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ON INTUITIONISTIC FUZZY SETS OVER UNIVERSES WITH HIERARCHICAL STRUCTURES

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1 Introduction

Let E be a fixed universe and let A be an Intuitionistic Fuzzy Set (IFS, see [1] over E.

Let F be another universe and let the set E be an IFS over F having the form:

$$E = \{\langle y, \mu_E(y), \nu_E(y)\rangle | y \in F\}.$$

Therefore the element $x \in E$ has the form (see [2]):

$$x = \langle y, \mu_E(y), \nu_E(y)\rangle,$$

i.e., $x \in F \times [0,1] \times [0,1]$,

$$\begin{aligned} A = \ & \{\langle\langle y, \mu_E(y), \nu_E(y)\rangle, \mu_A(\langle y, \mu_E(y), \nu_E(y)\rangle), \nu_A(\langle y, \mu_E(y), \nu_E(y)\rangle)\rangle | \\ & \langle y, \mu_E(y), \nu_E(y)\rangle \in E\} \end{aligned}$$

and there exists a bijection between the $E-$ and $F-$ elements of $x-$ and $y-$types, respectively.

Thus we can use the symbol "y" for both $y-$ and $x-$elements.

Let A/E stand for "A is an IFS over E".

If the degrees of membership and non-membership of an element y to a set A in the frames of a universe E are $\mu_A(y)$ and $\nu_A(y)$ and the element $\langle y, \mu_A(y), \nu_A(y)\rangle$ has degrees of membership and non-membership to the set E within the universe F are $\mu_E(y)$ and $\nu_E(y)$, then we define:

$$A = \{\langle y, \mu_E(y).\mu_A(y), \nu_E(y).\nu_A(y)\rangle | y \in F\}. \quad (1)$$

Obviously, from A/E and B/E follows that the sets A and B have equal $y-$elements.

All intuitionistic fuzzy operations, relations and operators can be transformed directly over the new objects. For example, the most general case, when the relations $A/E, B/F$, E/G, F/G hold, the IFS $A \cap B$ over the universe G has the form:

$$\{\langle y, \min(\mu_E(y).\mu_A(y), \mu_F(y).\mu_B(y)), \max(\nu_E(y).\nu_A(y), \nu_F(y).\nu_B(y))\rangle | y \in G\}.$$

2 Main results

When the universe is ordered, e.g., by relation $\leq$ the set A is called in [3] an *"IFS over an universe with hierarchical structure (H-IFS)".*

Here we shall extend the concept of H-IFS transforming some ideas and results from [1, 2].

First, we shall start with en example. Let E be a finite universe with the form

$$E = \{e_1, e_2, e_3, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_2, \{e_1, e_3\}\}\}.$$

Therefore, the IFS A over E will have the form

$$A = \{\langle e_1, \mu_A(e_1), \nu_A(e_1)\rangle, \langle e_2, \mu_A(e_2), \nu_A(e_2)\rangle, \langle e_3, \mu_A(e_3), \nu_A(e_3)\rangle,$$

$$\langle \{e_1, e_2\}, \mu_A(\{e_1, e_2\}), \nu_A(\{e_1, e_2\})\rangle, \langle \{e_1, e_3\}, \mu_A(\{e_1, e_3\}), \nu_A(\{e_1, e_3\})\rangle,$$

$$\langle \{e_1, e_2, \{e_1, e_3\}\}, \mu_A(\{e_1, e_2, \{e_1, e_3\}\}), \nu_A(\{e_1, e_2, \{e_1, e_3\}\})\rangle\}. \quad (2)$$

Obviously,

$$card(E) = 6.$$

Let the other set has the form

$$E_1 = \{e_1, e_2, e_3, \{f_1, f_2\}, \{f_1\}, \{g_1, g_2, \{g_1, g_3\}\}\}.$$

We can tell that elements e_1, e_2, e_3 are *"elements from first level"*, elements f_1, f_2, g_1, g_2 – *"elements from second level"* and elements g_1, g_3 – *"elements from third level"*.

Of course, one element can be element from two or more different types. For example for set E objects e_1, e_2, e_3 are elements from each one of the three types.

If there is an order between some of the elements of E, e.g., if for $i = 1, 2, 3$: $e_i = i$, this order ($\leq$ or $<$) cannot be extend over the rest E-elements. If the order is $\subset$, it will be valid for fourth and sixth elements of E, but will net be possible for the rest E-elements. Finally, the order $\in$ will be valid, e.g., for the fifth and sixth e-elements, but not, e.g. for the third and sixth elements.

Now, for H-IFS E that has n levels and for every natural number $i \leq n$ we can introduce set $support_i(E)$ that contains all E-elements that are from i-th level and that are not sets of elements of $(i+1)$-th level. Moreover,

$$support(E) = \bigcup_{i \leq n} support_i(E).$$

On the other hand we see that the e-elements are from *different hierarchical levels* and this is our reason to use the name of H-IFS for such sets. Obviously, this form of H-IFS is an extension of the first one.

We see that for the above set E with $card(E) = 6$: $card(support(E)) = 3$, while for set E_1 with the same cardinality ($card(E_1) = 6$) $card(support(E_1)) = 8$. Therefore, in the present case the bijection from [2] is not valid.

In [1] is formulated and proved (for the bijective case) following

Theorem 1.15.1: If $A/E, E/F$ and F/G, then:

(a) $A = \{\langle y, \mu_F(y).\mu_E(y).\mu_A(y), \nu_F(y).\nu_E(y).\nu_A(y)\rangle / y \in G\}$,

(b) $A/(E/F) = (A/E)/F$.

As it is noted in [1], all the above results can be transformed for the case of ordinary fuzzy sets, as follows: if A is a fuzzy set over universe E and E is a fuzzy set over universe F, then A is a fuzzy set over universe F of the form:

$$A = \{\langle y, \mu_E(y).\mu_A(y)\rangle | y \in F\}$$

where μ_A and μ_E are degrees of membership in the above sense.

Let E be a finite or infinite set and let for each its element e: $\mu_A(e)$ and $\nu_A(e)$ exist. By analogy with (1) we can construct the set $A/E/support(E)$. Extending the idea from [2], already we can use not only the multiplicative form of presentation of A in F, as it is in (1), but at least three other forms, that we will mention for the example with IFS A from (2), as follows

$$A/E[optimistic]support(E) = \{\langle e_1, \mu_A(e_1), \nu_A(e_1)\rangle, \langle e_2, \mu_A(e_2), \nu_A(e_2)\rangle, \langle e_3, \mu_A(e_3), \nu_A(e_3)\rangle,$$
$$\langle \{e_1, e_2\}, \max(\mu_A(e_1), \mu_A(e_2)), \min(\nu_A(e_1), \nu_A(e_2))\rangle,$$
$$\langle \{e_1, e_3\}, \max(\mu_A(e_1), \mu_A(e_3)), \min(\nu_A(e_1), \nu_A(e_3))\rangle,$$
$$\langle \{e_1, e_2, \{e_1, e_3\}\}, \max(\mu_A(e_1), \mu_A(e_2), \mu_A(e_3)), \min(\nu_A(e_1), \nu_A(e_2), \nu_A(e_3))\rangle\}$$

$$A/E[pessimistic]support(E) = \{\langle e_1, \mu_A(e_1), \nu_A(e_1)\rangle, \langle e_2, \mu_A(e_2), \nu_A(e_2)\rangle, \langle e_3, \mu_A(e_3), \nu_A(e_3)\rangle,$$
$$\langle \{e_1, e_2\}, \min(\mu_A(e_1), \mu_A(e_2)), \max(\nu_A(e_1), \nu_A(e_2))\rangle,$$
$$\langle \{e_1, e_3\}, \min(\mu_A(e_1), \mu_A(e_3)), \max(\nu_A(e_1), \nu_A(e_3))\rangle,$$
$$\langle \{e_1, e_2, \{e_1, e_3\}\}, \min(\mu_A(e_1), \mu_A(e_2), \mu_A(e_3)), \max(\nu_A(e_1), \nu_A(e_2), \nu_A(e_3))\rangle\}$$

$$A/E[averrage]support(E) = \{\langle e_1, \mu_A(e_1), \nu_A(e_1)\rangle, \langle e_2, \mu_A(e_2), \nu_A(e_2)\rangle, \langle e_3, \mu_A(e_3), \nu_A(e_3)\rangle,$$
$$\langle \{e_1, e_2\}, \frac{\mu_A(e_1) + \mu_A(e_2)}{2}, \frac{\nu_A(e_1) + \nu_A(e_2)}{2}\rangle,$$
$$\langle \{e_1, e_3\}, \frac{\mu_A(e_1) + \mu_A(e_3)}{2}, \frac{\nu_A(e_1) + \nu_A(e_3)}{2}\rangle,$$
$$\langle \{e_1, e_2, \{e_1, e_3\}\}, \frac{\mu_A(e_1) + \mu_A(e_2) + \frac{\mu_A(e_1) + \mu_A(e_3)}{2}}{2}, \frac{\nu_A(e_1) + \nu_A(e_2) + \frac{\nu_A(e_1) + \nu_A(e_3)}{2}}{2}\rangle\}$$
$$= \{\langle e_1, \mu_A(e_1), \nu_A(e_1)\rangle, \langle e_2, \mu_A(e_2), \nu_A(e_2)\rangle, \langle e_3, \mu_A(e_3), \nu_A(e_3)\rangle,$$
$$\langle \{e_1, e_2\}, \frac{\mu_A(e_1) + \mu_A(e_2)}{2}, \frac{\nu_A(e_1) + \nu_A(e_2)}{2}\rangle,$$
$$\langle \{e_1, e_3\}, \frac{\mu_A(e_1) + \mu_A(e_3)}{2}, \frac{\nu_A(e_1) + \nu_A(e_3)}{2}\rangle,$$
$$\langle \{e_1, e_2, \{e_1, e_3\}\}, \frac{3\mu_A(e_1) + \mu_A(e_2) + \frac{\mu_A(e_1) + \mu_A(e_3)}{2}}{2}, \frac{\nu_A(e_1) + \nu_A(e_2) + \frac{\nu_A(e_1) + \nu_A(e_3)}{2}}{2}\rangle\}$$

The following assertion is valid

Theorem: For each IFS A over E

$$A/E[pessimistic]support(E) \subset A/E[averrage]support(E) \subset A/E[optimistic]support(E).$$

References

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[3] Chountas, P., Representation of Null Values with the Aid H-IFS, Notes on IFS, Vol. 13, 2007, No. 1, 20-33.