Notes on Intuitionistic Fuzzy Sets
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New intuitionistic fuzzy extended modal operators

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Abstract: Seven new intuitionistic fuzzy operators are introduced. Some of their basic properties are discussed.
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1 Introduction

Following [3, 5], in the present paper we discuss extensions of some modal type of operators, defined over Intuitionistic Fuzzy sets (IFSs). We mention that each IFS A has the form

$$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where E is some universe, $A \subseteq E$ is a fixed set, $\mu_A(x)$ and $\nu_A(x)$ are the degrees of the membership and of the non-membership of $x \in E$ to A and $\mu_A(x), \nu_A(x), \mu_A(x) + \nu_A(x) \in [0, 1]$.

Below we will use only notation A^*, for brevity, and for this reason the asterisk will be omitted.

In [1], for a first time, extensions of the intuitionistic fuzzy modal operators $\Box$ and $\Diamond$ are introduced. They have the forms:

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1-\alpha).\pi_A(x)\rangle | x \in E\},$$

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

where $\alpha, \beta \in [0, 1]$ are fixed numbers and $\alpha + \beta \leq 1$.

Obviously, $D_\alpha(A) = F_{\alpha,1-\alpha}(A)$.

In [2], five other operators were defined and in [3], their basic properties were described. Their forms are the following:

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$
$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$
$$H^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle | x \in E\},$$
$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$
$$J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle | x \in E\}.$$

where $\alpha, \beta \in [0, 1]$ are fixed numbers.

In [4], these operators are extended, changing their parameters, being two real numbers, with whole IFSs. The respective definitions are:

$$F_B(A) = \{\langle x, \mu_A(x) + \mu_B(x).\pi_A(x), \nu_A(x) + \nu_B(x).\pi_A(x)\rangle \mid x \in E\};$$
$$G_B(A) = \{\langle x, \mu_B(x).\mu_A(x), \nu_B(x).\nu_A(x)\rangle \mid x \in E\};$$
$$H_B(A) = \{\langle x, \mu_B(x).\mu_A(x), \nu_A(x) + \nu_B(x).\pi_A(x)\rangle \mid x \in E\},$$
$$H^*_B(A) = \{\langle x, \mu_B(x).\mu_A(x), \nu_A(x) + \nu_B(x).(1 - \mu_B(x).\mu_A(x) - \nu_A(x))\rangle \mid x \in E\},$$
$$J_B(A) = \{\langle x, \mu_A(x) + \mu_B(x).\pi_A(x), \nu_B(x).\nu_A(x)\rangle \mid x \in E\},$$
$$J^*_B(A) = \{\langle x, \mu_A(x) + \mu_B(x).(1 - \mu_A(x) - \nu_B(x).\nu_A(x)), \nu_B(x).\nu_A(x)\rangle \mid x \in E\},$$

where B is an IFS.

Let

$$A \subseteq B \text{ if and only if } (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)),$$

$$A \supseteq B \text{ if and only if } B \subseteq A,$$

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\},$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in E\}.$$

As it is mentioned in [5], operator $F_B(A)$ totally extends operator $F_{\alpha,\beta}(A)$, while operators $G_B(A), H_B(A)$ etc. only partially extend operators $G_{\alpha,\beta}(A)$, $H_{\alpha,\beta}(A)$, etc., respectively.

In the present paper, we discuss a way for total extension of operators $G_B(A), H_B(A)$ etc. Obviously, the use of only one IFS as a parameter on the place of the two constants α and β is not possible. So, below we use two IFSs instead of the two parameters.

2 Main results

Here, we discuss step by step the respective intuitionistic fuzzy extended modal operators and discuss some of their propertiers. We give the proof of only the first two properties, formulated below, and the rest of the properties are proved in the same manner.

2.1 Operator $G_{B,C}$

It is defined by:

$$G_{B,C}(A) = \{\langle x, \mu_B(x)\mu_A(x), \nu_C(x)\nu_A(x)\rangle | x \in E\}$$

Theorem 1. For every six IFSs A, B, C, D, P, Q:

$$G_{\Box B, \Diamond C}(A) = G_{B,C}(A).$$
$$\neg G_{\neg C, \neg B}(\neg A) = G_{B,C}(A),$$
$$G_{B,C}(G_{P,Q}(A)) = G_{P,Q}(G_{B,C}(A)),$$
$$G_{B,C}(A \cap D) = G_{B,C}(A) \cap G_{B,C}(D),$$
$$G_{B,C}(A \cup D) = G_{B,C}(A) \cup G_{B,C}(D),$$
$$G_{B,C}(A) \subseteq G_{P,Q}(A),$$

where $B \subseteq P, C \subseteq Q$.

Proof. Let the IFSs A, B and C be given. Then, for the first equality we obtain that

$$\begin{aligned} G_{\Box B, \Diamond C}(A) &= G_{\Box \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}, \Diamond \{\langle x, \mu_C(x), \nu_C(x)\rangle | x \in E\}}(A) \\ &= G_{\langle x, \mu_B(x), 1-\mu_B(x)\rangle | x \in E\}, \Diamond \{\langle x, 1-\nu_C(x), \nu_C(x)\rangle | x \in E\}}(A) \\ &= \{\langle x, \mu_B(x)\mu_A(x), \nu_C(x)\nu_A(x)\rangle | x \in E\} \\ &= G_{B,C}(A), \end{aligned}$$

i.e., it holds. For the second equality we see:

$$\begin{aligned} \neg G_{\neg C, \neg B}(\neg A) &= \neg G_{\neg \{\langle x, \mu_C(x), \nu_C(x)\rangle | x \in E\}, \neg \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}}(\neg A) \\ &= \neg G_{\{\langle x, \nu_C(x), \mu_C(x)\rangle | x \in E\}, \{\langle x, \nu_B(x), \mu_B(x)\rangle | x \in E\}}(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}) \\ &= \neg \{\langle x, \nu_C(x)\nu_A(x), \mu_B(x)\mu_A(x)\rangle | x \in E\} \\ &= \{\langle x, \mu_B(x)\mu_A(x), \nu_C(x)\nu_A(x)\rangle | x \in E\} \\ &= G_{B,C}(A). \end{aligned}$$

This completes the proof. $\Box$

2.2 Operator $H_{B,C}$

It is defined by:

$$H_{B,C}(A) = \{\langle x, \mu_B(x)\mu_A(x), \nu_A(x) + \nu_C(x)\pi_A(x)\rangle | x \in E\}$$

Theorem 2. For every six IFSs A, B, C, D, P, Q, so that $B \subseteq P, C \subseteq Q$:

$$H_{\Box B, \Diamond C}(A) = H_{B,C}(A),$$
$$\neg H_{\neg C, \neg B}(\neg A) = J_{B,C}(A),$$

(see Subsection 2.5),

$$H_{B,C}(A \cap D) \subseteq H_{B,C}(A) \cap H_{B,C}(D),$$
$$H_{B,C}(A \cup D) \supseteq H_{B,C}(A) \cup H_{B,C}(D),$$
$$H_{B,C}(A) \subseteq H_{P,Q}(A).$$

2.3 Operator $H^*_{B,C}$

It is defined by:

$$H^*_{B,C}(A) = \{\langle x, \mu_B(x)\mu_A(x), \nu_A(x) + \nu_C(x)(1 - \mu_B(x)\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$

Theorem 3. For every three IFSs A, B, C:

$$H^*_{\square B, \Diamond C}(A) = H^*_{B,C}(A),$$
$$\neg H^*_{\neg C, \neg B}(\neg A) = J^*_{B,C}(A)$$

(see Subsection 2.6),

$$H^*_{B,C}(A \cap D) \subseteq H^*_{B,C}(A) \cap H_{B,C}(D),$$
$$H^*_{B,C}(A \cup D) \supseteq H^*_{B,C}(A) \cup H_{B,C}(D).$$

2.4 Operator $\overline{H}_{B,C}$

It is defined by:

$$\overline{H}_{B,C}(A) = \{\langle x, \mu_B(x)\mu_A(x), \nu_A(x) + \nu_C(x) - \nu_C(x)\nu_A(x)\rangle | x \in E\}$$

and it is an extension of operator

$$\overline{H}_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x), \nu_A(x) + \beta - \beta\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$ (see [6]

Theorem 4. For every five IFSs A, B, C, P, Q, so that $B \subseteq P, C \subseteq Q$:

$$\overline{J}_{\square B, \Diamond C}(A) = \overline{J}_{B,C}(A),$$
$$\neg\overline{H}_{\neg C, \neg B}(\neg A) = \overline{J}_{B,C}(A),$$

(see Subsection 2.7),

$$\overline{H}_{B,C}(A \cap D) \subseteq \overline{H}_{B,C}(A) \cap \overline{H}_{B,C}(D),$$
$$\overline{H}_{B,C}(A \cup D) \supseteq \overline{H}_{B,C}(A) \cup \overline{H}_{B,C}(D),$$
$$\overline{H}_{B,C}(A) \subseteq \overline{H}_{P,Q}(A).$$

2.5 Operator $J_{B,C}$

It is defined by:

$$J_{B,C}(A) = \{\langle x, \mu_A(x) + \mu_B(x)\pi_A(x), \nu_C(x)\nu_A(x)\rangle | x \in E\}$$

Theorem 5. For every five IFSs A, B, C, P, Q, so that $B \subseteq P, C \subseteq Q$:

$$J_{\square B, \Diamond C}(A) = J_{B,C}(A),$$
$$\neg J_{\neg C, \neg B}(\neg A) = H_{B,C}(A),$$
$$J_{B,C}(A \cap D) \supseteq J_{B,C}(A) \cap J_{B,C}(D),$$
$$J_{B,C}(A \cup D) \subseteq J_{B,C}(A) \cup J_{B,C}(D),$$
$$J_{B,C}(A) \subseteq J_{P,Q}(A).$$

2.6 Operator $J^*_{B,C}$

It is defined by:

$$J^*_{B,C}(A) = \{\langle x, \mu_A(x) + \mu_B(x)(1 - \mu_B(x) - \nu_C(x)\nu_A(x)), \nu_C(x)\nu_A(x)\rangle | x \in E\}$$

Theorem 6. For every three IFSs A, B, C:

$$J^*_{\Box B, \Diamond C}(A) = J^*_{B,C}(A),$$

$$\neg J^*_{\neg C, \neg B}(\neg A) = H^*_{B,C}(A),$$

$$J^*_{B,C}(A \cap D) \supseteq J^*_{B,C}(A) \cap J_{B,C}(D),$$

$$J^*_{B,C}(A \cup D) \subseteq J^*_{B,C}(A) \cup J_{B,C}(D).$$

2.7 Operator $\overline{J}_{B,C}$

It is defined by:

$$\overline{J}_{B,C}(A) = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_B(x)\mu_A(x), \nu_C(x)\nu_A(x)\rangle | x \in E\}$$

and it is an extension of operator (see [6]):

$$\overline{J}_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha - \alpha\mu_A(x), \beta\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

Theorem 7. For every five IFSs A, B, C, P, Q, so that $B \subseteq P, C \subseteq Q$:

$$\overline{J}_{\Box B, \Diamond C}(A) = \overline{J}_{B,C}(A),$$

$$\neg \overline{J}_{\neg C, \neg B}(\neg A) = \overline{H}_{B,C}(A),$$

$$\overline{J}_{B,C}(A \cap D) \supseteq \overline{J}_{B,C}(A) \cap \overline{J}_{B,C}(D),$$

$$\overline{J}_{B,C}(A \cup D) \subseteq \overline{J}_{B,C}(A) \cup \overline{J}_{B,C}(D),$$

$$\overline{J}_{B,C}(A) \subseteq \overline{J}_{P,Q}(A).$$

2.8 Remark on the new operators

Let

$$Y = \{\langle x, \alpha, \gamma\rangle | x \in E\},$$

$$Z = \{\langle x, \delta, \beta\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$. Then for each IFS A:

$$G_{\alpha,\beta}(A) = G_{Y,Z}(A),$$

$$H_{\alpha,\beta}(A) = H_{Y,Z}(A),$$

$$H^*_{\alpha,\beta}(A) = H^*_{Y,Z}(A),$$
$$\overline{H}_{\alpha,\beta}(A) = \overline{H}_{Y,Z}(A),$$
$$J_{\alpha,\beta}(A) = J_{Y,Z}(A),$$
$$J^*_{\alpha,\beta}(A) = J^*_{Y,Z}(A),$$
$$\overline{J}_{\alpha,\beta}(A) = \overline{J}_{Y,Z}(A).$$

Therefore, each of the new operators is an extension of its corresponding modal operator. It is obvious that the opposite representation is impossible.

3 Conclusion

In a next research, other properties of the newly defined operators will be studied. All of them can be used in a lot of areas of informatics and, especially, in the Artificial Intelligence for modification of intuitionistic fuzzy evaluations. Also, they can be used in Intercriteria Analysis procedures for changing of evaluations of the compared criteria.

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