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(α, β)-cut of intuitionistic fuzzy ideals

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Abstract:

For any Intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in E\}$ of a set E, we define a (α, β)-cut of A as the crisp subset $\{x \in E \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$ of E. In this paper some interesting properties of (α, β)-cut of Intuitionistic fuzzy ideals of a ring were discussed.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy ideal, (α,β)-cut.

Mathematics Subject Classification: 03E72.

1 Introduction

The idea of Intuitionistic fuzzy sets was initiated by K.T.Atanassov [1] and it was extended to Intuitionistic fuzzy ideals by Banerjee and Basnet [2, 3]. In theory of fuzzy sets the level subsets are very very important tools for development of the theory. The motivation of the paper is the same as that of level subsets in fuzzy set theory i.e. to establish some important links between Intuitionistic fuzzy sets and crisp sets. That might be a help for characterizing Intuitionistic fuzzy algebraic structures.

2 Preliminaries

Definition 2.1. [1] Let E be a fixed nonempty set. An Intuitionistic Fuzzy Set (IFS) A of E is an object of the following form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in E\}$ where $\mu_A : E \to [0, 1]$ and $\nu_A: E \to [0, 1]$ define the degree of membership and degree of nonmembership of the element $x \in E$ respectively and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2. [1] If $A = \{< x, \mu_A(x), \nu_A(x) >| x \in E\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >| x \in E\}$ be any two IFS of a set E then

- $A \subseteq B$ if and only if for all $x \in E$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
- $A = B$ if and only if for all $x \in E$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$;
- $A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) >| x \in E\}$, where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$;
- $A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) >| x \in E\}$, where $(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$.

Also we see that a fuzzy set has the form $\{< x, \mu_A(x), \mu^c{}_A(x) >| x \in E\}$, where $\mu^c{}_A(x) = 1 - \mu_A(x)$.

Definition 2.3.[2] An IFS A = $\{< x, \mu_A(x), \nu_A(x) >| x\in R\}$ of a ring R is said to be an Intutionistic Fuzzy Ideal (in short IFI) of R if

(i) $\mu_A(x - y) \geq \min \{ \mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$

(iii) $\nu_A(x - y) \leq \max \{\nu_A(x), \nu_A(y)\}$

(iv) $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\}$ for all $x, y \in R$.

Theorem 2.4.[2] If $A = \{< x, \mu_A(x), \nu_A(x) >| x\in R\}$ is an IFI of R then $\mu_A(0) \geq \mu_A(x)$, $\nu_A(0) \leq \nu_A(x)$, $\mu_A(-x) = \mu_A(x)$ and $\nu_A(-x) = \nu_A(x)$ for all $x\in R$.

Definition 2.5.[2] Let $A = \{< x, \mu_A(x), \nu_A(x) >| x\in R\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >| x\in R\}$ be two IFI's of a ring R then their sum $A + B$ is defined as

$$A + B = \{< x, (\mu_A+\mu_B)(x), (\nu_A+\nu_B)(x) >| x\in R\} \text{ where}$$

$$(\mu_A + \mu_B)(x) = \underset{x=a+b}{Sup}\{\min\{\mu_A(a), \mu_B(b)\}\} \text{ and } (\nu_A + \nu_B)(x) = \underset{x=a+b}{Inf}\{\max\{\nu_A(a), \nu_B(b)\}\}$$

Definition 2.6.[2] Let $A = \{< x, \mu_A(x), \nu_A(x) >| x\in R\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >| x\in R\}$ be two IFI's of a ring R then their product AB is defined as AB = $\{< x, (\mu_A\mu_B)(x), (\nu_A\nu_B)(x) >| x\in R\}$ where,

$$(\mu_A\mu_B)(x) = \underset{\substack{x=\sum a_i b_i \\ i<\infty}}{Sup}\{\min_i\{\min\{\mu_A(a_i), \mu_B(b_i)\}\}\}$$

and

$$(\nu_A\nu_B)(x) = \underset{\substack{x=\sum a_i b_i \\ i<\infty}}{Inf}\{\max_i\{\max\{\nu_A(a_i), \nu_B(b_i)\}\}\}$$

Theorem 2.7.[2] If A and B are two IFI's of a ring R then $A + B$ and AB are also IFI's of R.

3 (α, β)-cut of an IFS

Definition 3.1. For any Intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) >| x\in E\}$ of a set E, we define a (α, β)-cut of A as the crisp subset $\{x\in E \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$ of E and it is denoted by $C_{\alpha, \beta}(A)$.

Lemma 3.2. If A is an IFI of a ring R then $C_{\alpha, \beta}(A)$ is an ideal of R if $\mu_A(0) \geq \alpha$, $\nu_A(0) \leq \beta$.

Proof. Let $\mu_A(0) \geq \alpha$, $\nu_A(0) \leq \beta$. Clearly $C_{\alpha, \beta}(A) \neq \phi$. Let $x, y \in C_{\alpha,\beta}(A)$. Then $\mu_A(x), \mu_A(y) \geq \alpha$ and $\nu_A(x), \nu_A(y) \leq \beta$. Now $\mu_A(x - y) \geq \min \{ \mu_A(x), \mu_A(y)\} \geq \alpha$ and $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \beta$.
Thus $x - y \in C_{\alpha, \beta}(A)$. Also if $r\in R$ then $\mu_A(rx) \geq \max\{\mu_A(r), \mu_A(x)\} \geq \mu_A(x) \geq \alpha$,
$\mu_A(xr) \geq \max\{\mu_A(x), \mu_A(r)\} \geq \mu_A(x) \geq \alpha$ and $\nu_A(rx) \leq \min\{\nu_A(r), \nu_A(x)\} \leq \nu_A(x) \leq \beta$,
$\nu_A(xr) \leq \min\{\nu_A(x), \nu_A(r)\} \leq \nu_A(x) \leq \beta$. Therefore $rx, xr \in C_{\alpha, \beta}(A)$. Hence $C_{\alpha, \beta}(A)$ is an ideal of R.

Theorem 3.3. If A is an IFI of R then $C_{\alpha, \beta}(A) \subseteq C_{\gamma, \delta}(A)$ if $\alpha \geq \gamma$ and $\beta \leq \delta$.

Proof. Let $x \in C_{\alpha, \beta}(A)$, then $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$. Since $\alpha \geq \gamma$ and $\beta \leq \delta$, so $\mu_A(x) \geq \alpha \geq \gamma$ and $\nu_A(x) \leq \beta \leq \delta$. Therefore $x \in C_{\gamma, \delta}(A)$. Hence $C_{\alpha, \beta}(A) \subseteq C_{\gamma, \delta}(A)$.

Corollary 3.4. If $\alpha + \beta \leq 1$ then $C_{1-\beta, \beta}(A) \subseteq C_{\alpha, \beta}(A) \subseteq C_{\alpha, 1-\alpha}(A)$.

Theorem 3.5. If $C_{\alpha, \beta}(A)$ is an ideal of R for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ then $A = \{< x, \mu_A(x), \nu_A(x) >| x \in R\}$ is an IFI of R.

Proof. Let $x, y \in R$ and $\alpha = \min\{ \mu_A(x), \mu_A(y) \}$ and $\beta = \max\{\nu_A(x), \nu_A(y)\}$.
Then $\mu_A(x) \geq \alpha$, $\nu_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha$, $\nu_A(y) \leq \beta \Rightarrow x, y \in C_{\alpha, \beta}(A)$.
Since $C_{\alpha, \beta}(A)$ is an ideal of R so $x - y \in C_{\alpha, \beta}(A)$.
Consequently $\mu_A(x - y) \geq \alpha = \min\{ \mu_A(x), \mu_A(y) \}$ and $\nu_A(x - y) \leq \beta = \max\{\nu_A(x), \nu_A(y)\}$.
Next, let $\delta = \max\{ \mu_A(x), \mu_A(y) \}$ say $\delta = \mu_A(x)$.
Since $\mu_A(x) + \nu_A(x) \leq 1$, so $\nu_A(x) \leq 1 - \mu_A(x) = 1 - \delta$. Thus $x \in C_{\delta, 1-\delta}(A)$. Since $C_{\delta, 1-\delta}(A)$ is an ideal of R so $xy \in C_{\delta, 1-\delta}(A)$. Hence $\mu_A(xy) \geq \delta = \max\{ \mu_A(x), \mu_A(y) \}$. Similarly we can show $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\}$. Hence A is an IFI of R.

Theorem 3.6. If A and B are IFI's of a ring R then $C_{\alpha, \beta}(A \cap B) = C_{\alpha, \beta}(A) \cap C_{\alpha, \beta}(B)$.

Proof. We have $C_{\alpha, \beta}(A \cap B) = \{x \in R \mid (\mu_A \cap \mu_B)(x) \geq \alpha, (\nu_A \cup \nu_B)(x) \leq \beta\}$. Now
$x \in C_{\alpha, \beta}(A \cap B) \Leftrightarrow (\mu_A \cap \mu_B)(x) \geq \alpha$ and $(\nu_A \cup \nu_B)(x) \leq \beta$
$\Leftrightarrow \min\{ \mu_A(x), \mu_A(y) \} \geq \alpha$ and $\max\{\nu_A(x), \nu_A(y)\} \leq \beta$.
$\Leftrightarrow \mu_A(x), \mu_A(y) \geq \alpha$ and $\nu_A(x), \nu_A(y) \leq \beta \Leftrightarrow x \in C_{\alpha, \beta}(A)$ and $x \in C_{\alpha, \beta}(B)$
$\Leftrightarrow x \in C_{\alpha, \beta}(A) \cap C_{\alpha, \beta}(B)$. Therefore $C_{\alpha, \beta}(A \cap B) = C_{\alpha, \beta}(A) \cap C_{\alpha, \beta}(B)$.

Theorem 3.7. If $A \subseteq B$ then $C_{\alpha, \beta}(A) \subseteq C_{\alpha, \beta}(B)$, where A and B are IFI's of R.

Theorem 3.8. If A and B are IFI's of R then $C_{\alpha, \beta}(A \cup B) \supseteq C_{\alpha, \beta}(A) \cup C_{\alpha, \beta}(B)$.

Proof. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$ so by theorem 3.7, $C_{\alpha, \beta}(A) \subseteq C_{\alpha, \beta}(A \cup B)$ and $C_{\alpha, \beta}(B) \subseteq C_{\alpha, \beta}(A \cup B)$ and therefore $C_{\alpha, \beta}(A \cup B) \supseteq C_{\alpha, \beta}(A) \cup C_{\alpha, \beta}(B)$.

Remark.3.9. However the reverse inclusion does not hold, as shown by the following example: Consider the ring $(\boldsymbol{Z_4}, +_4, \times_4)$ where $\boldsymbol{Z_4} = \{ 0, 1, 2, 3\}$ and define
$\mu_A(0) = 0.8$, $\mu_A(2) = 0.5$, $\mu_A(1) = \mu_A(3) = 0.2$; $\nu_A(0) = 0.1$, $\nu_A(2) = 0.4$, $\nu_A(1) = \nu_A(3) = 0.6$
and $\mu_B(0) = 0.9$, $\mu_B(2) = 0.4$, $\mu_B(1) = \mu_B(3) = 0.3$; $\nu_B(0) = 0.1$, $\nu_B(2) = 0.2$, $\nu_B(1) = \nu_B(3) = 0.6$
Then $A = \{< x, \mu_A(x), \nu_A(x) >| x \in \boldsymbol{Z_4}\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >| x \in \boldsymbol{Z_4}\}$ are IFI's of $\boldsymbol{Z_4}$.
Now $C_{.5, .2}(A) = \{0\}$ and $C_{.5, .2}(B) = \{0\}$.
Also $(\mu_A \cup \mu_B)(0) = 0.9$, $(\mu_A \cup \mu_B)(2) = 0.5$, $(\mu_A \cup \mu_B)(1) = (\mu_A \cup \mu_B)(3) = 0.3$
and $(\nu_A \cap \nu_B)(0) = 0.1$, $(\nu_A \cap \nu_B)(2) = 0.2$, $(\nu_A \cap \nu_B)(1) = (\nu_A \cap \nu_B)(3) = 0.6$. Since $C_{.5, .2}(A \cup B) = \{0, 2\}$ so $C_{.5, .2}(A) \cup C_{.5, .2}(B) \neq C_{.5, .2}(A \cup B)$.

Remark 3.10. The equality holds if $\alpha + \beta = 1$ as shown below:
Let $x \in C_{\alpha, \beta}(A \cup B)$, then $(\mu_A \cup \mu_B)(x) \geq \alpha \Rightarrow \max\{\mu_A(x), \mu_B(x)\} \geq \alpha \Rightarrow \mu_A(x) \geq \alpha$ or $\mu_B(x) \geq \alpha$
If $\mu_A(x) \geq \alpha$ then $\nu_A(x) \leq 1 - \mu_A(x) \leq 1 - \alpha = \beta$ and so $x \in C_{\alpha, \beta}(A) \subseteq C_{\alpha, \beta}(A) \cup C_{\alpha, \beta}(B)$.
Similarly if $\mu_B(x) \geq \alpha$ then $x \in C_{\alpha, \beta}(B) \subseteq C_{\alpha, \beta}(A) \cup C_{\alpha, \beta}(B)$. Hence the equality follows.

Theorem 3.11. $C_{\alpha, \beta}(\cap\{A_i \mid i \in I\}) = \cap \{C_{\alpha, \beta}(A_i) \mid i \in I\}$.

Theorem 3.12. For any two IFI's A and B of a ring R, $C_{\alpha,\beta}(A) + C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A + B)$ and the equality holds if $\alpha + \beta = 1$.

Proof. Let $x = y + z \in C_{\alpha,\beta}(A) + C_{\alpha,\beta}(B)$, where $y \in C_{\alpha,\beta}(A)$ and $z \in C_{\alpha,\beta}(B)$.

Then $\mu_A(y) \geq \alpha$, $\nu_A(y) \leq \beta$, and $\mu_B(z) \geq \alpha$, $\nu_B(z) \leq \beta$

$\Rightarrow \min\{\mu_A(y), \mu_B(z)\} \geq \alpha$ and $\max\{\nu_A(y), \nu_B(z)\} \leq \beta$

$$\Rightarrow \quad \underset{x=y+z}{Sup}\{\min\{\mu_A(y), \mu_B(z)\}\} \geq \alpha$$

and

$$\underset{x=y+z}{Sup}\{\max\{\nu_A(y), \nu_B(z)\}\} \leq \beta$$

$\Rightarrow (\mu_A+\mu_B)(x) \geq \alpha$ and $(\nu_A+\nu_B)(x) \leq \beta$

$\Rightarrow x \in C_{\alpha,\beta}(A + B)$

Hence, $C_{\alpha,\beta}(A) + C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A + B)$.

For the other part let $\alpha + \beta = 1$ and $x \in C_{\alpha,\beta}(A + B)$.

Then $(\mu_A+\mu_B)(x) \geq \alpha$ and $(\nu_A+\nu_B)(x) \leq \beta$.

Now $(\mu_A+\mu_B)(x) \geq \alpha \quad \Rightarrow \underset{x=y+z}{Sup}\{\min\{\mu_A(y), \mu_B(z)\}\} \geq \alpha$

$\Rightarrow \min\{\mu_A(a), \mu_B(b)\}\} \geq \alpha$, for some $x = a + b$

$\Rightarrow \mu_A(a) \geq \alpha$ and $\mu_B(b) \geq \alpha$

$\Rightarrow \nu_A(a) \leq 1 - \mu_A(a) \leq 1 - \alpha = \beta$ and $\nu_B(b) \leq 1 - \mu_B(b) \leq 1 - \alpha = \beta$

$\Rightarrow a \in C_{\alpha,\beta}(A)$, $b \in C_{\alpha,\beta}(B)$

$\Rightarrow x = a + b \in C_{\alpha,\beta}(A) + C_{\alpha,\beta}(B)$

Thus $C_{\alpha,\beta}(A + B) \subseteq C_{\alpha,\beta}(A) + C_{\alpha,\beta}(B)$ and so the equality follows.

Remark 3.14. In the above theorem the equality does not hold as can be seen by the following example: consider the ring R and IFI's A and B of R as of the remark 3.9. Note that $A + B = \{< x, (\mu_A+\mu_B)(x), (\nu_A+\nu_B)(x) >| x \in R\}$ where

$$(\mu_A + \mu_B)(x) = \underset{x=a+b}{Sup}\{\min\{\mu_A(a), \mu_B(b)\}\}$$

and

$$(\nu_A + \nu_B)(x) = \underset{x=a+b}{Inf}\{\max\{\nu_A(a), \nu_B(b)\}\}$$

Therefore we get

$(\mu_A+\mu_B)(0) = Sup\{\min\{\mu_A(0), \mu_B(0)\}, \min\{\mu_A(2), \mu_B(2)\}\}$

$= Sup\{\min\{0.8, 0.9\}, \min\{0.5, 0.4\}\}$

$= 0.8$

next

$(\mu_A+\mu_B)(1) = Sup\{\min\{\mu_A(0), \mu_B(1)\}, \min\{\mu_A(1), \mu_B(0)\}, \min\{\mu_A(2), \mu_B(3)\}, \min\{\mu_A(3), \mu_B(2)\}\}$

$= Sup\{\min\{0.8, 0.3\}, \min\{0.2, 0.9\}, \min\{0.5, 0.3\}, \min\{0.2, 0.4\}\}$

$= 0.3$

Similarly $(\mu_A+\mu_B)(2) = 0.5$ and $(\mu_A+\mu_B)(3) = 0.3$

Similarly $(\nu_A+\nu_B)(0) = 0.1$, $(\nu_A+\nu_B)(1) = 0.6$, $(\nu_A+\nu_B)(2) = 0.2$, $(\nu_A+\nu_B)(3) = 0.6$

Now $C_{.5,.2}(A+B) = \{0, 2\}$ and from remark 3.9, we get $C_{.5,.2}(A) = \{0\}$ and $C_{.5,.2}(B) = \{0\}$

Therefore $C_{.5,.2}(A) + C_{.5,.2}(B) = \{0\} \neq \{0, 2\} = C_{.5,.2}(A+B)$.

Theorem 3.13. For any two IFI's A and B of a ring R, $C_{\alpha,\beta}(A)C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(AB)$ and the equality holds if $\alpha + \beta = 1$.

Proof. We have $AB = \{< x, (\mu_A\mu_B)(x), (\nu_A\nu_B)(x) >| x\in R\}$ where,

$$(\mu_A\mu_B)(x) = \underset{x=\sum_{i<\infty} a_ib_i}{Sup} \{\min_i\{\mu_A(a_i), \mu_B(b_i)\}\}$$

and

$$(\nu_A\nu_B)(x) = \underset{x=\sum_{i<\infty} a_ib_i}{Inf} \{\max_i\{\nu_A(a_i), \nu_B(b_i)\}\}$$

Let $x = \sum_{i<\infty} a_ib_i \in C_{\alpha,\beta}(A)C_{\alpha,\beta}(B)$, where $a_i\in C_{\alpha,\beta}(A)$ and $b_i\in C_{\alpha,\beta}(B)$, for all i.

Then $\mu_A(a_i) \geq \alpha$, $\nu_A(a_i) \leq \beta$, and $\mu_B(b_i) \geq \alpha$, $\nu_B(b_i) \leq \beta$, for all i.

$\Rightarrow \min\{\mu_A(a_i), \mu_B(b_i)\} \geq \alpha$ and $\max\{\nu_A(a_i), \nu_B(b_i)\} \leq \beta$, for all i.

$\Rightarrow \min_i\{\min\{\mu_A(a_i), \mu_B(b_i)\}\} \geq \alpha$ and $\max_i\{\max\{\nu_A(a_i), \nu_B(b_i)\}\} \leq \beta$

$$\Rightarrow \underset{x=\sum_{i<\infty} a_ib_i}{Sup} \{\min_i\{\min\{\mu_A(a_i), \mu_B(b_i)\}\}\} \geq \alpha$$

and

$$\underset{x=\sum_{i<\infty} a_ib_i}{Inf} \{\max_i\{\max\{\nu_A(a_i), \nu_B(b_i)\}\}\} \leq \beta$$

$\Rightarrow (\mu_A\mu_B)(x) \geq \alpha$ and $(\nu_A\nu_B)(x) \leq \beta$

$\Rightarrow x \in C_{\alpha,\beta}(AB)$ and so the result follows.

For the second part let $\alpha + \beta = 1$ and $x \in C_{\alpha,\beta}(AB)$

Then $(\mu_A\mu_B)(x) \geq \alpha$ and $(\nu_A\nu_B)(x) \leq \beta$

Now $(\mu_A\mu_B)(x) \geq \alpha$

$$\Rightarrow \underset{x=\sum_{i<\infty} a_ib_i}{Sup} \{\min_i\{\min\{\mu_A(a_i), \mu_B(b_i)\}\}\} \geq \alpha$$

$\Rightarrow \min_i\{\min\{\mu_A(a_i), \mu_B(b_i)\}\} \geq \alpha$ for some $x = \sum_{i<\infty} a_ib_i$

$\Rightarrow \min\{\mu_A(a_i), \mu_B(b_i)\} \geq \alpha$, for all i.

$\Rightarrow \mu_A(a_i) \geq \alpha$, $\mu_B(b_i) \geq \alpha$, for all i.

$\Rightarrow \nu_A(a_i) \leq 1 - \mu_A(a_i) \leq 1 - \alpha = \beta$ and $\nu_B(b_i) \leq 1 - \mu_B(b_i) \leq 1 - \alpha = \beta$, for all i.

$\Rightarrow a_i\in C_{\alpha,\beta}(A)$ and $b_i\in C_{\alpha,\beta}(B)$, for all i.

$\Rightarrow x = \sum_{i<\infty} a_ib_i \in C_{\alpha,\beta}(A)C_{\alpha,\beta}(B)$

Hence $C_{\alpha,\beta}(AB) \subseteq C_{\alpha,\beta}(A)C_{\alpha,\beta}(B)$ and so the equality follows.

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