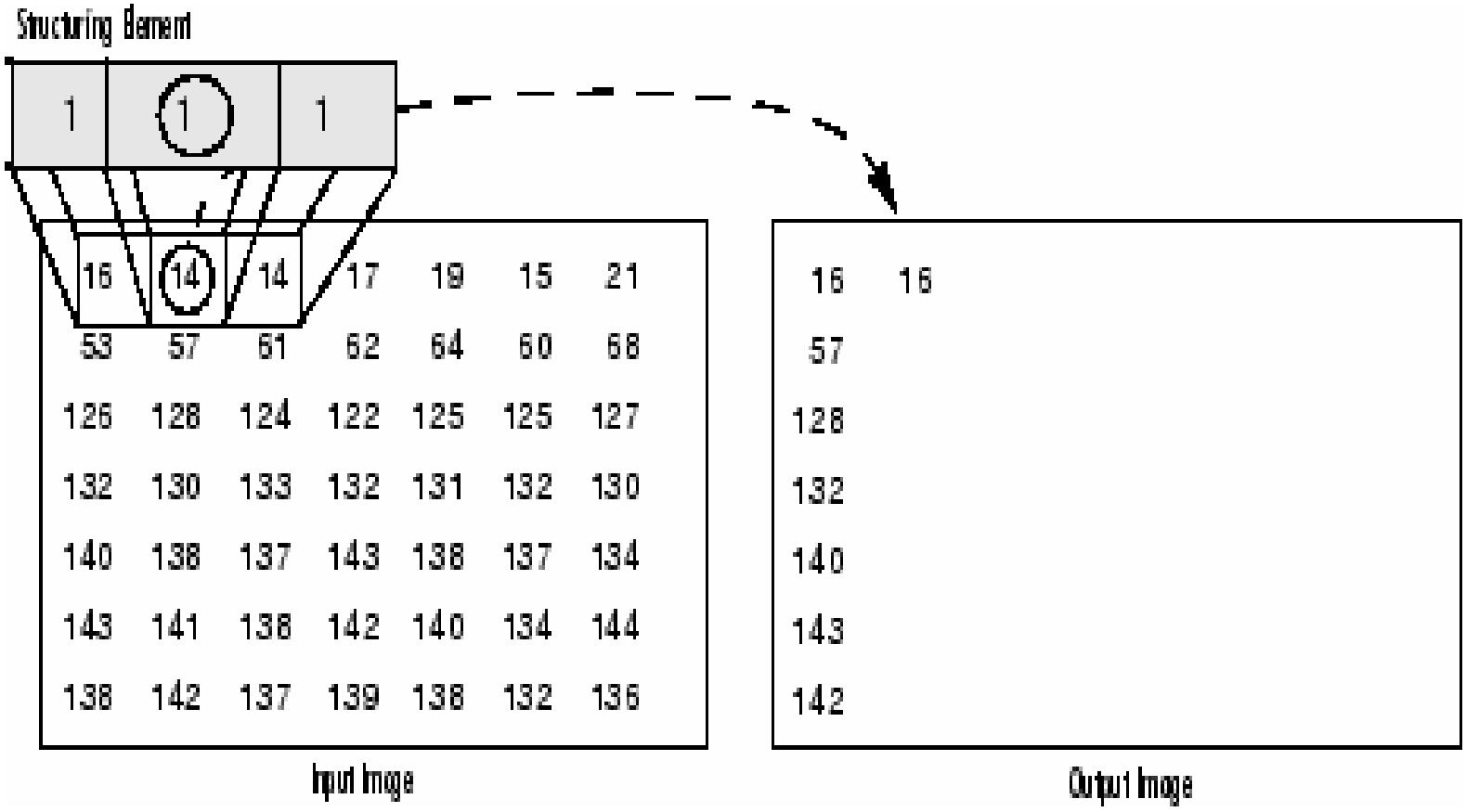
Удебеляване и изтъняване

Dilation of a binary image



Удебеляване на монохромно изображение

Dilation of a grayscale image



Удебеляване на изображение от сивата гама

Intuitionistic fuzzy morphological operators

Интуиционистки размити морфологични оператори

DILATION

- Let X be a nonempty set and let A and B be two IFSsST on X. Then the *dilation* of A by B, denoted by DIL (A, B), is defined as DIL (A, B) = $\{\langle x, \mu_{DILAB}(x), \gamma_{DILAB}(x)\rangle : x \in X\}$ where

$$\mu_{DILAB}(x) = \max\ (\mu_A(x), \mu_B(x))$$

$$\gamma_{DILAB}(x) = min\ (1 - \sqrt{1 - \gamma_A(x)^2}, 1 - \sqrt{1 - \gamma_B(x)^2})$$

Удебеляване

EROSION (CONCENTRATION)

- Let X be a nonempty set and let A and B be two IFSsST on X. Then the *erosion* of A by B, denoted by ERO (A, B), is defined as

ERO (A, B) = $\{\langle x, \mu_{EROAB}(x), \gamma_{EROAB}(x)\rangle : x \in X\}$

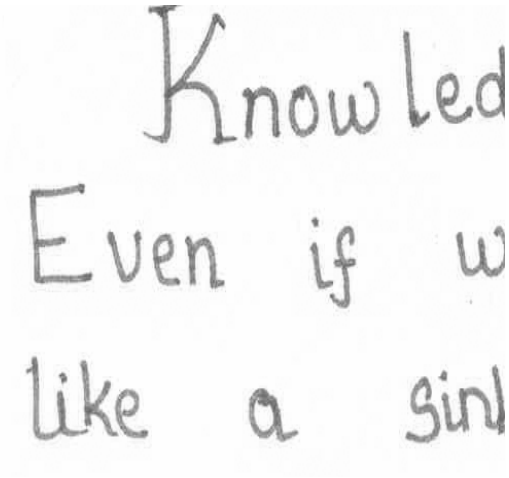
where

$$\mu_{EROAB}(x) = min\ \left(\mu_A(x)^4, \gamma_A(x)^4\right)$$

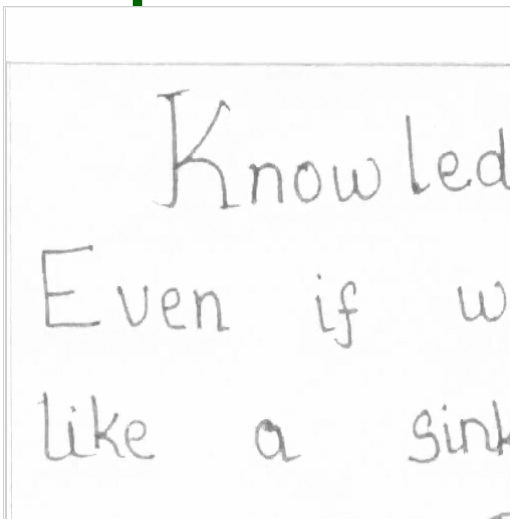
$$\gamma_{EROAB}(x) = max\,(1-(1-\gamma_A(x)^2)^2, 1-\gamma_B(x)^8)$$

Концентриране (изтъняване)

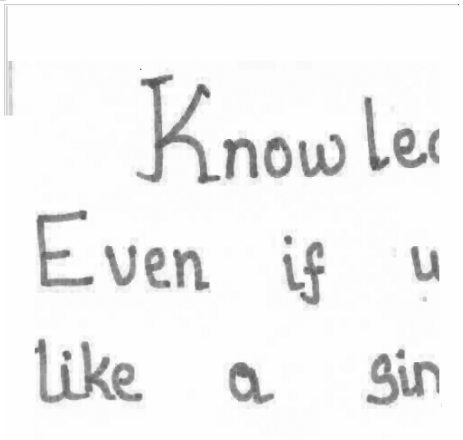
Morphological Operators



Original image



Eroded image



Dilated image

Adv. in Fuzzy Sets and Systems , 3(1), 2008,p87 – 97.

- **NORMALIZATION**
- **FUZZIFICATION**
- .....

Нормализация, размиване (фъзификация)

THANK YOU...

paarvathis@rediffmail.com