# A new topological operator over intuitionistic fuzzy sets 

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#### Abstract

A new intuitionistic fuzzy topological opertor is defined and some of its properties are studied. It is different than the defined by the moment topological operators over intuitionistic fuzzy sets.


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## 1 Introduction

The first topological operators defined over Intuitionistic Fuzzy Sets (IFSs, see [1, 2]) were introduced 30 years ago. About 15 years later they were extended and modified (see [2]). Now, a new operator is introduced. It has essentially different properties than previous ones.

Here, we give the definition of the new operator and study some of its basic properties.
Initially, we give some basic definitions, related to the IFSs, following [2].
Let a set $E$ be fixed. An IFS $A$ in $E$ is an object of the following form:

$$
A=\left\{\left\langle x, \mu_{A}(x), \nu_{A}(x)\right\rangle \mid x \in E\right\},
$$

where the functions $\mu_{A}: E \rightarrow[0,1]$ and $\nu_{A}: E \rightarrow[0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$ :

$$
0 \leq \mu_{A}(x)+\nu_{A}(x) \leq 1 .
$$

For every two IFSs $A$ and $B$ a lot of operations, relations and operators have been defined (see, e.g. [2]), the most important of which, related to the present research, are:

$$
\begin{aligned}
& A \subset B \quad \text { iff } \quad(\forall x \in E)\left(\mu_{A}(x) \leq \mu_{B}(x) \& \nu_{A}(x) \geq \nu_{B}(x)\right), \\
& A=B \quad \text { iff } \quad(\forall x \in E)\left(\mu_{A}(x)=\mu_{B}(x) \& \nu_{A}(x)=\nu_{B}(x)\right), \\
& \neg A \quad=\quad\left\{\left\langle x, \nu_{A}(x), \mu_{A}(x)\right\rangle \mid x \in E\right\}, \\
& \square A \quad=\quad\left\{\left\langle x, \mu_{A}(x), 1-\mu_{A}(x)\right\rangle \mid x \in E\right\}, \\
& \diamond A \quad=\quad\left\{\left\langle x, 1-\nu_{A}(x), \nu_{A}(x)\right\rangle \mid x \in E\right\} .
\end{aligned}
$$

Here, $\neg A$ is the classical negation. Let $U^{*}=\{\langle x, 0,0\rangle \mid x \in E\}$.

## 2 Main results

For every IFS $A \neq U^{*}$, the new topological operator has the form

$$
T(A)=\left\{\left.\left\langle x, \frac{\mu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}, \frac{\nu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right\rangle \right\rvert\, x \in E\right\} .
$$

Therefore, operator $T$ decreases the degree of uncertainty, increasing both the degrees of membership and non-membership.

Theorem 1. For every IFS $A \neq U^{*}: T(T(A))=T(A)$.
Proof. Let $A$ be an IFS. Then

$$
\begin{aligned}
& T(T(A))=T\left(\left\{\left.\left\langle x, \frac{\mu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}, \frac{\nu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right\rangle \right\rvert\, x \in E\right\}\right) \\
& =\left\{\left.\left\langle x, \frac{\frac{\mu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}}{\sup _{z \in E}\left(\frac{\mu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}+\frac{\nu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right)}, \frac{\frac{\nu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}}{\sup _{z \in E}\left(\frac{\mu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}+\frac{\nu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right)}\right\rangle \right\rvert\, x \in E\right\} \\
& =\left\{\left.\left\langle x, \frac{\frac{\mu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}}{\sup _{z \in E}\left(\frac{\mu_{A}(z)+\nu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right)}, \frac{\frac{\nu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}}{\sup _{z \in E}\left(\frac{\mu_{A}(z)+\nu_{A}(z)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right)}\right\rangle \right\rvert\, x \in E\right\}
\end{aligned}
$$

$$
\begin{aligned}
& =\left\{\left.\left\langle x, \frac{\mu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}, \frac{\nu_{A}(x)}{\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)}\right\rangle \right\rvert\, x \in E\right\} \\
& =T(A) \text {. }
\end{aligned}
$$

This completes the proof.

Theorem 2. For every IFS $A \neq U^{*}: \neg T(\neg A)=T(A)$.
Theorem 3. For every IFS $A \neq U^{*}$ :
(a) $T(\square A)=\square A$,
(b) $\square A \subset \square T(A)$,
(c) $T(\diamond A)=\diamond A$,
(d) $\diamond T(A) \subset \diamond A$.

Theorems 2 and 3 are proved by analogy.
As it is discussed in [1, 2], the IFSs have different geometrical interpretations. One of them (probably, the most important among the IFS interpretations) is shown in Fig. 1, where $x \in E$ is an arbitrary element of the universe.

Now, we see that operator $T$ transforms element $x$ with respect to its degrees $\mu_{A}(x)$ and $\nu_{A}(x)$, if $\mu_{A}(x)+\nu_{A}(x)=\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)$ and element $z \in E$ with $\mu_{A}(z)+\nu_{A}(z)<$ $\sup _{y \in E}\left(\mu_{A}(y)+\nu_{A}(y)\right)$, as it is shown in Fig. 2. $y \in E$


Figure 1.


Figure 2.

## 3 Conclusion

In future, other properties of the operator $T$ will be studied. It can obtain concrete applications for solving of different problems, e.g., in the area of the intercriteria analysis (see [3]). In this case, as we mentioned above, the degree of uncertainty for the discussed data will be decreased.

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