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On two modifications of the intuitionistic fuzzy implication $\rightarrow_{@}$

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Abstract: Two new intuitionistic fuzzy implications are constructed. The are modifications of the intuitionistic fuzzy implication $\rightarrow_{@}$ introduced by the author and extended by P. Dworniczak. Some of the basic properties of the new implications are discussed.
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1 Introduction

In [3], a new type of intuitionistic fuzzy implications (marked by $\rightarrow_{@}$) was introduced by the author, and some of its basic properties were discussed in [4]. This implication was extended by Piotr Dworniczak in [5, 6, 7, 8]. Here, two new modifications of implication $\rightarrow_{@}$ are proposed, and some of their basic properties are discussed.

In the beginning, we must note that the concept of Intuitionistic Fuzzy Propositional Calculus (IFPC) was introduced about 25 years ago (see, e.g., [1, 2]). In IFPC, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x.

Below, we shall assume that for the three variables x, y and z the equalities: $V(x) = \langle a, b \rangle$, $V(y) = \langle c, d \rangle$, $V(z) = \langle e, f \rangle$ $(a, b, c, d, e, f, a + b, c + d, e + f \in [0, 1])$ hold.

For the needs of the discussion below, following the definition from [1], we shall define the notion of Intuitionistic Fuzzy Tautology (IFT) by:

$$x \text{ is an IFT, if and only if for } V(x) = \langle a, b\rangle \text{ holds: } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables x and y the operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1]) by:

$$V(x\&y) = \langle \min(a, c), \max(b, d)\rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d)\rangle.$$

For two variables x and y the relation $\leq$ is defined (see [1]) by:

$$V(x) \leq V(y) \ \text{ if and only if } \ a \leq c \ \text{ and } \ b \geq d.$$

2 Main results

In [3], we introduced the implication $\rightarrow_{@}$ by

$$V(x \rightarrow_{@} y) = \langle \frac{b+c}{2}, \frac{a+d}{2}\rangle.$$

It generates the negation

$$V(\neg_{@} x) = \langle \frac{b}{2}, \frac{a+1}{2}\rangle.$$

Now, we modify this operation to the following two forms:

$$V(x \rightarrow'_{@} y) = \langle \frac{b+c+\min(b,c)}{3}, \frac{a+d+\max(a,d)}{3}\rangle, \quad (1)$$

$$V(x \rightarrow''_{@} y) = \langle \frac{b+c+\max(b,c)}{3}, \frac{a+d+\min(a,d)}{3}\rangle. \quad (2)$$

First, we see that the values of both implications are intuitionistic fuzzy pairs. Really, for example, for the first implication we have

$$\begin{aligned} 0 &\leq \frac{b+c+\min(b,c)}{3} + \frac{a+d+\max(a,d)}{3} \\ &\leq \frac{a+b+c+d+\min(b,c)+\max(a,d)}{3} \\ &\leq \frac{2+\min(b,c)+\max(a,d)}{3} \\ &\leq \frac{2+\min(b,c)+\max(1-b,1-c)}{3} \\ &= \frac{2+\min(b,c)+1-\min(b,c)}{3} = 1. \end{aligned}$$

Now, we can see that

$$V(x \rightarrow'_{@} y) \leq V(x \rightarrow_{@} y) \leq V(x \rightarrow''_{@} y).$$

Second, using the well-known formula (see, e.g. [9])

$$\neg x = x \to 0,$$

in the present case we can construct the following two negations:

$$V(\neg'_{@} x) = \langle a, b\rangle \to \langle 0, 1\rangle = \langle \frac{b}{3}, \frac{a+2}{3}\rangle, \tag{3}$$

$$V(\neg''_{@} x) = \langle a, b\rangle \to \langle 0, 1\rangle = \langle \frac{2b}{3}, \frac{a}{3}\rangle. \tag{4}$$

Obviously, they are intuitionistic fuzzy pairs.

Now, we can see that

$$V(\neg'_{@} x) \leq V(\neg_{@} x) \leq V(\neg''_{@} x).$$

Theorem 1. Implication $\to'_{@}$

(a) does not satisfy Modus Ponens in the case of tautology,

(b) satisfies Modus Ponens in the IFT-case.

Proof: (a) Let x and $x \to'_{@}$ be tautologies, i.e.,

$$a = 1, b = 0, \frac{b + c + \min(b, c)}{3} = 1, \frac{a + d + \max(a, d)}{3} = 0.$$

Therefore, from second and third equalities we obtain that $c = 1$, but from first and fourth equalities we obtain that $2 + d = 0$, that is impossible.

(b). Let x and $x \to_{@} y$ be IFTs. Then

$$a \geq b$$

and

$$\frac{b + c + \min(b, c)}{3} \geq \frac{a + d + \max(a, d)}{3},$$

i.e.,

$$c - d \geq a - b + \max(a, d) - \min(b, c) \geq \max(a, d) - \min(b, c) \geq a - b \geq 0.$$

Therefore, y is an IFT.

Theorem 2. Implication $\to''_{@}$

(a) does not satisfy Modus Ponens in the case of tautology,

(b) does not satisfy Modus Ponens in the IFT-case.

Theorem 3. For the two new intuitionistic fuzzy implications and negations, none of the following three properties:

Property P1: $A \to'_{@} \neg'_{@}\neg'_{@} A$ and $A \to''_{@} \neg''_{@}\neg''_{@} A$,

Property P2: $\neg'_{@}\neg'_{@} A \to'_{@} A$ and $\neg''_{@}\neg''_{@} A \to''_{@} A$,

Property P3: $\neg'_{@}\neg'_{@}\neg'_{@} A = \neg'_{@} A$ and $\neg''_{@}\neg''_{@}\neg''_{@} A = \neg''_{@} A$

is valid.

Now, similarly to the case with negation $\neg_{@}$, the question about the form of expressions $\neg'_{@}\neg'_{@} ... \neg'_{@} A$ and $\neg''_{@}\neg''_{@} ... \neg''_{@} A$ is interesting.

Let us define:

$$\neg'^{1}_{@}A = \neg'_{@}A$$

$$\neg'^{n+1}_{@}A = \neg'_{@}\neg'^{n}_{@}A.$$

Theorem 4. Let $n \geq 1$ be a natural number.

$$\neg'^{2n-1}_{@}\langle a, b\rangle = \langle \frac{2^n}{3^{2n-1}}.b,\ \frac{2^{n-1}}{3^{2n-1}}.a\rangle,$$

$$\neg^{2n}_{@}\langle a, b\rangle = \langle \frac{2^n}{3^{2n}}.a,\ \frac{2^n}{3^{2n}}.b\rangle.$$

$$\neg'^{2n-1}_{@}\langle a, b\rangle = \langle \frac{4b + 3.(3^{2n-2} - 1)}{4.3^{2n-1}}, \frac{4a + 3^{2n} - 1}{4.3^{2n-1}}\rangle,$$

$$\neg''^{2n}_{@}\langle a, b\rangle = \langle \frac{4a + 3^{2n} - 1}{4.3^{2n}}, \frac{4b + 3.(3^{2n} - 1)}{4.3^{2n}}\rangle.$$

Corollary:

$$\lim_{n\to\infty} \neg'^{n}_{@}\langle a, b\rangle = \langle 0, 0\rangle,$$

$$\lim_{n\to\infty} \neg''^{n}_{@}\langle a, b\rangle = \langle \frac{1}{4}, \frac{3}{4}\rangle.$$

3 Conclusion

The new implications and negations have some unique properties with respect to the other implications and negations, already existing in intuitionistic fuzzy logics. A part of these properties have analogues only with the properties of implication $\rightarrow_{@}$ and negation $\neg_{@}$

There exists an interesting relation between the new negation and one of the intuitionistic fuzzy modal like operators – operator $\boxplus$ (see [2]).

In a next research, intuitionistic fuzzy set analogues of the new intuitionistic fuzzy logic implication and negation will be discussed.

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