Alpha generalized Homeomorphism in Intuitionistic Fuzzy Topological Spaces

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Abstract: This paper introduces intuitionistic fuzzy alpha generalized homeomorphism and intuitionistic fuzzy M - alpha generalized homeomorphism in intuitionistic fuzzy topological spaces. Also they are related to the fundamental concepts of intuitionistic continuous mappings and intuitionistic fuzzy open mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy alpha generalized closed set, intuitionistic fuzzy alpha generalized continuous mapping, intuitionistic fuzzy alpha generalized homeomorphism, intuitionistic fuzzy M-alpha generalized homeomorphism.

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1 Introduction

The concept of fuzzy sets was introduced by Zadeh [12] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy alpha generalized homeomorphism and intuitionistic fuzzy M-alpha generalized homeomorphism. Also they are related to the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings. We provide some characterizations of intuitionistic fuzzy alpha generalized homeomorphism.

2 Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

\[ A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \]

where the functions \( \mu_A(x): X \rightarrow [0, 1] \) and \( \nu_A(x): X \rightarrow [0, 1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \). Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form
A = \{ x, \mu_A(x), \nu_A(x) \} / x \in X \} and B = \{ x, \mu_B(x), \nu_B(x) \} / x \in X \}. Then

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)
(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
(c) \( A^c = \{ x, \nu_A(x), \mu_A(x) \} / x \in X \}
(d) \( A \cap B = \{ x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) \} / x \in X \}
(e) \( A \cup B = \{ x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) \} / x \in X \}.

For the sake of simplicity, we shall use the notation \( A = \{ x, \mu_A, \nu_A \} \) instead of \( A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \} \). Also for the sake of simplicity, we shall use the notation \( A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \} \) instead of \( A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle \).

The intuitionistic fuzzy sets \( 0_\infty = \{ \langle x, 0, 1 \rangle / x \in X \} \) and \( 1_\infty = \{ \langle x, 1, 0 \rangle / x \in X \} \) are respectively the empty set and the whole set of \( X \).

**Definition 2.3:**[3] An intuitionistic fuzzy topology (IFT in short) on \( X \) is a family \( \tau \) of IFSs in \( X \) satisfying the following axioms.

(i) \( 0_\infty, 1_\infty \in \tau \)
(ii) \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \)
(iii) \( \bigcup_{i \in J} G_i \in \tau \) for any family \( \{ G_i / i \in J \} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS in short) in \( X \). The complement \( A^c \) of an IFOS \( A \) in IFTS \( (X, \tau) \) is called an intuitionistic fuzzy closed set (IFCS in short) in \( X \).

**Definition 2.4:**[3] Let \( (X, \tau) \) be an IFTS and \( A = \{ x, \mu_A, \nu_A \} \) be an IFS in \( X \). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

\[ \text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \} \],
\[ \text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \} \].

**Definition 2.5:**[7] An IFS \( A = \{ x, \mu_A, \nu_A \} \) in an IFTS \( (X, \tau) \) is said to be an

(i) intuitionistic fuzzy \( \alpha \)-open set (IF\( \alpha \)OS in short) if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \),
(ii) intuitionistic fuzzy \( \alpha \)-closed set (IF\( \alpha \)CS in short) if \( \text{cl}(\text{int}((\text{cl}(A))) \subseteq A \).

The family of all IFCS (resp. IF\( \alpha \)CS, IF\( \alpha \)OS) of an IFTS \( (X, \tau) \) is denoted by IFC(X) (resp. IF\( \alpha \)C(X), IFO(X), IF\( \alpha \)O(X)).

**Definition 2.6:**[10] Let \( A \) be an IFS in an IFTS \( (X, \tau) \). Then

\[ \text{acl}(A) = \bigcup \{ G / G \text{ is an IFaOS in } X \text{ and } G \subseteq A \} \],
\[ \text{acl}(A) = \bigcap \{ K / K \text{ is an IFaCS in } X \text{ and } A \subseteq K \} \],
\[ \text{agcl}(A) = \bigcap \{ M / M \text{ is an IFaGCS in } X \text{ and } A \subseteq M \} \].

**Result 2.7:**[10] Let \( A \) be an IFS in \( (X, \tau) \). Then

(i) \( \alpha \text{cl}(A) = A \cup \text{cl}(\text{int}(\text{cl}(A))) \)
(ii) \( \alpha \text{int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A))) \)

**Definition 2.8:**[10] An IFS \( A \) in an IFTS \( (X, \tau) \) is an
(i) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.

(ii) *intuitionistic fuzzy alpha generalized closed set* (IF$\alpha$GCS in short) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $X$.

**Result 2.9:** [10] Every IFCS, IF$\alpha$CS is an IF$\alpha$GCS but the converses may not be true in general.

**Definition 2.10:** [7] Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be

(i) *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

(ii) *intuitionistic fuzzy$\alpha$-continuous* (IF$\alpha$ continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$.

**Result 2.11:** [7] Every IF continuous mapping is an IF$\alpha$-continuous mapping but the converse may not be true in general.

**Definition 2.12:** [9] Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be

(i) *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B) \in \text{IFGC}(X)$ for every IFCS $B$ in $Y$.

(ii) *intuitionistic fuzzy$\alpha$-generalized continuous* (IF$\alpha$G continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{GC}(X)$ for every IFCS $B$ in $Y$.

**Definition 2.13:** [9] Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be an *intuitionistic fuzzy$\alpha$ generalized alpha open mapping* (IF$\alpha$G open mapping in short) if $f(A) \in \text{IF}\alpha\text{GOS}(X)$ for every IFOS $A$ in $X$.

**Definition 2.14:** [9] Let $f$ be a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then the map $f$ is said to be an *intuitionistic fuzzy$\alpha$ generalized irresolute* (IF$\alpha$G irresolute in short) if $f^{-1}(B) \in \text{IF}\alpha\text{GCS}(X)$ for every IF$\alpha$GCS $B$ in $Y$.

**Definition 2.15:** Let $f$ be a bijection mapping from an IFTS $(X, \tau)$ into an IFTS $(Y, \sigma)$. Then $f$ is said to be

(i) *intuitionistic fuzzy homeomorphism* (IF homeomorphism in short) if $f$ and $f^{-1}$ are IF continuous mappings.

(ii) *intuitionistic fuzzy$\alpha$ homeomorphism* (IF$\alpha$ homeomorphism in short) if $f$ and $f^{-1}$ are IF$\alpha$ continuous mappings.

(iii) *intuitionistic fuzzy generalized homeomorphism* (IFG homeomorphism in short) if $f$ and $f^{-1}$ are IFG continuous mappings.

**Result 2.16:** [9] Every IF continuous mapping, IF$\alpha$ continuous mapping is an IF$\alpha$G continuous but the converse may not be true in general.
Definition 2.17: [9] An IFTS \((X, \tau)\) is said to be an intuitionistic \(\alpha T_{1/2}\) (in short IF\(\alpha T_{1/2}\)) space if every IF\(\alpha\)GCS in \(X\) is an IFCS in \(X\).

Definition 2.18: [9] An IFTS \((X, \tau)\) is said to be an intuitionistic \(\beta T_{1/2}\) (in short IF\(\beta T_{1/2}\)) space if every IF\(\alpha\)GCS in \(X\) is an IFGCS in \(X\).

Definition 2.19: An IFTS \((X, \tau)\) is said to be an intuitionistic \(\delta T_{1/2}\) (in short IF\(\delta T_{1/2}\)) space if every IF\(\alpha\)GCS in \(X\) is an IF\(\alpha\)CS in \(X\).

3 Intuitionistic fuzzy alpha generalized homeomorphisms

In this section we introduce intuitionistic fuzzy alpha generalized homeomorphism and study some of its properties.

Definition 3.1: A bijection mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy alpha generalized homeomorphism (IF\(\alpha\)G homeomorphism in short) if \(f\) and \(f^{-1}\) are IF\(\alpha\)G continuous mappings.

Example 3.2: Let \(X = \{a, b\}\), \(Y = \{u, v\}\) and \(T_1 = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle\), \(T_2 = \langle y, (0.4, 0.7), (0.4, 0.2) \rangle\). Then \(\tau = \{0\,\text{e}, T_1, 1\,\text{e}\}\) and \(\sigma = \{0\,\text{e}, T_2, 1\,\text{e}\}\) are IFTs on \(X\) and \(Y\) respectively. Define a bijection mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = u\) and \(f(b) = v\). Then \(f\) is an IF\(\alpha\)G continuous mapping and \(f^{-1}\) is also an IF\(\alpha\)G continuous mapping. Therefore \(f\) is an IF\(\alpha\)G homeomorphism.

Theorem 3.3: Every IF homeomorphism is an IF\(\alpha\)G homeomorphism but not conversely.

Proof: Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be an IF homeomorphism. Then \(f\) and \(f^{-1}\) are IF continuous mappings. This implies \(f\) and \(f^{-1}\) are IF\(\alpha\)G continuous mappings. That is the mapping \(f\) is IF\(\alpha\)G homeomorphism.

Example 3.4: Let \(X = \{a, b\}\), \(Y = \{u, v\}\) and \(T_1 = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle\), \(T_2 = \langle y, (0.5, 0.4), (0.4, 0.2) \rangle\). Then \(\tau = \{0\,\text{e}, T_1, 1\,\text{e}\}\) and \(\sigma = \{0\,\text{e}, T_2, 1\,\text{e}\}\) are IFTs on \(X\) and \(Y\) respectively. Define a bijection mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = u\) and \(f(b) = v\). Then \(f\) is an IF\(\alpha\)G homeomorphism but not an IF homeomorphism since \(f\) and \(f^{-1}\) are not an IF continuous mappings.

Theorem 3.5: Every IF\(\alpha\) homeomorphism is an IF\(\alpha\)G homeomorphism but not conversely.

Proof: Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be an IF\(\alpha\) homeomorphism. Then \(f\) and \(f^{-1}\) are IF\(\alpha\) continuous mappings. This implies \(f\) and \(f^{-1}\) are IF\(\alpha\)G continuous mappings. That is the mapping \(f\) is an IF\(\alpha\)G homeomorphism.

Example 3.6: Let \(X = \{a, b\}\), \(Y = \{u, v\}\) and \(T_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle\), \(T_2 = \langle y, (0.2, 0.2), (0.7, 0.8) \rangle\). Then \(\tau = \{0\,\text{e}, T_1, 1\,\text{e}\}\) and \(\sigma = \{0\,\text{e}, T_2, 1\,\text{e}\}\) are IFTs on \(X\) and \(Y\) respectively. Define a mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = u\) and \(f(b) = v\). Then \(f\) is an IF\(\alpha\)G homeomorphism. Consider an IFCS \(A = \langle x, (0.7, 0.8), (0.2, 0.2) \rangle\) in \(Y\). Then \(f^{-1}(A) = \langle y, (0.7, 0.8), (0.2, 0.2) \rangle\) in \(Y\).
0.8), (0.2, 0.2) is not an IFαCS in X. This implies f is not an IFα continuous mapping. Hence f is not an IFα homeomorphism.

**Theorem 3.7:** Let f: (X, τ) → (Y, σ) be an IFαG homeomorphism, then f is an IF homeomorphism if X and Y are IFαT_{1/2} space.

**Proof:** Let B be an IFCS in Y. Then f^{-1}(B) is an IFαGCS in X, by hypothesis. Since X is an IFαT_{1/2} space, f^{-1}(B) is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is an IFαG continuous mapping. Let A be an IFCS in X. Then (f^{-1})^{-1}(A) = f(A) is an IFαGCS in Y, by hypothesis. Since Y is an IFαT_{1/2} space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

**Theorem 3.8:** Let f: (X, τ) → (Y, σ) be an IFαG homeomorphism, then f is an IFG homeomorphism if X and Y are IFαbT_{1/2} space.

**Proof:** Let B be an IFCS in Y. Then f^{-1}(B) is an IFαGCS in X, by hypothesis. Since X is an IFαbT_{1/2} space, f^{-1}(B) is an IFGCS in X. Hence f is an IFG continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is an IFαG continuous mapping. Let A be an IFCS in X. Then (f^{-1})^{-1}(A) = f(A) is an IFαGCS in Y, by hypothesis. Since Y is an IFαbT_{1/2} space, f(A) is an IFGCS in Y. Hence f^{-1} is an IFG continuous mapping. Therefore the mapping f is an IFG homeomorphism.

**Theorem 3.9:** Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFαG continuous mapping, then the following are equivalent.

(i) f is an IFαG closed mapping
(ii) f is an IFαG open mapping
(iii) f is an IFαG homeomorphism.

**Proof:** (i) → (ii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFαG closed mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFαG continuous mapping. That is every IFOS in X is an IFαGOS in Y. Hence f^{-1} is an IFαG open mapping.

(ii) → (iii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f is an IFαG open mapping. This implies f^{-1}: (Y, σ) → (X, τ) is IFαG continuous mapping. Hence f and f^{-1} are IFαG continuous mappings. That is f is an IFαG homeomorphism.

(iii) → (i): Let f is an IFαG homeomorphism. That is f and f^{-1} are IFαG continuous mappings. Since every IFCS in X is an IFαGCS in Y, f is an IFαG closed mapping.

**Remark 3.10:** The composition of two IFαG homeomorphisms need not be an IFαG homeomorphism in general.

**Example 3.11:** Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let T_1 = ⟨x, (0.8, 0.6), (0.2, 0.4)⟩, T_2 = ⟨y, (0.6, 0.1), (0.4, 0.3)⟩ and T_3 = ⟨z, (0.4, 0.4), (0.6, 0.2)⟩. Then τ = {0~, T_1, 1~}, σ = {0~, T_2, 1~} and Ω = {0~, T_1, 1~} are IFTs on X, Y and Z respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f(a) = c, f(b) = d and g: (Y, σ) → (Z, Ω) by f(c) = u, f(d) = v. Then f and f^{-1} are IFαG continuous mappings. Also g and g^{-1} are IFαG continuous mappings. But the composition g.f: X → Z is not an IFαG homeomorphism since g.f is not an IFαG continuous mapping.
4  Intuitionistic fuzzy M - alpha generalized homeomorphisms

**Definition 4.1:** A bijection mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an *intuitionistic fuzzy M-alpha generalized homeomorphism* (IF\( \alpha \)G homeomorphism in short) if \( f \) and \( f^{-1} \) are IF\( \alpha \)G irresolute mappings.

**Theorem 4.2:** Every IF\( \alpha \)G homeomorphism is an IF\( \alpha \)G homeomorphism but not conversely.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \alpha \)G homeomorphism. Let \( B \) be IFCS in \( Y \). This implies \( B \) is an IF\( \alpha \)GCS in \( Y \). By hypothesis \( f^{-1}(B) \) is an IF\( \alpha \)GCS in \( X \). Hence \( f \) is an IF\( \alpha \)G continuous mapping. Similarly we can prove \( f^{-1} \) is an IF\( \alpha \)G continuous mapping. This implies the mapping \( f \) is an IF\( \alpha \)G homeomorphism.

**Example 4.3:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( T_1 = (\langle x, (0.4, 0.3), (0.6, 0.7) \rangle, T_2 = (\langle y, (0.2, 0.1), (0.4, 0.5) \rangle \). Then \( \tau = \{0, T_1, 1\} \) and \( \sigma = \{0, T_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a bijection mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \alpha \)G homeomorphism. Let us consider an IFS \( G = (\langle y, (0.3, 0.2), (0.7, 0.7) \rangle \) in \( Y \). Clearly \( G \) is an IF\( \alpha \)GCS in \( Y \). But \( f^{-1}(G) \) is not an IF\( \alpha \)GCS in \( X \). That is \( f \) is not an IF\( \alpha \)G irresolute mapping. Hence \( f \) is not an IF\( \alpha \)G homeomorphism.

**Theorem 4.4:** If the mapping \( f: X \rightarrow Y \) is an IF\( \alpha \)G homeomorphism, then \( \alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B)) \) for every IFS \( B \) in \( Y \).

**Proof:** Let \( B \) be an IFS in \( Y \). Then \( \alpha cl(B) \) is an IF\( \alpha \)GCS in \( Y \). This implies \( \alpha cl(B) \) is an IF\( \alpha \)GCS in \( Y \). Since the mapping \( f \) is an IF\( \alpha \)G irresolute mapping, \( f^{-1}(\alpha cl(B)) \) is an IF\( \alpha \)GCS in \( X \). This implies \( \alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)) \). Now \( \alpha cl(f^{-1}(B)) \subseteq \alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)) \). Hence \( \alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B)) \) for every IFS \( B \) in \( Y \).

**Theorem 4.5:** If \( f: X \rightarrow Y \) is an IF\( \alpha \)G homeomorphism, then \( \alpha cl(f^{-1}(B)) = f^{-1}(\alpha cl(B)) \) for every IFS \( B \) in \( Y \).

**Proof:** Since \( f \) is an IF\( \alpha \)G homeomorphism, \( f \) is an IF\( \alpha \)G irresolute mapping. Consider an IFS \( B \) in \( Y \). Clearly \( \alpha cl(B) \) is an IF\( \alpha \)GCS in \( Y \). This implies \( \alpha cl(B) \) is an IF\( \alpha \)GCS in \( Y \). By hypothesis \( f^{-1}(\alpha cl(B)) \) is an IF\( \alpha \)GCS in \( X \). Since \( f^{-1}(B) \subseteq f^{-1}(\alpha cl(B)) \), \( \alpha cl(f^{-1}(B)) \subseteq \alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B)) \). This implies \( \alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B)) \). Since \( f \) is an IF\( \alpha \)G homeomorphism, \( f^{-1}: Y \rightarrow X \) is an IF\( \alpha \)G irresolute mapping. Consider an IFS \( f^{-1}(B) \) in \( X \). Clearly \( \alpha cl(f^{-1}(B)) \) is an IF\( \alpha \)GCS in \( X \). Hence \( \alpha cl(f^{-1}(B)) \) is an IF\( \alpha \)GCS in \( X \). This implies \( f^{-1}(\alpha cl(f^{-1}(B))) = f(\alpha cl(f^{-1}(B))) \) is an IF\( \alpha \)GCS in \( Y \). Clearly \( B = f^{-1}(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(f^{-1}(B))) \). Therefore \( \alpha cl(B) \subseteq \alpha cl(f(\alpha cl(f^{-1}(B)))) = f(\alpha cl(f^{-1}(B))) \), since \( f^{-1} \) is an IF\( \alpha \)G irresolute mapping. Hence \( f^{-1}(\alpha cl(B)) \subseteq f^{-1}(f(\alpha cl(f^{-1}(B))) = \alpha cl(f^{-1}(B)) \). That is \( f^{-1}(\alpha cl(B)) \subseteq f^{-1}(\alpha cl(B)) \). This implies \( \alpha cl(f^{-1}(B)) = f^{-1}(\alpha cl(B)) \).

**Theorem 4.6:** If \( f: X \rightarrow Y \) is an IF\( \alpha \)G homeomorphism, then \( \alpha cl(f(B)) = f(\alpha cl(B)) \) for every IFS \( B \) in \( X \).

**Proof:** Since \( f \) is an IF\( \alpha \)G homeomorphism, \( f^{-1} \) is an IF\( \alpha \)G homeomorphism. Let us consider an IFS \( B \) in \( X \). By theorem(4.6) \( \alpha cl(f(B)) = f(\alpha cl(B)) \) for every IFS \( B \) in \( X \).
Remark 4.7: The composition of two IFMαG homeomorphisms is IFMαG homeomorphism in general.

Proof: Let f: X → Y and g: Y → Z be any two IFMαG homeomorphisms. Let A be an IFαGCS in Z. Then by hypothesis, g⁻¹(A) is an IFαGCS in Y. Then by hypothesis, f⁻¹(g⁻¹(A)) is an IFαGCS in X. Hence (g.f)⁻¹ is an IFαG irresolute mapping. Now let B be an IFαGCS in X. Then by hypothesis, f(B) is an IFαGCS in Y. Then by hypothesis g(f(B)) is an IFαGCS in Z. This implies g.f is an IFαG irresolute mapping. Hence g.f is an IFMαG homeomorphism. That is the composition of two IFMαG homeomorphisms is an IFMαG homeomorphism in general.

References