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Intercriteria analysis using special type of intuitionistic fuzzy implications

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Abstract: The possibility for changes in the algorithm of InterCriteria Analysis using an intuitionistic fuzzy implications from a special type instead of relations, is discussed. This new development of the theory of the InterCriteria Analysis method is a prerequisite for application of the method in advanced procedures of decision making under uncertainty.

Keywords: Intercriteria analysis, Intuitionistic fuzzy implication, IF index matrix.
AMS Classification: 03E72.

1 Introduction

The concept of InterCriteria Analysis was introduced in [2, 5]. It is based on the apparatus of the Index Matrices (IMs, see [2]) and of Intuitionistic Fuzzy Logics (IFSs, see, e.g., [3]).

The present paper is based on [7], where, for the first time we discussed the possibility, the data, that will be processed by intercriteria analysis, to be Intuitionistic Fuzzy Pairs (IFP, see [6]), variables or formulas, or more generally - intuitionistic fuzzy data (see [8]). In the procedure, the relations $<$, $>$ and $=$ are used. Here, we change these relations with intuitionistic fuzzy implications from a special type.

First, we mention that the Intuitionistic Fuzzy Pair (IFP) is an object in the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process and which components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc.

The concept of Index Matrix (IM) was discussed in a series of papers collected in [2].

Let I be a fixed set of indices and $\mathcal{R}$ be the set of the real numbers. By IM with index sets K and L $(K, L \subset I)$, we denote the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\},$ for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

In [1, 2], different operations, relations and operators are defined over IMs. For the needs of the present research, we will introduce the definitions of some of them.

When elements a_{k_i,l_j} are some variables, propositions or formulas, we obtain an extended IM with elements from the respective type. Then, we can define the evaluation function V that juxtaposes to this IM a new one with elements – IFPs $\langle \mu, \nu\rangle$, where $\mu, \nu, \mu + \nu \in [0, 1]$. The new IM, called Intuitionistic Fuzzy IM (IFIM), contains the evaluations of the variables, propositions, etc., i.e., it has the form

$$V([K, L, \{a_{k_i,l_j}\}]) = [K, L, \{V(a_{k_i,l_j})\}] = [K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}]$$

$$= \begin{array}{c|ccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1}\rangle & \dots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j}\rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_i & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1}\rangle & \dots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle & \dots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n}\rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1}\rangle & \dots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j}\rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n}\rangle \end{array},$$

where for every $1 \leq i \leq m, 1 \leq j \leq n$: $[V(a_{k_i,l_j}) = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle$ and $0 \leq \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \leq 1$.

2 Intercriteria analysis using intuitionistic fuzzy implications from a special type

In [4], the intuitionistic fuzzy implication $\rightarrow$ is called *tautologically asymmetric* if for every two different IFPs x and y (i.e., $a \neq c$ or $b \neq d$):

$$x \rightarrow y \text{ is a tautology iff } y \rightarrow x \text{ is not a tautology.}$$

Now, following [7], here we describe shortly the intercriteria analysis, but changes in the algorithm the relations $<$, $>$ and $=$ with intuitionistic fuzzy implications that are tautologically asymmetric.

In [4] was shown that there are 112 different intuitionistic fuzzy implications that are tautologically asymmetric and they are $\rightarrow_s$ for $s = 1, 4, ..., 7, 9, 10, 12, ..., 15, 17, ..., 19, 21, 24, ..., 26, 28, 29, 46, ..., 56, 58, 60, 61, 64, 66, 67, 69, 71, ..., 72, 75, 78, 80, 81, 91, ..., 96, 98, ..., 100, 102, 106, 108, ..., 113, 119, ..., 128, 134, ..., 152, 154, ..., 166, 169, 169, 175, 179, 184, 186, ..., 189$. In [4], their list is given.

Let below $\rightarrow_s$ is one of these implications.

Let us have the set of objects $O = \{O_1, O_2, ..., O_n\}$ that must be evaluated by criteria from the set $C = \{C_1, C_2, ..., C_m\}$.

Let us have an IM

$$A = \begin{array}{c|ccccccc}
 & O_1 & \cdots & O_i & \cdots & O_j & \cdots & O_n \\
\hline
C_1 & a_{C_1,O_1} & \cdots & a_{C_1,O_i} & \cdots & a_{C_1,O_j} & \cdots & a_{C_1,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_k & a_{C_k,O_1} & \cdots & a_{C_k,O_i} & \cdots & a_{C_k,O_j} & \cdots & a_{C_k,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_l & a_{C_l,O_1} & \cdots & a_{C_l,O_i} & \cdots & a_{C_l,O_j} & \cdots & a_{C_l,O_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_m & a_{C_m,O_1} & \cdots & a_{C_m,O_i} & \cdots & a_{C_m,O_j} & \cdots & a_{C_m,O_n}
\end{array},$$

where for every p, q $(1 \leq p \leq m,\ 1 \leq q \leq n)$:

(1) C_p is a criterion, taking part in the evaluation,

(2) O_q is an object, being evaluated.

(3) $a_{C_p,O_q} = \langle \alpha_{C_p,O_q}, \beta_{C_p,O_q} \rangle$ is an intuitionistic fuzzy pair.

Let IFPs $\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle$, $\langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$, $\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle$ and $\langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle$ satisfy conditions:

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \neq \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle \tag{1}$$

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \neq \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle. \tag{2}$$

Let $S^{\mu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \rightarrow_s \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle \text{ is a tautology, and}$$

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \rightarrow_s \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle \text{ is a tautology,}$$

or

$$\langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle \rightarrow_s \langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \text{ is a tautology, and}$$

$$\langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle \rightarrow_s \langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \text{ is a tautology}$$

are simultaneously satisfied.

Let $S^{\nu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle \rightarrow_s \langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \text{ is a tautology, and}$$

$$\langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle \rightarrow_s \langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \text{ is a tautology,}$$

or

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \rightarrow_s \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle \text{ is a tautology, and}$$

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \rightarrow_s \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle \text{ is a tautology}$$

are simultaneously satisfied.

Obviously,

$$S^{\mu}_{k,l} + S^{\nu}_{k,l} \le \frac{n(n-1)}{2}.$$

The number $S^{\pi}_{k,l}$ corresponds to the cases, when (1) and (2) are not valid and to the cases, when in the above formulas there is not tautology.

Now, for every k, l, such that $1 \le k < l \le m$ and for $n \ge 2$, we define

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

Hence,

$$\mu_{C_k,C_l} + \nu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)} + 2\frac{S^{\nu}_{k,l}}{n(n-1)} \le 1.$$

Therefore, $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle$ is an IFP. Now, we can construct the IM

$$\begin{array}{c|ccc} & C_1 & \cdots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \cdots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \cdots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

that determines the degrees of correspondence between criteria $C_1, \ldots, C_m$.

3 Conclusion

In the presented research, an Intercriteria Analysis over intuitionistic fuzzy data is discussed, for the case, in which relations are changed with intuitionistic fuzzy implications. It extends the area of objects and rules which the Intercriteria Analysis uses.

In a next research of the authors, the above described constructions will be extended to the case of 3-dimensional IMs. This new development of the theory of the InterCriteria Analysis method can improve the applicability of the method in problems requiring reasoning and decision making under uncertainty.

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