Fifth Int. Conf. on IFSs, Sofia, 22-23 Sept. 2001
NIFS 7 (2001), 3, 8-13

IF-Bags in Decision Analysis

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Abstract

In the present paper, the author discusses the concept of IF-Bags, and she applies this notion in case of dealing with a decision analysis problem under uncertainty. It is observed that a confidence factor and a psychological hesitation factor can be derived in case of a multi-attribute grading system using the notions of Intuitionistic fuzziness.

Keywords: IF-sets, Bags, IF-Bags, Confidence Factor, Psychological Hesitation Factor.

1 Introduction

The notions of IF-sets, as introduced by Atanassov[1], generalizes the concept of fuzzy sets, and with their wide-variety of applications in the areas concerning real-life applications, they provide a major alternative to the traditional concept of set membership where the degrees of membership as well as the degrees of non-membership play essential roles in the quantification of uncertainty.

In his paper [12], Yager has defined the notion of *bag* which is a type of collection, where the redundancy of objects in the collection becomes a functional property of the objects and plays an important role in case of knowledge representation and decision analysis problems. Chakrabarty, Biswas and Nanda [6] defined the concepts of *bag complements* for any fixed information system and *Cartesian product* of bags in [6]. While dealing with Object Definition Language(ODL), a type system can be formed from a basis of types and the bags can act as one of the type constructors with the help of which the basic types can be combined into structured types.

The notion of IF-Bags have been introduced by Chakrabarty, Biswas and Nanda in [5]. Some characterization of IF-Bags has also been done in [5]. The present paper deals with the concept of IF-Bags, and this notion is applied in case of a decision analysis problem involving uncertainty in multi-attribute grading. The notions of *relative cardinality* and *pseudo-relative cardinality* of IF-bags are introduced and it is observed that the confidence factor and the psychological hesitation factor can be derived in case of a multi-attribute grading system, using these notions. The relations between the *concentration of opinion, confidence co-efficient*, and *hesitation co-efficient* are established as well as the *derivability* of a decision in the context of the concerned information system is considered.

2 Preliminaries

In this section, we briefly discuss the notion of IF-bags as presented in [5].

Let X be a non-empty set. Then an IF-bag ψ drawn from X is characterized by a function

$$CM_\psi : X \longrightarrow Q,$$

where Q is the set of all crisp bags drawn from $I \times I$, where I represents the continuum [0,1].
Thus for any $x \in X$, $CM_\psi(x)$ is a crisp bag drawn from $I \times I$ and

$$C_{CM_\psi(x)} : I \times I \longrightarrow N$$

which is the characterizing count function for the bag $CM_\psi(x)$.

There exists an IF-bag ϕ drawn from any set X, such that for each $x \in X$, $CM_\phi(x)$ is an empty bag. i.e. $C_{CM_\phi(x)}(\alpha,\beta) = 0$, for each $x \in X$ and $(\alpha,\beta) \in I \times I$. This IF-bag is called the null IF-bag.

Two IF-bags ψ_1 and ψ_2 drawn from a set X are equal if $\forall\, x \in X$ and $\forall\, (\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1}(x)}(\alpha,\beta) = C_{CM_{\psi_2}(x)}(\alpha,\beta).$$

If ψ_1 and ψ_2 be two IF-bags drawn from the set X, then, ψ_1 is called a sub-bag of ψ_2, if for all $x \in X$, $(\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1}(x)}(\alpha,\beta) \leq C_{CM_{\psi_2}(x)}(\alpha,\beta).$$

The following are defined for any two IF-bags ψ_1 and ψ_2 drawn from the set X:

1. The addition of ψ_1 and ψ_2 results in the IF-bag $\psi_1 \oplus \psi_2$ such that $\forall\, x \in X$ and $\forall\, (\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1 \oplus \psi_2}(x)}(\alpha,\beta) = C_{CM_{\psi_1}(x)}(\alpha,\beta) + C_{CM_{\psi_2}(x)}(\alpha,\beta).$$

2. The removal of ψ_2 from ψ_1 results in the IF-bag $\psi_1 \ominus \psi_2$ such that $\forall\, x \in X$ and $\forall\, (\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1 \ominus \psi_2}(x)}(\alpha,\beta) = max\{C_{CM_{\psi_1}(x)}(\alpha,\beta) - C_{CM_{\psi_2}(x)}(\alpha,\beta), 0\}.$$

3. The insertion of an element $y/(a,b)$ into an IF-bag ψ results in the IF-bag ψ^o such that for $(\alpha,\beta) \neq (a,b)$ and $x \neq y$

$$C_{CM_{\psi^o}(x)}(\alpha,\beta) = C_{CM_\psi(x)}(\alpha,\beta)$$

and otherwise

$$C_{CM_{\psi^o}(y)}(a,b) = C_{CM_\psi(y)}(a,b) + 1.$$

4. The union of ψ_1 and ψ_2 is an IF-bag $\psi_1 \sqcup \psi_2$ such that $\forall\, x \in X$ and $\forall\, (\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1 \sqcup \psi_2}(x)}(\alpha,\beta) = max\{C_{CM_{\psi_1}(x)}(\alpha,\beta), C_{CM_{\psi_2}(x)}(\alpha,\beta)\}.$$

Their intersection is the IF-bag $\psi_1 \sqcap \psi_2$ such that $\forall\, x \in X$ and $\forall\, (\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi_1 \sqcap \psi_2}(x)}(\alpha,\beta) = min\{C_{CM_{\psi_1}(x)}(\alpha,\beta), C_{CM_{\psi_2}(x)}(\alpha,\beta)\}.$$

For any fixed information system, if U be the universal bag and X be any set, then the universal IF-bag $IF(U)$ for this information system is an IF-bag in which $\forall\, x \in X$, $(\alpha,\beta) \in I \times I$, the following conditions hold for each IF-bag I drawn from X:

(I) $\sum_{(\alpha,\beta)} C_{CM_{IF(U)(x)}}(\alpha,\beta) = C_U(x)$,

(II) $C_{CM_{I(x)}}(\alpha,\beta) \le C_{CM_{IF(U)(x)}}(\alpha,\beta)$.

For any IF-bag ψ, the complement of ψ is the IF-bag ψ^c such that for all $x \in X$ and $(\alpha,\beta) \in I \times I$,

$$C_{CM_{\psi^c(x)}}(\alpha,\beta) = C_{CM_{F(U)(x)}}(\alpha,\beta) - C_{CM_{\psi(x)}}(\alpha,\beta).$$

The intuitionistic fuzzy supporting set of ψ is an IF-set of X denoted by ψ^* whose membership function μ and non-membership function ν are given as below:

$$\begin{aligned}
\mu_{\psi^*}(x) &= \max_{\alpha}[min\{\max_{\beta}(C_{CM_{\psi(x)}}(\alpha,\beta)),\alpha\}] \\
\nu_{\psi^*}(x) &= \min_{\beta}[min\{\max_{\alpha}(C_{CM_{\psi(x)}}(\alpha,\beta)),\beta\}]
\end{aligned}$$

For any IF-bag ψ drawn from a set X we have

(a) $(\psi^c)^c = \psi$ (b) $\psi^* \cup (\psi^c)^* = IF(U)^*$.

(c)$\psi^* \neq (\psi^c)^*$, in general.

3 Peak-value and Cardinality Types

Let ψ be an IF-bag drawn from a set X. Then the peak-value of ψ is denoted by $\pi(\psi)$ and is given by

$$\pi(\psi) = \max_{x\in X,(\alpha,\beta)\in I\times I}\{C_{CM_{\psi(x)}}(\alpha,\beta)\}$$

The grade of concentration of ψ is denoted as $\gamma_c(\psi)$ and is given by

$$\gamma_c(\psi) = \frac{\pi(\psi)}{(\max_{\alpha\in I})_{\psi}(\alpha) - (\min_{\alpha\in I})_{\psi}(\alpha)}$$

where $(\max_{\alpha\in I})_{\psi}(\alpha)$ and $(\min_{\alpha\in I})_{\psi}(\alpha)$ denote the maximum and minimum values of α occuring in ψ.

The cardinality of an IF-bag ψ drawn from a set X is denoted by $\#(\psi)$ and is given as

$$\#(\psi) = \sum_{x\in X}\sum_{(\alpha,\beta)\in I\times I}\{C_{CM_{\psi(x)}}(\alpha,\beta) \star max[(\alpha-\beta),0]\}$$

where '$\star$' denotes the usual multiplication.

The absolute cardinality of ψ is denoted by $\#^o(\psi)$ and is given by

$$\#^o(\psi) = \sum_{x\in X}\sum_{(\alpha,\beta)\in I\times I} C_{CM_{\psi(x)}}(\alpha,\beta)$$

The pseudo cardinality of ψ is denoted by $\#_o(\psi)$ and is given by

$$\#_o(\psi) = \sum_{x \in X} \sum_{(\alpha,\beta) \in I \times I} \{C_{CM_\psi(x)}(\alpha, \beta) \star [1 - (\alpha + \beta)]\}$$

The relative cardinality of ψ is denoted by $\rho(\#)(\psi)$ and is defined as

$$\rho(\#)(\psi) = \frac{\#(\psi)}{\#^o(\psi)}.$$

The pseudo-relative cardinality of ψ is denoted by $\underline{\rho}(\#)(\psi)$ and is defined as

$$\underline{\rho}(\#)(\psi) = \frac{\#_o(\psi)}{\#^o(\psi)}.$$

4 IF-bags in Decision Analysis

Let us consider a situation when three of the recently released movies are evaluated independently by two experts ragarding their suitability for viewing by the audience of ages less than or equal to 18. Each expert has got some pre-defined attribute (e.g., violence, horror, adult content etc) and the number of attributes pre-assigned by each expert might vary. Let us assume that the first expert has pre defined two, and the second expert has pre defined three attributes for evaluating the movies. We consider the gradings to be intuitionistic in nature, i.e., they indicate the possibilistic degrees of membership and non-membership that can be associated with the conceptual grading for the specific attribute values for each movie by each expert.

For each movie $M_i (i = 1, 2, ...n)$, we calculate the confidence factor $\sigma(M_i^j)$ accoring to the kth expert E_k $(k = 1, 2, ...m)$ by using

$$\sigma_k(M_i^j) = \frac{1}{\xi_k} \sum_{j=1}^{\xi_k} \mu_{ij}^k$$

where μ represents the degree of membership, and ξ_k is the number of pre defined attribute decided by the kth expert.

The psychological hesitation factor $\tau(M_i^j)$, according to the kth expert E_k $(k = 1, 2, ...m)$ is calculated as

$$\tau_k(M_i^j) = \frac{1}{\xi_k} \sum_{j=1}^{\xi_k} [1 - (\mu_{ij}^k + \nu_{ij}^k)].$$

where ν represents the degree of non-membership.

The data representation of a hypothetical observation is furnished below:

M_i		E_1		E_2		
		A_1^1	A_2^1	A_1^2	A_2^2	A_2^3
M_1	μ	0.8	0.3	0.6	0.9	0.2
	ν	0.1	0.4	0.1	0.1	0.3
M_2	μ	0.6	0.2	0.5	0.8	0.4
	ν	0.4	0.5	0.5	0.2	0.4
M_3	μ	1.0	0.6	0.6	0.7	0.7
	ν	0	0.1	0.3	0.2	0.3

The data representation of confidence factor and psychological hesitation factor is shown below:

M_i		E_1	E_2
M_1	σ	0.55	0.56
	τ	0.2	0.27
M_2	σ	0.4	0.57
	τ	0.15	0.07
M_3	σ	0.8	0.67
	τ	0.15	0.07

Considering σ and τ from the above table, the IC-bag ψ_σ^τ is obtained such that

$$\begin{aligned} \psi_\sigma^\tau &= \{M_1/\{(0.55, 0.2)/1, (0.56, 0.27)/1\}, M_2/\{(0.2, 0.4)/1, \\ &\quad (0.27, 0.57)/1\}, M_3/\{(0.8, 0.15)/1, (0.67, 0.07)/1\}. \end{aligned}$$

It is observed that the relative cardinality of ψ represents the overall confidence factor ω_c and the pseudo-relative cardinality of ψ represents the overall psychological hesitation factor ω_h.

Thus, $\omega_c(\psi)$ and $\omega_h(\psi)$ are obtained as below:

$$\omega_c(\psi) = \rho(\#)(\psi) = 0.315$$

$$\omega_h(\psi) = \underline{\rho}(\#)(\psi) = 0.215$$

If for any ψ, $\omega_c(\psi) > \omega_h(\psi)$, then the decision is called *derivable* in the context of the concerned information system.

Otherwise, if $\omega_c(\psi) \leq \omega_h(\psi)$, then the decision is called *pseudo-derivable* in the context of the concerned information system.

Hence, in our case discussed above, the decision is derivable.

The more the value of confidence factor (or less the value of psychological hesitation factor), the the more *dependable* the decision is.

We define the concentration of opinion $\kappa(\psi)$ as below

$$\begin{aligned}\kappa(\psi) &= \vartheta \star \omega_c(\psi) \\ &= \frac{\varepsilon}{\omega_h(\psi)}\end{aligned}$$

where ϑ and ε are respectively called the *confidence co-efficient*, and *hesitation co-efficient* for the concerned information system.

Conclusion

This paper extends the concept of IF-bags as introduced in [5] and applies this notion in case of a decision analysis problem where the confidence factors and the psychological hesitation factors play significant roles in describing the contextual derivability of the decision in the context of the concerned information system.

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