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Derivatives Related to Intuitionistic Fuzzy Sets

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In this paper we propose a way to define derivative of a function of a special kind. This kind is described by the function $F_A(\alpha)$ defined further down.

Let $E = E(+, ||, ||_E)$ be a universe over the set of real numbers closed with respect to the operation $+$. Let

$$P = \{< x, \mu_P(x), \nu_P(x) > / x \in E\}$$

be an IFS (see[1]).

We shall introduce an operator F_A using following

DEFINITION 0: If

$$A = \{< x, \mu_A(x), \nu_A(x) > / x \in E\},$$

then

$$F_A(\alpha) = \{< x, \mu_A(x + \alpha), \nu_A(x + \alpha) > / x \in E\},$$

where a given $\alpha \in E$.

PROPOSITION 1: $F_A(\alpha)$ is an IFS.

Proof: A is an IFS. Therefore

$$0 \leq \mu_A(x + \alpha) + \nu_A(x + \alpha) \leq 1.$$

So $F_A(\alpha)$ is an IFS.

Let t be a function defined over E.

$$H_p^s := \{t(x)/(\forall x, \Delta x \in E)(||\Delta x|| \leq s \;\&\; ||\Delta x|| \leq s|t(x + \Delta x) - t(x)| \leq p(x)||\Delta x||_E)\}.$$

Let

$$M := \{t(x)/(\forall x \in E)(\exists\, t'(x) := \lim_{||\Delta x||_E \to 0} |t(x + \Delta x) - t(x)|/||\Delta x||_E\}$$

DEFINITION 1: If $\mu_A(x) \in H^s_{\mu_P}$ and $\nu_A(x) \in H^s_{\nu_P}$ and $||\Delta\alpha||_E \leq s \leq 1$ we define

$$\Delta F_A(\alpha) := F_A(\alpha + \Delta\alpha) - F_A(\alpha) :=$$

$$\{< x, |\mu_A(x + \alpha + \Delta\alpha) - \mu_A(x + \alpha)|, |\nu_A(x + \alpha + \Delta\alpha) - \nu_A(x + \alpha)| > / x \in E\}$$

PROPOSITION 2: $\Delta F_A(\alpha)$ is an IFS.

Proof: From $\mu_A(x) \in H^s_{\mu_P}$ and $\nu_A(x) \in H^s_{\nu_P}$ and $||\Delta\alpha||_E \leq s \leq 1$ we have that

$$0 \leq |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)| + |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|$$

$$\leq (\mu_P(x) + \nu_P(x))||\Delta\alpha||_E$$
$$\leq ||\Delta\alpha||_E \leq s \leq 1$$

because $0 \leq \mu_P(x) + \nu_P(x) \leq 1$. Therefore, $\Delta F_A(\alpha)$ is an IFS.

DEFINITION 2: If $\mu_A(x) \in H^s_{\mu_P}$ and $\nu_A(x) \in H^s_{\nu_P}$ and $||\Delta\alpha||_E \leq s \leq 1$ Then

$$\Delta F_A(\alpha)/||\Delta\alpha||_E :=$$

$$\{< x, |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E,$$
$$|\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E > /x \in E\}$$

PROPOSITION 3: $\Delta F_A(\alpha)/||\Delta\alpha||_E$ is an IFS.

Proof: From $\mu_A(x) \in H^s_{\mu_P}$ and $\nu_A(x) \in H^s_{\nu_P}$ and $||\Delta\alpha||_E \leq s \leq 1$ we have that

$$0 \leq |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E + |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E$$

$$\leq (\mu_P(x) + \nu_P(x)) \leq 1(0 \leq \mu_P(x) + \nu_P(x) \leq 1)$$

DEFINITION 3: If $\mu_A(x) \in H^s_{\mu_P}$, $\nu_A(x) \in H^s_{\nu_P}$, $\mu_A(x) \in M$, $\nu_A(x) \in M$ and $||\Delta\alpha||_E \leq s \leq 1$

Then

$$F'_A(\alpha) := \lim_{||\Delta\alpha||_E \to 0} \Delta F_A(\alpha)/||\Delta\alpha||_E :=$$

$$\{< x, \lim_{||\Delta\alpha||_E \to 0} |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E,$$
$$\lim_{||\Delta\alpha||_E \to 0} |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E > /x \in E\}$$

PROPOSITION 4: $F'_A(\alpha)$ is an IFS.

Proof: From $\mu_A(x) \in H^s_{\mu_p}$, $\nu_A(x) \in H^s_{\nu_P}$ we have

$$0 \leq |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E \leq \mu_P(x) \leq 1$$

and

$$0 \leq |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E \leq \nu_P(x) \leq 1.$$

From $\mu_A(x) \in M$, $\nu_A(x) \in M$ we have

$$0 \leq \lim_{||\Delta\alpha||_E \to 0} |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E \leq \mu_P(x) \leq 1$$

and

$$0 \leq \lim_{||\Delta\alpha||_E \to 0} |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E \leq \nu_P(x) \leq 1.$$

Therefore

$$\begin{aligned} 0 \leq & \lim_{||\Delta\alpha||_E \to 0} |\mu_A(x+\alpha+\Delta\alpha) - \mu_A(x+\alpha)|/||\Delta\alpha||_E \\ & + \lim_{||\Delta\alpha||_E \to 0} |\nu_A(x+\alpha+\Delta\alpha) - \nu_A(x+\alpha)|/||\Delta\alpha||_E \\ & \leq \mu_P(x) + \nu_P(x) \leq 1, \end{aligned}$$

i.e., P is an IFS.

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References

[1] Atanassov K., Intuitionistic Fuzzy Sets, Physica Verlag, 1999.

[2] Rudin W., Principles of Mathematical Analysis, Real and Complex Analysis, McGraw-Hill, 1986.