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# REMARKS ON THE INTUITIONISTIC FUZZY SETS. II

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ABSTRACT: Four intuitionistic fuzzy interpretations of the Zimermann-Zysno's grade of compensation are given.

KEY WORDS: fuzzy set, intuitionistic fuzzy set, Zimmermann-Zysno's grade of compensation are given.

Let E be a fixed universe and let A be an Intuitionistic Fuzzy Set (IFS) [1] over E with the form:

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \gamma_A(x) \le 1.$$

The Zimermann-Zysno's grade of compensation in the sense of [2, 3] in the intuitionistic fuzzy case can be represented for two IFSs A and B about parameters $\Gamma, \partial \in [0, 1]$ and $\Gamma + \partial \le 1$, and for every $x \in E$ by four ways:

$$Z^1_{\Gamma,\partial}(A, B)(x) = \min(\mu_A(x), \mu_B(x))^{\partial} . (1 - \min(\gamma_A(x), \gamma_B(x)))^{\Gamma},$$

$$Z^2_{\Gamma,\partial}(A, B)(x) = \min(\mu_A(x), \mu_B(x))^{\partial} . (1 - \max(\gamma_A(x), \gamma_B(x)))^{\Gamma},$$

$$Z^3_{\Gamma,\partial}(A, B)(x) = \max(\mu_A(x), \mu_B(x))^{\partial} . (1 - \min(\gamma_A(x), \gamma_B(x)))^{\Gamma},$$

$$Z^4_{\Gamma,\partial}(A, B)(x) = \max(\mu_A(x), \mu_B(x))^{\partial} . (1 - \max(\gamma_A(x), \gamma_B(x)))^{\Gamma}.$$

Obviously, for every two IFSs A and B, for every $\Gamma, \partial \in [0, 1]$ for which $\Gamma + \partial \le 1$, for every i $(1 \le i \le 4)$ and for every $x \in E$:

$$Z^i_{\Gamma,\partial}(A, B)(x) \in [0, 1].$$

When for $x \in E$: $\gamma_A(x) = 1 - \mu_A(x)$ and $\gamma_B(x) = 1 - \mu_B(x)$ and $\partial = 1 - \Gamma$, we obtain:

$$Z^1_{\Gamma,1-\Gamma}(A, B)(x) = \min(\mu_A(x), \mu_B(x))^{1-\Gamma}.(1 - \min(\gamma_A(x), \gamma_B(x)))^{\Gamma}$$

$$= \min(\mu_A(x), \mu_B(x))^{1-\Gamma}.(1 - \min(1 - \mu_A(x), 1 - \mu_B(x)))^{\Gamma}$$

$$= \min(\mu_A(x), \mu_B(x))^{1-\Gamma}.\max(\mu_A(x), \mu_B(x))^{\Gamma}.$$

Therefore, in the particular case of the fuzzy sets, this grade of compensation coincides with the Zimermann-Zysno's one.

THEOREM: For every two IFSs A and B, for every $x \in E$ and for every $\Gamma, \partial \in [0, 1]$ and $\Gamma + \partial \leq 1$:

(a) $Z^2_{\Gamma,\partial}(A, B)(x) \leq Z^1_{\Gamma,\partial}(A, B)(x) \leq Z^3_{\Gamma,\partial}(A, B)(x)$,

(b) $Z^2_{\Gamma,\partial}(A, B)(x) \leq Z^4_{\Gamma,\partial}(A, B)(x) \leq Z^3_{\Gamma,\partial}(A, B)(x)$.

When the IFSs A and B coincide, we obtain:

$$\bar{Z}_{\Gamma,\partial}(A)(x) = Z^1_{\Gamma,\partial}(A, A)(x) = \mu_A(x)^{\partial}.(1 - \gamma_A(x))^{\Gamma}$$

$$= Z^2_{\Gamma,\partial}(A, A)(x) = Z^3_{\Gamma,\partial}(A, A)(x) = Z^4_{\Gamma,\partial}(A, A)(x)$$

and $\bar{Z}_{\Gamma,\partial}$ can be used as a norm of the element $x \in E$ about set A.

Let for every two IFSs A and B and for every i ($1 \leq i \leq 4$):

$$Z^i_{\Gamma,\partial}(A, B) = \{Z^i_{\Gamma,\partial}(A, B)(x) \;/\; x \in E\}.$$

Then, $Z^i_{\Gamma,\partial}(A, B) \subset [0, 1]$.

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