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ON INTUITIONISTIC FIZZY NEGATIONS AND INTUITIONISTIC FIZZY LEVEL OPERATORS

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Abstract

Some relations between intuitionistic fuzzy negations and intuitionistic fuzzy level operations $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$ are studied.

1 On some previous results

The concept of the Intuitionistic Fuzzy Set (IFS, see [?]) was introduced in 1983 as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the IFS case. One of them - operartion "negation" now there is 24 different forms (see [?]). In [?] the relations between the "classical" negation and the two standard modal operators "necessity" and "possibility" are given. Here, we shall study the relations between the intuitionistic fuzzy negations and the intuitionistic fuzzy extended modal operations $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$.

In some definitions we shall use functions sg and $\overline{\text{sg}}$:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

For two IFSs A and B the following relations are valid:

$$A \subset B \text{ iff } (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \nu_A(x) \geq \nu_B(x)),$$

$$A \supset B \text{ iff } B \subset A,$$

$$A = B \text{ iff } (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)).$$

Let A be a fixed IFS. In [?] definitions of some level operators are given. The most important (and global) of them are:

$$P_{\alpha,\beta}(A) = \{\langle x, \max(\mu_A(x), \alpha), \min(\nu_A(x), \beta)\rangle | x \in E\},$$

$$Q_{\alpha,\beta}(A) = \{\langle x, \min(\mu_A(x), \alpha), \max(\nu_A(x), \beta)\rangle | x \in E\},$$

where $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$.

In [**?**, **?**, **?**, **?**, **?**] the following 27 different negations are described.

$$\neg_1 A = \{\langle \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_2 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_3 A = \{\langle \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^2\rangle | x \in E\},$$

$$\neg_4 A = \{\langle \nu_A(x), 1 - \nu_A(x)\rangle | x \in E\},$$

$$\neg_5 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_6 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_7 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \mu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_8 A = \{\langle 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_9 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_{10} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), 1 - \nu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_{11} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{12} A = \{\langle \nu_A(x).(\mu_A(x) + \nu_A(x)), \mu_A(x).(\mu_A(x) + \nu_A(x)^2)\rangle | x \in E\},$$

$$\neg_{13} A = \{\langle \text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{14} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{15} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{16} A = \{\langle \overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(1 - \mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{17} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(\nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{18} A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{19} A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{20} A = \{\langle x, \nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{21} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^n\rangle | x \in E\},$$

where real number $n \in [2, \infty)$,

$$\neg_{22} A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\text{sg}}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{23} A = \{\langle x, (1 - \mu_A(x)).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(1 - \nu_A(x))\rangle | x \in E\},$$

$$\neg_{24} A = \{\langle x, (1 - \mu_A(x)).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{25} A = \{\langle x, 1 - \nu_A(x), 0\rangle | x \in E\},$$

$$\neg^{\varepsilon} A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \varepsilon)\rangle | x \in E\},$$

where $\varepsilon \in [0,1]$,

$$\neg^{\varepsilon,\eta} A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta)\rangle | x \in E\},$$

where $0 \leq \varepsilon \leq \eta \leq 1$.

2 Main results

Now, following and extending the idea from [**?**, **?**, **?**] we shall prove following

Theorem 1: For every IFS A and for every $\alpha, \beta \in [0, 1]$ so that $\alpha + \beta \leq 1$, the following properties are valid:

(1) $\neg_1 P_{\alpha,\beta}(A) = Q_{\beta,\alpha}(\neg_1 A)$,
(2) $\neg_2 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_2 A)$,
(3) $\neg_4 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_4 A)$,
(4) $\neg_5 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_5 A)$,
(5) $\neg_6 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_6 A)$,
(6) $\neg_7 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_7 A)$,
(7) $\neg_8 P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_8 A)$,
(8) $\neg_9 P_{\alpha,\beta}(A) \supset P_{\alpha,\beta}(\neg_9 A)$,
(9) $\neg_{10} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{10} A)$,
(10) $\neg_{11} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{11})$,
(11) $\neg_{13} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{13} A)$,
(12) $\neg_{14} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{14} A)$,
(13) $\neg_{15} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{15} A)$,
(14) $\neg_{16} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{16} A)$,
(15) $\neg_{17} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{17} A)$,
(16) $\neg_{20} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg_{20} A)$,
(17) $\neg_{25} P_{\alpha,\beta}(A) \supset P_{\alpha,\beta}(\neg_{25} A)$,
(18) $\neg^{\varepsilon} P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg^{\varepsilon} A)$,
(19) $\neg P_{\alpha,\beta}(A) \subset P_{\alpha,\beta}(\neg^{\varepsilon,\eta} A)$.

Proof: Let $\alpha, \beta \in [0, 1]$ be given so that $\alpha + \beta \leq 1$, and let A be an IFS. Then we obtain directly that:

$$\begin{aligned}\neg_1 P_{\alpha,\beta}(A) &= \neg_1 \{\langle x, \max(\mu_A(x), \alpha), \min(\nu_A(x), \beta)\rangle | x \in E\} \\ &= \{\langle x, \min(\nu_A(x), \beta), \max(\mu_A(x), \alpha)\rangle | x \in E\} \\ &= Q_{\beta,\alpha}(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}) \\ &= Q_{\beta,\alpha}(\neg_1 A).\end{aligned}$$

Therefore equality (1) is valid.

The rest assertions can be proved by another manner. Let us prove, for example (4).

Let $\alpha, \beta \in [0, 1]$ be given so that $\alpha + \beta \leq 1$, and let A be an IFS. Then:

$$\neg_5 P_{\alpha,\beta}(A) = \neg_5 \{\langle x, \max(\mu_A(x), \alpha), \min(\nu_A(x), \beta)\rangle | x \in E\}$$

$$= \{\langle \overline{\text{sg}}(1 - \min(\nu_A(x), \beta)), \text{sg}(1 - \min(\nu_A(x), \beta))\rangle | x \in E\}$$

and

$$P_{\alpha,\beta}(\neg_5 A) = P_{\alpha,\beta}(\{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_A(x))\rangle | x \in E\}$$

$$\{\langle \max(\overline{\text{sg}}(1 - \nu_A(x)), \alpha), \min(\text{sg}(1 - \nu_A(x)), \beta)\rangle | x \in E\}.$$

Now, let

$$X \equiv \max(\overline{\text{sg}}(1 - \nu_A(x)), \alpha) - \overline{\text{sg}}(1 - \min(\nu_A(x), \beta)).$$

If $\nu_A(x) = 1$, then

$$\begin{aligned}X &= \max(\overline{\text{sg}}(0), \alpha) - \overline{\text{sg}}(1 - \min(1, \beta)) \\ &= \max(1, \alpha) - \overline{\text{sg}}(1 - \beta) = 1 - \overline{\text{sg}}(1 - \beta) \geq 0.\end{aligned}$$

If $\nu_A(x) < 1$, then

$$X = \max(0, \alpha) - \overline{\mathrm{sg}}(1 - \min(\nu_A(x), \beta))$$
$$= \alpha \geq 0.$$

Let

$$Y \equiv \mathrm{sg}(1 - \min(\nu_A(x), \beta)) - \min(\mathrm{sg}(1 - \nu_A(x)), \beta).$$

If $\nu_A(x) = 1$, then

$$Y \equiv \mathrm{sg}(1 - \min(1, \beta)) - \min(\mathrm{sg}(0), \beta)$$
$$= \mathrm{sg}(1 - \beta) - \min(0, \beta) = \mathrm{sg}(1 - \beta) \geq 0.$$

If $\nu_A(x) < 1$, then $1 - \min(\nu_A(x), \beta) > 0$ and

$$Y = 1 - \min(1, \beta) = 1 - \beta \geq 0.$$

Therefore inclusion (4) is valid.

Theorem 2: For every IFS A and for every $\alpha, \beta \in [0, 1]$ so that $\alpha + \beta \leq 1$, the following properties are valid:
(1) $\neg_1 Q_{\alpha,\beta}(A) = P_{\beta,\alpha}(\neg_1 A)$,
(2) $\neg_2 Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_2 A)$,
(3) $\neg_4 Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_4 A)$,
(4) $\neg_6 Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_6 A)$,
(5) $\neg_7 Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_7 A)$,
(6) $\neg_8 Q_{\alpha,\beta}(A) \subset Q_{\alpha,\beta}(\neg_8 A)$,
(7) $\neg_9 Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_9 A)$,
(8) $\neg_{10} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{10} A)$,
(9) $\neg_{11} Q_{\alpha,\beta}(A) \subset Q_{\alpha,\beta}(\neg_{11})$,
(10) $\neg_{13} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{13} A)$,
(11) $\neg_{14} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{14} A)$,
(12) $\neg_{15} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{15} A)$,
(13) $\neg_{16} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{16} A)$,
(14) $\neg_{17} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{17} A)$,
(15) $\neg_{20} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{20} A)$,
(16) $\neg_{24} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{24} A)$,
(17) $\neg_{25} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg_{25} A)$,
(18) $\neg^{\varepsilon} Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg^{\varepsilon} A)$,
(19) $\neg Q_{\alpha,\beta}(A) \supset Q_{\alpha,\beta}(\neg^{\varepsilon,\eta} A)$.

The validity of these assertions is checked analogously as above ones.

3 Conclusion

In a next research authors will study the above properties for the case of other extended intuitionistic fuzzy modal operators and for the intuitionistic fuzzy topological operators.

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